Ground Effect in Flight*

(Birds, Fishes and High-Speed Vehicle)

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The present paper aims to analyze the propulsion of birds and fishes undergoing the ground effect as well as the lift of high-speed ground vehicle. Applying the analytical method which was developed for flutter of a soft plate placed at an arbitrary position in subsonic channel flows, calculations are carried out first for non-oscillatory case in compressible flow and then for oscillating cases of birds and fishes in incompressible flow. The results obtained show that the ground effect acts to increase not only the lift in steady flight but also the thrust and propulsive efficiency in oscillating modes. This method holds not only in the case of very close proximity to the ground but also in compressible flow case, so it would be applicable to the flutter analysis of high-speed ground vehicle with wings.

Key Words: Ground Effect, Bird, Fish, Ground Vehicle, Lift, Propulsion, Subsonic Flow

1. Introduction

The ground effect has been studied mostly on non-oscillatory wings which are not too close to the ground as in the case of aircraft (Ref.(1) is a good source of references). Regarding high-speed transportation, Widnall and Barrows⁽¹⁾ investigated two- and three-dimensional wings in close proximity to the ground in incompressible flow and predicted that the ground effect enormously increases the lift. Besides, it is wellknown that the big water birds use the ground effect when gliding near the water surface. The ground effect may also act on flatfish swimming near the sea bottom.

The propulsion of birds and fishes has attracted many scientists. Azuma⁽²⁾ analyzed the flight of birds theoretically and predicted the flapping conditions which give either maximum thrust or maximum efficiency (the details are also referred to Ref.(3)). Wu⁽⁴⁾ and Siekmann⁽⁵⁾ investigated the propulsion of fishes theoretically, assuming a two-dimensional thin meandering plate, and calculated the thrust and propulsive efficiency for the wide range of swimming

mode. Siekmann⁽⁵⁾ also carried out experiments and measured the thrusts, which are in good agreement with his own theoretical results. Except the wind-tunnel interaction on an aerofoil⁽⁶⁾, however, the ground effect is not considered yet for an oscillating plate, as in the cases of birds and fishes, as well as for a plate in compressible flow, as in the case of high-speed transportation.

The present paper aims to analyze the propulsion of birds and fishes undergoing the ground effect as well as the lift of high-speed ground transportation. Applying an analytical method which was developed for flutter of a soft plate placed at an arbitrary position in subsonic channel flows^{(7),(8)}, calculations are carried out first for non-oscillatory case of an aerofoil in compressible flow and then for oscillating cases of birds and fishes in incompressible flow.

2. Formulation

2.1 Analytical model and procedures

Figure 1 shows a fundamental model, in which a thin plate placed at an arbitrary position in two-dimensional channel oscillates with small amplitude in subsonic isentropic flow. Here, U is the flow velocity, a_∞ the speed of sound, M the Mach number (U/a_∞), ρ_∞ the density of fluid, b the half chord length of plate, d the channel height, and (x, y) and (ξ, η) are the coordi-

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Fig. 1 Model and nomenclatures

ates, in which (ξ, η) is related to the plate. Further, p is the pressure and t is the time.

Assume that the plate oscillates steadily with a frequency ω , then it can be represented by a sheet of doublets of strength $A(\xi) \exp(i\omega t)$, with their axes perpendicular to the chord. The doublet strength is directly proportional to the pressure jump across the plate. That is,

$$A(\xi) \exp(i\omega t) = \frac{p_u - p_l}{\rho_\infty U^2} \quad (1)$$

where p_u and p_l denote the pressures on the upper and lower surfaces of the plate.

Solving the linearized Euler equation with the continuity equation, the upwash velocity induced by a doublet is deduced. In the present case, the images of the plate due to the wall reflection should be considered such that the doublets $A(\xi) \exp(i\omega t)$ are on $(\xi, \eta + 2md)$ and $-A(\xi) \exp(i\omega t)$ on $(\xi, -\eta + 2md)$, where $m=0, \pm 1, \pm 2, \dots$. This channel flow analysis leads to the transformation of the kernel function into much simpler form, and the upwash velocity induced on the plate ($y=\eta$) by a doublet and its images is given as (the details are given in Refs. (7) and (8))

$$\begin{aligned} v_d(x, \eta; \xi, \eta) &= \frac{UA(\xi)}{d} \exp(i\omega t) \left[\exp \left\{ i \frac{Mk}{1-M^2} (x-\xi) \right\} \right. \\ &\quad \times (S_0 + S_1 + S_2) + \{1 + \operatorname{sgn}(x-\xi)\} \\ &\quad \times \exp \left\{ -i \frac{k}{M} (x-\xi) \right\} \cdot S_3 \left. \right] \quad (2) \end{aligned}$$

where

$$q = \frac{\pi |x-\xi|}{d\sqrt{1-M^2}}, \quad p_\eta = -\frac{\pi\eta}{d}$$

$$k = \frac{\omega}{a_\infty}, \quad w = \frac{kd}{\pi\sqrt{1-M^2}}$$

$$S_0 = \frac{1}{4} \operatorname{sgn}(x-\xi) \times \sinh q \left\{ \frac{1}{\cosh q - 1} - \frac{1}{\cosh q - \cos 2p_\eta} \right\}$$

$$S_1 = i \frac{w}{4M} \log \left(\frac{\cosh q - \cos 2p_\eta}{\cosh q - 1} \right)$$

$$\begin{aligned} S_2 &= \sum_{\nu=1}^{\infty} \left[\frac{M\nu^2}{\sqrt{\nu^2 - w^2}} \frac{\exp(-q\sqrt{\nu^2 - w^2})}{\operatorname{sgn}(x-\xi) \cdot M\sqrt{\nu^2 - w^2} - i\nu} \right. \\ &\quad \left. - \left\{ \operatorname{sgn}(x-\xi) + i \frac{w}{M\nu} \right\} \exp(-\nu q) \right] \sin^2 \nu p_\eta \\ S_3 &= \frac{\pi}{4} a \left\{ \coth \pi a - \frac{\cosh a(\pi - 2p_\eta)}{\sinh \pi a} \right\}; \\ a^2 &\equiv \left(\frac{1-M^2}{M^2} \right) w^2 \end{aligned}$$

Here, S_0 , S_1 and S_2 are of the orders of magnitude $O(w^0)$, $O(w^1)$ and $O(w^2)$, respectively, so S_0 gives the quasi-steady component. S_3 describes the shed wake vortex conveyed downstream by the primary flow.

Integrating Eq. (2) over the plate gives the velocity induced on $y=\eta$ by the whole doublets of the plate.

$$v(x, \eta) = \int_{-b}^b v_d(x, \eta; \xi, \eta) d\xi \quad -b \leq x \leq b \quad (3)$$

Assuming that the plate is oscillating with the displacement $f(x) \exp(i\omega t)$, the boundary condition is given as

$$v(x, \eta) = \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \{ f(x) \exp(i\omega t) \} \quad (4)$$

From Eqs. (2), (3) and (4), the doublet distribution $A(\xi)$ can be obtained for a given mode of oscillation. According to the usual way of calculation, the doublet distribution function $A(\xi)$ is expressed by the Glauert series, which satisfies the singularity at the leading edge and the Kutta condition at the trailing edge of the plate. That is,

$$\begin{aligned} A(\xi) &= A_0 \tan \frac{\theta}{2} + 2 \sum_{n=1}^{\infty} A_n \sin n\theta; \\ \frac{\xi}{b} &= \cos \theta, \quad 0 \leq \theta \leq \pi \quad (5) \end{aligned}$$

Calculations are carried out for a finite number (N) of control points, and the first N coefficients of the Glauert series are obtained. For a hard plate such as an aerofoil, the unsteady lift and moment about the mid-chord of the plate are easily obtained as

$$\begin{cases} \frac{L}{\pi \rho_\infty U^2 b} = -(A_0 + A_1) \exp(i\omega t) \\ \frac{M}{\pi \rho_\infty U^2 b^2} = -\frac{1}{2} (A_0 + A_2) \exp(i\omega t) \end{cases} \quad (6)$$

In general, the work done by the plate for one cycle of oscillation is obtained by

$$\begin{aligned} \frac{W}{\rho_\infty U^2} &= \int_0^{2\pi/\omega} \int_{-b}^b [A(\xi) \exp(i\omega t)]_R \\ &\quad \times \left[\frac{\partial}{\partial t} \{ f(\xi) \exp(i\omega t) \} \right]_R d\xi dt \quad (7) \end{aligned}$$

where the suffix R denotes the real part of complex function.

The time-averaged thrust induced on the plate consists of two components. The one is induced by the streamwise component of the surface pressure force, given as

$$\frac{T_d}{\rho_\infty U^2} = -\frac{\omega}{2\pi} \int_0^{2\pi/\omega} \int_{-b}^b [A(\xi) \exp(i\omega t)]_R$$

$$\times \left[\frac{\partial}{\partial \xi} \{ f(\xi) \exp(i\omega t) \} \right]_R d\xi dt \quad (8)$$

The other is the suction force at the leading edge. Applying the Blasius theorem to the flow around the leading edge⁽⁴⁾, or from the quasi-steady consideration, the suction force is given by

$$\frac{T_s}{\rho_\infty U^2 b} = \frac{\pi}{4} |A_0|^2 \quad (9)$$

Hence, the thrust is

$$T = T_d + T_s \quad (10)$$

Then, the coefficient of thrust is defined as

$$C_T = \frac{T}{\pi \rho_\infty U^2 b} \quad (11)$$

The effective work done by the thrust for one cycle is

$$W_T = \frac{2\pi U}{\omega} T \quad (12)$$

so the propulsive efficiency is defined by

$$\eta_p = \frac{W_T}{W} \quad (13)$$

In the following, the channel height is taken as large as possible, e.g. $d/b=10$, to simulate the infinite (free) or semi-infinite flow field. It was confirmed for such a large channel height that the aerodynamic characteristics of the plate placed on the channel axis were in good agreement with those of an isolated aerofoil in the range of moderate frequency. To examine the ground effect, keeping the channel height constant, the plate is allocated at an arbitrary position in proximity to the ground (the lower wall of the channel).

2.2 Modes of flight

2.2.1 Stationary mode This is the case where a flat plate is placed at rest with an arbitrary angle of attack. The calculation can be performed for quasi-steady torsional oscillation of infinitesimal frequency in the bird mode described below.

2.2.2 Bird mode It is assumed that the wing of birds oscillates with two degrees of freedom of bending (heaving) and torsion, not being accompanied by the deformation of its profile (flat plate), as shown in

(a) Bird mode

(b) Fish mode

Fig. 2 Modes of oscillation

Fig. 2(a). ϕ is the phase lag of torsional mode behind bending one.

$$f(x) \exp(i\omega t) = \{h + ax \exp(-i\phi)\} \exp(i\omega t) \quad (14)$$

2.2.3 Fish mode The oscillating mode of fish is not defined uniquely, so a linear mode will be considered here (cf. Refs. (4) and (5)), as shown in Fig. 2(b), in which the amplitude of oscillation increases linearly along the plate from its leading edge to the trailing edge. The plate oscillates generally with a travelling-wave mode, as given by Eq. (15) where V is the travelling-wave velocity and then the phase lag of the trailing edge behind the leading edge is given by $\phi = 2\omega b/V$.

$$\begin{aligned} f(x) \exp(i\omega t) &= \frac{c}{2} \left(1 + \frac{x}{b}\right) \exp\left\{-i\frac{\omega b}{V} \left(1 + \frac{x}{b}\right)\right\} \exp(i\omega t) \end{aligned} \quad (15)$$

3. Results

3.1 Stationary mode (compressible flow)

Stationary mode generates lift but no thrust. Figure 3 shows the ground effect on steady lift, where the lift L is normalized by the lift L_∞ of an isolated plate in incompressible flow ($M=0.01$ in the present calculations) after being corrected by the Prandtl-Glauert similarity law. The ground acts to suppress the downwash generated by the plate, or to create the upwash by its image, so that the lift increases as the plate approaches the ground, particularly within a distance corresponding to the plate chord length $\eta/b < 2.0$.

In order to examine the ground effect in very close proximity, the lift is replotted in Fig. 4 for $\eta/b < 1.0$ after Widnall and Barrows⁽¹⁾. The present result

Fig. 3 Ground effect on steady lift

Fig. 4 Steady lift in close proximity to the ground
----: Widnall & Barrows⁽¹⁾

for incompressible flow is in very good agreement with the numerical solution of Widnall and Barrows (broken line, partly seen on the left-hand side in the figure) even in very close proximity, where very high lift could be achieved.

As shown in Figs. 3 and 4, the Prandtl-Glauert correction deteriorates as the plate approaches the ground and the Mach number increases, so that it leads to underestimate of lift in compressible flow. Further, Fig. 3 shows that the aerodynamic center (AC), which is measured from the leading edge (divided by the semi-chord length), moves downstream slightly from the quarter chord position as the plate approaches the ground.

3.2 Bird mode (incompressible flow)

The oscillation mode of the outer half of bird wing, which mainly generates the thrust, is likely to be similar to that shown in Fig. 2(a), where the amplitude ratio ba/h is around 0.3, the phase angle ϕ between bending and torsion is around 90 degree and the reduced frequency $\omega b/U$ is usually less than one. So, consider the plate flapping with no mean angle of attack, when mean thrust is generated but no mean lift.

Figure 5 shows the thrust and propulsive efficiency for the amplitude ratio $ba/h=0.2$, when the plate is placed on the channel axis, simulating free flight. For a given phase angle, as the reduced frequency $\omega b/U$ decreases, the propulsive efficiency tends to increase towards unity, but for positive phase angle ($\phi > 0$) it vanishes abruptly at a critical reduced frequency, below which flutter occurs giving negative thrust (drag). The flutter boundary is predicted approximately by the quasi-steady analysis as

$$\frac{\omega b}{U} < \frac{ba}{h} \sin \phi \quad (16)$$

Fig. 5 Thrust and propulsive efficiency in free flight
(Bird mode: $ba/h=0.2$)

Fig. 6 Ground effect in bird mode
($ba/h=0.2, \phi=1.5$)

The thrust changes approximately in proportion to the square of the reduced frequency. As the reduced frequency increases, the phase angle giving maximum thrust is likely to change from -2.0 to 0 . The maximum propulsive efficiency is achieved for the phase angle around -2.0 in lower reduced frequency and around 2.0 in higher reduced frequency. Hence, roughly speaking, moderate flight of high propulsive efficiency can be achieved with ϕ being around 2.0 , while strong flight of high thrust can be done with $\phi = 0$ to -2.0 , but being accompanied by low propulsive efficiency. These results agree well with the Azuma's⁽²⁾. The above-mentioned characteristics are nearly the same for other amplitude ratios, although the flutter boundary depends on the amplitude ratio,

Fig. 7 Thrust in close proximity to the ground
(Bird mode: $ba/h=0.2$, $\phi=1.5$)

as is given by Eq.(16).

Figure 6 shows that, as the plate approaches the ground ($\eta/b < 2.0$), both thrust and propulsive efficiency increase. The thrust is replotted in Fig. 7 following Fig. 4 for steady lift. Comparing with Fig. 4, the ground effect is larger for thrust than for lift. The ground effect is smaller for larger reduced frequency, when the streamwise wave length of the free vortices becomes shorter, that may weaken the ground effect. Assuming the drag being constant, the ground effect may lead to smaller amplitude of oscillation, that saves power of flight incorporating with higher propulsive efficiency.

3.3 Fish mode (incompressible flow)

Figure 8 gives the thrust and propulsive efficiency when the fish swims with the linear mode of oscillation, comparing with the Siekmann's theoretical results in free flow field. It is shown that the thrust is larger for smaller phase angle and increases with increasing reduced frequency, becoming positive in the range beyond a critical reduced frequency. The thrust obtained agrees well with the Siekmann's for larger reduced frequency and phase angle, but it tends to deviate as the reduced frequency increases, particularly for smaller phase angle. For large reduced frequency, the virtual mass effect of fluid becomes predominant in the fluid force acting on the plate, so the thrust would change nearly in proportion to $(\omega b/U)^2$. In the present calculations, the channel height is limited, so the virtual mass effect may be suppressed in the case of small phase angle ϕ or when the wave length of oscillation is large as compared with the plate length. The real fishes, however, swim with large reduced frequency $\omega b/U$ and phase angle ϕ of the same order, e.g. eight for the sturgeon⁽⁹⁾, so the channel wall effects on the thrust could be neglected if

Fig. 8 Thrust and propulsive efficiency in free flight
(Fish mode), ----: Siekmann⁽⁵⁾

d/b is taken as 10 or more, as is seen in Fig. 8.

Figure 8 also shows that the propulsive efficiency becomes maximum at a reduced frequency just above the critical one and decreases gradually as $\omega b/U$ increases. The present results of propulsive efficiency agree quite well with the Siekmann's in spite of the deviation in the thrust. This is presumably because the virtual mass effect acts as inertia or potential, which is not related to energy loss. These results suggest that the fishes swimming with large reduced frequency and large phase angle, such as 6 to 10, can perform effective swimming with very high propulsive efficiency such as 80%.

The flatfish swims near the surface of the sea bottom, so it may undergo the ground effect just similarly as the birds. Figures 9 and 10 show the ground effect on the plate oscillating with linear mode. As in the case of bird, the ground effect acts to increase the thrust, whereas the propulsive efficiency is kept nearly constant even in close proximity to the ground. The ground effect on the thrust is rather small as compared with the case of bird, that may be because the oscillation of large travelling-wave number eliminates local ground effects with each other along the plate.

4. Concluding Remarks

The present method assumes the channel flow in principle, so it leads to simple calculations and is easily applied to the ground effect on both steady and oscillating flights. The results obtained show that the ground effect acts to increase not only the lift in steady flight mode but also the thrust and propulsive efficiency in oscillating modes. The ground effect is

Fig. 9 Ground effect in fish mode

Fig. 10 Thrust in close proximity to the ground (Fish mode)

likely to be much more effective for birds than for fishes.

This method holds even in the case where the plate is in close proximity to the ground in compressible flow, so it would be applicable not only to the flights of birds and fishes but also to the steady and unsteady characteristics of high-speed ground vehicle with wings.

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