Experiments on Flow-Induced In-Line Oscillation of a Circular Cylinder in a Water Tunnel*

(2nd Report, Influence of the Aspect Ratio of a Cantilevered Circular Cylinder)

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The flow-induced in-line oscillation of a cantilevered circular cylinder was experimentally studied through free-oscillation tests in a water tunnel. The response displacement amplitude at a circular cylinder tip was measured at reduced velocity from 1.0 to 4.0. A cantilevered cylinder was supported by a plate spring mounted on the water tunnel wall. The cylinder aspect ratio was varied from 5 to 21 to investigate the effect of aspect ratio on the response displacement. It is found that cylinders with aspect ratios of 5 and 10 have one excitation region, while cylinders with aspect ratios of 14 and 21 have two excitation regions. The aspect ratio, therefore, affects the amplitude of the excitation regions. The influence of end-effect was also investigated using cylinders with an end plate attached to the free end. Since the cylinders with an end plate show two excitation regions, even at an aspect ratio of 5, the flow around the free end of a cantilevered cylinder causes the end-effect. The mechanism of vibration was investigated using a cylinder with a splitter plate in wake to prevent alternate vortices. The amplitude is greater than those of a normal cylinder without a splitter plate, especially at $V_r=2.3$ to 3.0, where a cylinder with an end plate shows the second excitation region. In other words, the alternate vortices suppress the amplitude in this range. The maximum amplitude of each excitation region decreases in proportion to C_n and the amplitude of the first excitation is more sensitive to C_n .

Key Words: Flow-Induced Vibration, In-Line Oscillation, Circular Cylinder, Cantilevered Cylinder, Water Tunnel, Aspect Ratio, Reduced Mass-Damping Parameter, Lock-In

1. Introduction

Flow-induced vibration (FIV) may occur when a structure is exposed perpendicularly to a flow. To avoid FIV, the Japan Society of Mechanical Engineers established "Guideline for Evaluation of Flow-Induced Vibration of a Cylindrical Structure in a Pipe," Standard JSME S012-1998. Okajima et al.⁽¹⁾ provided

important data for this guideline, relating to circular cylinders supported at both ends in a wind tunnel at sub-critical Reynolds numbers. They observed two in-line (streamwise) oscillation regions at reduced velocity V_r from 1.0 to 4.0. The cylinder sheds symmetrical vortices in the first region and sheds alternate vortices in the second region. The maximum amplitudes of displacement decrease more sensitive to the reduced mass-damping parameter C_n in the first region than in the second region. Based on this data, the guideline states that the criterion $V_r < 1.0$ is effective for avoiding in-line oscillation, while the criteria $V_r < 3.3$ and $C_n > 2.5$ are effective in suppressing in-line oscillation.

One application of this guideline is a thermometer well inserted in the pipe that has following character-

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istics :

(a) Cantilevered, circular, cylindrical structure ; or a somewhat tapered cylinder.

(b) The surrounding fluid is water and the mass ratio $m/\rho D^2$ is lower than in the air.

(c) The well is welded to the pipe and the damping ratio is very low.

(d) The flow speed is high ; the Reynolds number is between the sub-critical region to the super-critical region.

It is important to evaluate the influence of these characteristics on the oscillation because the free end of a cantilevered cylinder may affect the width of the vibration range or the sensitivity of C_n in proportion to the aspect ratio L/D . King et al.⁽²⁾ carried out experiments on the in-line oscillation of a cantilevered circular cylinder in a water channel. The cantilevered cylinder in these experiments had an aspect ratio of 41 and a free end above the water surface. Tanaka et al.⁽³⁾ examined a cantilevered cylinder with an aspect ratio of 6 in a water tunnel at sub-critical Reynolds numbers, with the free end of the cylinder being close to the opposite wall of the pipe. Kondo and Anoda⁽⁴⁾ examined a long cylinder with an aspect ratio of 65. King, Tanaka and Kondo observed two excitation regions of in-line oscillation. Ishihara et al.⁽⁵⁾ examined a cylinder with an aspect ratio of 3, with the free end being close to the opposite wall of the water tunnel, and observed one excitation region.

Sakai et al.⁽⁶⁾ conducted experiments on some types of cylinders in water to evaluate the displacement of a thermometer well in a plant pipe. They observed one excitation region from $V_r=1.0$ to 4.0 in most cases. Kawamura et al.⁽⁷⁾ conducted experiments on a cylinder with a top end plate at a certain turbulence intensity and observed one or two excitation regions. Hagiwara et al.⁽⁸⁾ examined some tapered cylinders in the water at Reynolds numbers from 10^5 to 10^6 . These experiments were conducted to evaluate the vibration at super-critical Reynolds numbers, but it was difficult to find consistent characteristics because the aspect ratio, turbulence intensity and Strouhal number were different in each investigation. It is, therefore, important to investigate the effects of these conditions individually.

In this study, some experiments were conducted in a water tunnel at sub-critical Reynolds numbers, using cantilevered circular cylinders with aspect ratios from 5 to 21. The reduced mass-damping parameter was varied to suppress the maximum in-line direction amplitude. To investigate the mechanism of vibration and vortex structure, an end plate was attached to the free end of a cylinder, or a splitter plate was placed in the wake.

Nomenclature

C_n :	Reduced mass-damping parameter, $2m\delta/\rho D^2$
D :	Cylinder diameter (m)
f :	Frequency of cylinder in fluid (Hz)
f_c :	Natural frequency of cylinder in air (Hz)
f_w :	Frequency of wake (Hz)
L :	Span length (m)
m :	Mass per unit span length with added mass in fluid (kg/m)
m_a :	Added fluid mass per unit span length (kg/m)
m_s :	Structural mass per unit span length (kg/m)
Re :	Reynolds number, UD/ν
St :	Strouhal number, $f_w D/U$
U :	Average flow velocity (m/s)
V_r :	Reduced velocity, U/fD
y :	Cylinder tip displacement (m)
δ_s :	Logarithmic decrement of cylinder structure
ν :	Kinematic viscosity (m^2/s)
ρ :	Fluid mass density (kg/m^3)
ϕ_0 :	Fundamental vibration mode shape
$\xi_{\max}$:	Non-dimensional zero-peak value of in-line cylinder tip displacement
ξ_{rms} :	Non-dimensional root-mean-square value of in-line cylinder tip displacement
ζ_f :	Added damping ratio in viscous fluid
Subscripts	
c :	cylinder
f :	fluid
s :	structure
w :	wake
x :	in-line direction
y :	cross flow direction

2. Experimental Arrangements

The present experiment was performed in a water tunnel⁽⁹⁾ with a rectangular test section of $780 \times 400 \times 167$ mm ($L \times H \times W$). The flow velocity was measured with a Pitot-tube and the turbulence intensity was measured using an I-type hot-film probe. When the water velocity U ranges from 0.2 to 1.8 m/s, the turbulence intensity is less than 2.0%. The Reynolds number $Re = UD/\nu$ (where D is the diameter of the model and ν is the kinematic viscosity of water) is in the range of 0.8×10^4 to 4.0×10^4 . The uniformity of the velocity profiles is less than $\pm 0.5\%$ when the mean velocity is 1.5 m/s.

Figure 1 shows in detail the set-up of a cantilevered circular cylinder with an aspect ratio of 10 ; the cylinder diameter is 20 mm and the span length is 200 mm. The cylinders, of stainless steel, were elastically supported by a plate-spring attached to the wall of the tunnel. The natural frequency f_c of the cylinder was adjusted between 20 Hz to 30 Hz, since

Fig. 1 Experimental apparatus of a circular cantilever supported at single end, test cylinder S2 with aspect ratio of 10, unit: mm

the reduced velocity was varied from 0.9 to 4.0 to investigate in-line oscillation. The aluminum plates and rubber plates were clamped with the plate-spring, in order to vary the reduced mass-damping parameter. Table 1 shows the details of all test cylinders, S1 - S4, with aspect ratios of 5 to 21.

An acceleration sensor, mounted on the free end of the cylinder, was used to measure the displacement of the oscillating cylinder, using an integrator. A hot-film probe, located around the outer edge of the wake, measured the fluctuating water velocity for calculating the non-dimensional vortex shedding frequencies, where Strouhal number $St_w = f_w D / U$ (f_w is the vortex frequency). The measurements were made over 40 seconds, 5 minutes after the velocity was increased, when the vibration became stable.

The structural logarithmic decrements δ_s for all combinations of cylinders and plate-springs were measured more than 10 times before the experiments by impact tests in the air. The added damping ratio in a viscous fluid is given by Eq. (1) (Blevins⁽¹⁰⁾).

$$\zeta_f = \frac{\pi}{2} \frac{\rho D^2}{m} \left(\frac{\nu}{\pi f D^2} \right)^{1/2} \quad (1)$$

The added mass-damping parameter C_{nadded} in the fluid is given by Eq. (2) using Eq. (1).

$$C_{nadded} = \frac{4\pi m \zeta_f}{\rho D^2} \quad (2)$$

The reduced mass-damping parameter C_n in the fluid is given by Eq. (3).

$$C_n = \frac{2m\delta_s}{\rho D^2} + C_{nadded} \quad (3)$$

In Eq. (3), m is the mass per unit span length with added mass in fluid and given by Eq. (4)

$$m = m_s + m_a, \quad m_s = \frac{\int_L m(x) \phi_0^2(x) dx}{\int_L \phi_0^2(x) dx}, \quad m_a = \frac{\pi \rho D^2}{4} \quad (4)$$

Table 1 Details of tested circular cylinders

	Length: L [mm]	Diameter: D [mm]	Aspect Ratio: L/D [-]	Structural mass per unit span length: m_s [kg/m]	Mass ratio: $m_r / \rho D^2$ [-]	Material
S1	100	20	5	2.381	5.96	Stainless Steel
S2	200	20	10	1.500	3.76	
S3	200	14	14	1.128	5.77	
S4	300	14	21	1.141	5.83	

Table 2 The results of Eq. (2) and added reduced mass-damping parameter in water

aspect ratio	C_n in water	C_n in air	$C_{nadded} :$ C_n in water - C_n in air	$C_{nadded} :$ Equation (2)
5	0.248	0.130	0.118	0.109
10	0.272	0.151	0.122	0.127
14	0.271	0.127	0.144	0.145
21	0.275	0.100	0.175	0.171

To verify Eq. (2) for the cantilevered cylinder, the logarithmic decrements in still water were measured by impact tests. Table 2 shows the logarithmic decrements of all cylinders in the air and in still water with a plate-spring that is scraped at its center to decrease structural damping ratio. In Table 2, Eq. (2) shows good agreement with the added reduced mass-damping parameter measured in the fluid. The reduced mass-damping parameter in the fluid C_n is, therefore, defined by Eq. (3) in this paper.

3. Experimental Results

3.1 Amplitudes of the cantilevered cylinders with aspect ratio of 5 to 21

Figure 2 shows the wake vortex frequency measured along the span of a stationary cylinder with an aspect ratio of 10. The hot-film probe measured the wake velocity fluctuation behind the cylinder along the span length of the cylinder. The Strouhal number St_w of the stationary cylinder is 0.2 in this water tunnel. The lower frequency was measured at the free end ($L=180$ to 200 mm) and at the supported end ($L=0$ to 50 mm) near the wall of the water tunnel.

Figure 3 (a) shows the non-dimensional amplitude of in-line tip displacement $\xi_{rms} = y_{rms}/D$, (where y_{rms} is the root-mean-square response displacement) against the reduced velocity $V_r = U/fD$ with a finite length aspect ratio of 10. The Strouhal number St_w of the wake vortex frequency is also shown in Fig. 3 (a), and the reduced mass-damping parameter C_n is 0.31. It is found that the response amplitude of the cylinder shows one excitation region in the range $V_r = 1.2$ to 3.4 . There is an increase in in-line amplitude at $V_r = 1.2$ to 2.2 and a decrease to $V_r = 3.4$. The maximum value of ξ_{rms} is 0.090 at $V_r = 2.2$. The lock-in phenomenon of St_w to $1/2 St_c$ occurs at $V_r = 2.2$ to 3.0 , where $St_c = fD/U$.

The response amplitude is very different from the response curve of a two-dimensional cylinder presented by Okajima⁽¹⁾ because the two-dimensional cylinder shows two excitation regions in the same V_r range

Fig. 2 The vortex frequency in wake using a stationary cylinder with aspect ratio of 10, the free end is $L=200$ mm and the supported end is $L=0$ mm. The size of $\blacksquare$ shows the strength of power spectrum of f_w .

(a) The relationship of the in-line amplitude ξ_{rms} and the Strouhal number St_w of wake frequency at $C_n=0.31$

(b) The influence of reduced mass-damping parameter C_n to the amplitude ξ_{rms}

Fig. 3 The non-dimensional in-line tip amplitude of cantilevered cylinder S2 with aspect ratio of 10

(see Fig. 5). The lowest point of the amplitude is at approximately $V_r=2.5$ in Okajima's data. Also, the lock-in of St_w to $1/2 St_c$ occurs in the second excita-

(a) The relationship of the amplitude ξ_{rms} and the Strouhal number St_w of wake frequency at $C_n=0.36$

(b) The influence of reduced mass-damping parameter C_n to the amplitude ξ_{rms}

Fig. 4 The non-dimensional in-line tip amplitude of cantilevered cylinder S4 with aspect ratio of 21

tion region at $V_r=2.5$ to 3.5 .

Figure 3(b) shows that the amplitudes ξ_{rms} decreases in proportion to an increase in the reduced mass-damping parameter C_n . All of the lines show one excitation region, with maximum points at $V_r=1.9$ to 2.1 .

Figure 4(a) shows the variation of the non-dimensional in-line tip amplitude ξ_{rms} against reduced velocity V_r with an aspect ratio of 21. The Strouhal number St_w of the wake vortex frequency is also shown in Fig. 4 (a), where the reduced mass-damping parameter C_n is 0.36. It is found that the response amplitudes of the cylinder show two excitation regions in the range $V_r=1.2$ to 3.1 , similar to the result of Okajima⁽¹⁾. There is an increase in in-line amplitude from $V_r=1.2$ to 2.1 and a decrease to $V_r=2.4$, and this V_r range is the first excitation. There is a valley of the amplitude at about $V_r=2.5$. There is another increase from $V_r=2.5$ to 2.7 and then a rapid drop at $V_r=3.1$, so this V_r range is the second excitation region. The maximum points of the first excitation region and second excitation region are 0.087 at $V_r=2.1$ and 0.081 at $V_r=2.7$ respectively. The lock-in of St_w to $1/2 St_c$ occurs at $V_r=2.2$ to 3.0 , almost in the second excitation region. The St_w number in the

first excitation region, however, is an almost constant value of 0.21, similar to the Strouhal number for a stationary cylinder, $St_n=0.20$ in Fig. 2.

Figure 4(b) shows that the amplitudes ξ_{rms} decreases in proportion to an increase in the reduced mass-damping parameter C_n . There are two excitation regions in the range $C_n=0.28$ to 1.39, but there is only one excitation region at $C_n=1.65$.

Figure 4 show that there are two excitation regions for the cylinder with an aspect ratio of 21, in contrast to the cylinder of aspect ratio 10 in Fig. 3. It is considered that this vibration characteristic, number of excitation regions, is produced by the aspect ratio. The two-dimensional cylinder presented by Okajima^{(1),(9)} shows two excitation regions in a wind tunnel and in a water tunnel (see first report). Since a two-dimensional cylinder is ideally considered as an infinitely long cantilevered cylinder, it would be expected that a long cantilevered cylinder would have two excitation regions, like a two-dimensional cylinder.

Figure 5 shows the effect of aspect ratio on the amplitude of cantilevered cylinders S1 - 4 at almost the same C_n . In the case of S1 with an aspect ratio of 5, it is found that the response characteristics of the cylinder show one excitation region in the range $V_r=1.7$ to 3.2. There is an increase in in-line amplitude from $V_r=1.7$ to 2.2 and a rapid decrease at $V_r=2.4$, with the maximum value of $\xi_{rms}=0.097$ at $V_r=2.2$. On the other hand, the cylinders with aspect ratios of 14 and 21 have two excitation regions. The maximum amplitudes of the first excitation regions and the amplitudes between $V_r=1.7$ to 2.1 are almost equal with every aspect ratios. The amplitudes of the second excitation regions are low with low aspect ratios. It seems the aspect ratio affects the second excitation region more than the first excitation region. The St_w could not be measured because there was no clear frequency peak in the wake.

3.2 Amplitudes of cantilevered cylinders with an end plate or with a splitter plate

To clarify the effect of aspect ratio on excitation regions, an end plate was attached to the free end of the cantilevered cylinder and a splitter plate was placed in the wake. An end plate makes the flow parallel to the cylinder and reduces the flow going over the free end. An end plate, therefore, reduces the influence of end-effect.

Figure 6 shows the effect of an end plate of 60 mm diameter attached to the free end of cantilevered cylinders with aspect ratios of 5 and 21. Figure 6 (a) shows that the cylinder S1 with an end plate has two excitation regions, in contrast to S1 without an end plate in Fig. 5. There are the first excitation region from $V_r=1.6$ to 2.1, a valley from $V_r=2.1$ to 2.4 and

Fig. 5 The effect of aspect ratio on the amplitudes of cantilevered cylinders S1-4; ×, Okajima⁽¹⁾

(a) Aspect ratio of 5 (at $C_n=0.40$)

(b) Aspect ratio of 21 (at $C_n=0.38$)

Fig. 6 The effect of an end plate on the free end of cantilevered cylinders with aspect ratios of 5 and 21

the second excitation region from $V_r=2.5$ to 3.0. Figure 6 (b) shows that the cylinder S4 with an end plate also has two excitation regions, and the depth of the amplitude valley is greater than S4 without an end plate in Fig. 4 (a). It is clear that the influence of end-effect is related to the steepness of a valley and to the second excitation region. The lock-in phenomenon of St_w to $1/2St_c$ occurs from $V_r=2.1$ to 3.0 for an aspect ratio of 5 and from $V_r=2.2$ to 3.3 for an aspect ratio of 21, respectively. That is, the lock-in phenomena begin at the end of the first excitation region and finish at the end of the second excitation region.

Fig. 7 Experimental apparatus of cylinder S1 with a splitter plate in the wake, unit: mm

Fig. 8 The effect of a splitter plate in the wake of cantilevered cylinder S1 with aspect ratio of 5, compared with a free end cylinder and an end-plate-cylinder

The splitter plate was placed in the wake to prevent alternate vortices and to clarify the excitation mechanism of the single excitation region that occurs with an aspect ratio of 5. Figure 7 shows an experimental apparatus with a splitter plate 200 mm wide. Figure 8 shows the effect of a splitter plate on the cylinder S1 with an aspect ratio of 5, compared with a free-end-cylinder and an end-plate-cylinder.

It is found that the response amplitude of the cylinder with a splitter plate shows one excitation region in the range of V_r from 1.1 to 3.2. There is an increase of amplitude from $V_r=1.1$ to 2.5 and a rapid decrease from $V_r=2.6$ to 3.0, with a maximum ξ_{rms} of 0.12 at $V_r=2.5$. The amplitude of the cylinder with a splitter plate is greater than that of a normal cylinder from $V_r=1.1$ to 1.7 and from 2.3 to 3.0, but is almost equal from $V_r=1.7$ to 2.3. The amplitude of the cylinder with an end plate shows that the second excitation region is from $V_r=2.3$ to 3.0 and that the lock-in of St_w to $1/2 St_c$ occurs. Clearly, symmetrical vortices create the vibrations from $V_r=1.7$ to 2.2 and

the alternate vortices suppress the amplitude at $V_r=2.3$ to 3.0 with a normal cylinder.

4. Discussion

4.1 Effect of aspect ratio on excitation regions

In contrast to a two-dimensional cylinder, a cantilevered cylinder rotates at the supported end, with the flow going over the free end. If the angle of rotation defines the difference between a cantilevered cylinder and a two-dimensional cylinder, the excitation regions should change in proportion to the maximum amplitude when C_n is varied. Figure 3 (b) and Fig. 4 (b) show that the excitation regions are almost the same at any C_n and that the angle of rotation does not affect response characteristics. Figure 5 shows that the aspect ratio affects the excitation regions, particularly the depth of valley and the amplitude of the second excitation region. That is, the longer the cylinder, the greater the aspect ratio and the less the end-effect influence. If the flow is parallel to a cylinder with an end plate, there are two excitation regions, as shown in Fig. 6. It follows, then, that it is the flow around the free end of a cantilevered cylinder that causes the end-effect influence and affects the valley and the second excitation region of in-line response amplitude.

4.2 Effect of reduced mass-damping parameter on maximum amplitudes

Figure 9 shows the variation of the maximum peak amplitude of excitation regions against the reduced mass-damping parameter. The single excitation regions, which are observed with test cylinders S1 and S2, are shown in Fig. 9(a) as the first excitation region. Also the cantilevered cylinders presented by Ishihara⁽⁵⁾ and Tanaka⁽³⁾, and the two-dimensional cylinders examined by Okajima^{(1),(9)} are shown. It is found that the maximum amplitudes for each excitation region decrease in proportion to C_n and that the maximum amplitudes in the first excitation regions are more sensitive to C_n . This sensitivity of the first excitation region is also revealed in the experimental results on two-dimensional cylinders by Okajima^{(1),(9)} in Fig. 9, but the maximum amplitudes in the first excitation regions of the cantilevered cylinders show more deviation than those in the second excitation regions.

5. Conclusions

The effect of aspect ratio on the response amplitudes of the in-line tip displacement of cantilevered cylinders was investigated in a water tunnel at sub-critical Reynolds numbers. The aspect ratio was varied from 5 to 21 and the reduced mass-damping parameter was varied for cylinders with aspect ratios

(a) The 1st region

(b) The 2nd region

Fig. 9 The maximums of the non-dimensional peak amplitude of cantilevered cylinders against the reduced mass-damping parameter, also two-dimensional cylinders are shown as Okajima^{(1),(9)}

of 10 and 21. The vibration mechanism was investigated using a cylinder with an end plate or with a splitter plate in the wake. It can be concluded from these experiments that:

(1) The response characteristics of cylinders with aspect ratios of 5 and 10 show single excitation region in the range of V_r from 1.0 to 4.0 in contrast to the cylinders with aspect ratios of 14 and 21. Most notably, the aspect ratio affects the depth of valley between the first and second excitation regions and the amplitudes of the second excitation regions.

(2) The cylinders with an end plate on the free end show two excitation regions, even with an aspect ratio of 5. The valley for a cylinder with an aspect ratio of 21 is steeper than that for a cylinder without an end plate. The flow around the free end of a cantilevered cylinder produces this end-effect.

(3) The amplitude of the cylinder with a splitter plate in the wake is greater than that of a normal cylinder, especially at $V_r=2.3$ to 3.0, and is almost equal between $V_r=1.7$ to 2.2. This clearly shows that

symmetrical vortices create the vibrations from $V_r=1.7$ to 2.2 and that alternate vortices suppress the amplitude of a normal cylinder from $V_r=2.3$ to 3.0.

(4) The maximum amplitudes of each excitation region decrease in proportion to C_n . And the maximum amplitudes in the first excitation region are more sensitive to C_n than that in the second excitation region. This sensitivity of the first excitation region is in agreement with the experimental results on two-dimensional cylinders presented by Okajima^{(1),(9)}.

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