Necklace Vortex Excitation of Upstream Cylinder in Crisscross Circular Cylinder System*

(Influence of Cylinder Diameter, Gap and Damping Factor)

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The necklace vortex excitation, one of the longitudinal vortex excitations, appeared on the upstream cylinder in a crisscross circular cylinder system in uniform flow. The influence of the diameter ratio, gap-to-diameter ratio s/d and the logarithmic damping factor on the oscillation behavior caused by the necklace vortex excitation was investigated using a wind tunnel. The amplitude of the oscillation increases with increasing diameter ratio. The substantial fluctuating lift coefficient of the necklace vortex was introduced based on the spanwise dimension of the vortex, $(C_{LR})'_{rms}$, and it was evaluated using the measurement of the oscillation amplitude. In the necklace vortex excitation region, the relationship between $(C_{LR})'_{rms}$ and the gap-to-diameter ratio collapses on a single curve irrespective of the diameter ratio of the two cylinders, and $(C_{LR})'_{rms}$ decreases with increasing gap-to-diameter ratio.

Key Words: Crisscross Circular Cylinders, Necklace Vortex, Vortex Excitation, Wind Tunnel Experiment, Substantial Fluctuating Lift Coefficient

1. Introduction

When a cylinder is located perpendicular to a uniform flow and is supported elastically, the periodic shedding of Karman vortex from the cylinder sometimes induces a cross-flow oscillation on it (Karman vortex excitation)⁽¹⁾⁻⁽⁴⁾. Shirakashi and co-workers⁽⁵⁾⁻⁽⁷⁾ and Takahashi et al.⁽⁸⁾ investigated the oscillation-suppressing effect by adding another cylinder set downstream in crisscross arrangement as shown in Fig. 1. They reported that the Karman vortex excitation was eliminated when the diameter of the downstream cylinder was equal to the diameter of the upstream cylinder and the gap between them was less than half of the diameter. However, an additional oscillation was also observed by them which occurs when the uniform flow velocity is increased to a value approximately three times the

velocity at which the Karman vortex excitation occurs. Fox⁽⁹⁾ reported periodic shedding of the longitudinal vortices around fixed cruciform cylinders. Recent investigations by Shirakashi et al.⁽⁷⁾ show that this newly discovered oscillation is caused by one of the two types of vortices which shed periodically around the crossing of the two cylinders, as seen in

Fig. 1 Arrangement of two cylinders and coordinate system

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(a) Trailing vortex ($U=12.8$ cm/s, $s/d=0.08$, $Re=860$)

(b) Necklace vortex ($U=12.5$ cm/s, $s/d=0.28$, $Re=900$)

Fig. 2 Photographs of longitudinal vortices shedding from two cylinders in crisscross circular cylinder arrangement (in water tunnel, $d=10$ mm)⁽¹⁰⁾

Fig. 2⁽¹⁰⁾. Since they have axes parallel to the free flow, they are called the longitudinal vortices. They are termed the “trailing vortex” (Fig. 2(a)) and the “necklace vortex” (Fig. 2(b)) according to their geometry. The trailing vortex appears when the gap-to-diameter ratio is less than 0.25 and the necklace vortex appears when the gap-to-diameter ratio is in the range from 0.25 to 0.5, when the diameters of the cylinders are equal⁽⁶⁾. The features of the two longitudinal vortices and the oscillation behaviors caused by them are quite different. Takahashi et al.⁽⁸⁾ reported the relationship between the Strouhal number St_0 of the longitudinal vortices and the Reynolds number Re when both cylinders are supported rigidly. St_0 for the shedding frequency of the trailing vortex increases continuously over a range of Re from 2 500 to 40 000, while St_0 in the case of the necklace vortex shedding is almost constant over the same range of Re . Moreover, while the dependence of St_0 of the trailing vortex on the gap-to-diameter ratio s/d is not consistent, St_0 of the necklace vortex decreases slightly with increasing s/d .

The amplitude of the oscillation caused by the trailing vortex excitation is larger than that caused by the Karman vortex excitation. The necklace vortex excitation is more dangerous since the oscillation amplitude of this excitation phenomenon can attain three times the trailing vortex excitation. Therefore,

we focus on the necklace vortex in this study as a first step in the investigation of these longitudinal vortex excitations which occur in the case of the crisscross arrangement of the cylinders. In this work, the oscillation amplitude of the necklace vortex excitation is measured for several conditions of the diameter ratio of the two cylinders, the gap-to-diameter ratio and the logarithmic damping factor of the upstream cylinder, and the excitation conditions of the necklace vortex are obtained. The fluctuating lift force acting on the upstream cylinder is evaluated based on the oscillation amplitude, and the influences of the diameter ratio of the two cylinders, the gap-to-diameter ratio and the logarithmic damping factor are clarified.

2. Nomenclature

A : oscillation amplitude of upstream cylinder [m]

$(C_L)_{rms}$: substantial fluctuating lift coefficient (rigid system), Eq. (1)

$(C_{LR})_{rms}$: substantial fluctuating lift coefficient (elastically supported system), Eq. (2)

d_1 : diameter of upstream cylinder [m]

d_2 : diameter of downstream cylinder [m]

d : cylinder diameter in the case of $d_1=d_2$ [m]

f_v : vortex shedding frequency [Hz]

f_{v0} : vortex shedding frequency (rigid system) [Hz]

f_n : natural frequency of upstream cylinder [Hz]

f_L : frequency of fluctuating lift force [Hz]

F_L : fluctuating lift force [N]

Re : Reynolds number $=dU/\nu$

R_T : trailing vortex shedding region along upstream cylinder [m]

R_N : necklace vortex shedding region along upstream cylinder [m]

s : gap between cylinders [m]

Sc : Scruton number $=2m\delta/\rho d^2 L$

S_u : spectrum of x -component of velocity fluctuation

St : Strouhal number $=f_v d/U$

St_c : nondimensional oscillation frequency of upstream cylinder $=f_n d_1/U$

U : free stream velocity [m/s]

Ur : reduced velocity $=U/f_n d$

Ur_2 : reduced velocity $=U/f_n d_2$

Ur'_2 : reduced velocity in case of $f_n=f_{v0} = U/f_n d_2$

Z_{rms} : root-mean-square value of z -direction displacement of upstream cylinder [m]

δ : logarithmic damping factor

3. Experimental Apparatus and Procedures

The schematic diagram of the experimental apparatus and coordinate system is shown in Fig. 3. The

Fig. 3 Schematic diagram of experimental apparatus and coordinate system

wind tunnel is of the blow down type. The attainable maximum velocity is 40 m/s and the turbulence intensity is less than 0.4% at 10 m/s. The test section is 320 mm $\times$ 320 mm in cross section and 1 000 mm in length. The upstream cylinder is set horizontally and perpendicularly to the free flow at the center of the test section and it passes through the slots on the side-walls of the test section. The end plates are mounted on the cylinder near the side-walls to prevent flow through the slots. The effective length of the upstream cylinder is 318 mm. The downstream cylinder is mounted vertically on the traversing device in order to make the gap s adjustable within an error of 0.01 mm. One of five cylinders which have different diameters (18, 22, 26, 30, and 32 mm) is used as the downstream cylinder, while the upstream cylinder is 26 mm in diameter.

In order to measure the vortex shedding characteristics and the fluctuating lift force in the absence of the effect of the oscillation, experiments were carried out using the rigid system in which both cylinders are fixed rigidly. In this paper, the diameter of the cylinders is simply denoted as d when both diameters are the same, and the diameter of both cylinders is 26 mm whenever d is used. The fluctuating lift force acting on the upstream cylinder supported rigidly is measured using a strain-gauge-type load-cell.

The investigation of the influences of the diameter ratio d_2/d_1 and the gap-to-diameter ratio s/d_2 on the longitudinal vortex excitation is carried out using the elastically supported system. In the elastically

Table 1 Hot wire probe position

	probe position		
	x/d_1	y/d_2	z/d_1
Trailing vortex	1.00	-1.25	0.25
Necklace vortex	1.00	-0.75	0.25

supported system, the upstream cylinder is supported by plate springs that are mounted outside of the test section as shown in Fig. 3, in order to make the upstream cylinder motion parallel to the vertical (z -) direction.

Two eddy-current dampers are attached on both the ends of the upstream cylinder to control the logarithmic damping factor δ . δ and the natural frequency f_n of the upstream cylinder are determined from the decay of displacement amplitudes during a free damping oscillation in the otherwise still air. During the measurement, f_n is kept constant at about 15 Hz. The z -displacement of the upstream cylinder is measured by noncontacting laser beam displacement detectors which are attached on both ends of the upstream cylinder. The free stream velocity U is measured by a ring-type Karman vortex anemometer developed in our laboratory⁽¹¹⁾.

A hot wire anemometer is used to detect the x -component of velocity fluctuation u . To detect the velocity fluctuation caused by the periodic vortex shedding, the hot wire probe is set near the crossing part of the cylinders. The vortex shedding frequency f_v is determined from the spectrum S_u of the velocity fluctuation signal $u(t)$. The region where the vortex sheds depends on the downstream cylinder diameter d_2 . Therefore, the hot wire probe is set at the optimal position where the velocity fluctuation signal $u(t)$ can be detected by high sensitivity for each value of d_2 . The optimal probe position for the experiments is given in Table 1.

4. Experimental Results and Discussion

4.1 Fluctuating lift force acting on upstream cylinder

The relationship between the free stream velocity U and the frequency of the fluctuating lift force f_L is shown in Fig. 4. In this experiment, the downstream cylinder diameter d_2 is equal to the upstream cylinder diameter d_1 and both cylinders are supported rigidly. f_L is measured by a load-cell that is set on the cylinder support. The trailing vortex occurs at $s/d=0.08$ and the necklace vortex occurs at 0.28. In Fig. 4, the solid and broken lines indicate the shedding frequency f_{v0} of the trailing vortex and the necklace vortex, respectively. They are evaluated by analyzing the velocity fluctuation measured by the hot wire anemometer. In both cases, f_L shows good agreement with f_{v0} over the

Fig. 4 Relationship between free stream velocity and frequency of fluctuating lift force ($d=26$ mm, rigid system)

entire U range of measurements. It is clarified that the fluctuating lift force F_L is the resultant of the fluctuating pressure acting on the upstream cylinder surface caused by the periodical shedding of the longitudinal vortices.

Bae et al.⁽¹²⁾ measured the configuration of the longitudinal vortices on the upstream cylinder surface and reported that the fluctuating pressure due to the longitudinal vortices acts on a limited part of the upstream cylinder. In the present paper, the spanwise length of this limited part is defined as R_N for the necklace vortex. Taking this length into consideration, the substantial fluctuating lift coefficient $(C_L)'_{rms}$ for the necklace vortex is newly defined by Eq.(1).

$$(C_L)'_{rms} = \left((F_L)_{rms} / \frac{1}{2} \rho U^2 d \times R_N \right) \quad (1)$$

Here, ρ is the air density, U is free stream velocity and d is the cylinder diameter. In the case of the trailing vortex, R_N in Eq.(1) is replaced by R_T . The uncertainty of $(C_L)'_{rms}$ is estimated at about 6%. The relationship between Re and $(C_L)'_{rms}$ is shown in Fig. 5. $(C_L)'_{rms}$ of the necklace vortex, that is $s/d=0.28$, is nearly constant over the entire range of Re , from 5 000 to 30 000, while that of the trailing vortex, $s/d=0.08$, decreases with increasing Re . Figure 6 shows the relationship between s/d_2 and $(C_L)'_{rms}$ for three conditions of d_2/d_1 . The necklace vortex appears in the regime $s/d_2 > 0.25$, and $(C_L)'_{rms}$ is not affected by d_2/d_1 in this regime, as seen in Fig. 6. The relationship between the diameter ratio d_2/d_1 and R_T/d_1 , and R_N/d_1 is shown in Fig. 7. R_T/d_1 and R_N/d_1 are approximately proportional to d_2/d_1 . Therefore, it is clear that the fluctuating lift force caused by the necklace vortex is nearly proportional to its spanwise length R_N along the upstream cylinder, and is hence proportional to d_2/d_1 because of the relationship given in Eq.(1).

4.2 Influence of diameter ratio

Figure 8 shows the nondimensional amplitude of

Fig. 5 Substantial fluctuating lift coefficient due to longitudinal vortices ($d=26$ mm, rigid system), $s/d=0.08$: trailing vortex, $s/d=0.28$: necklace vortex

Fig. 6 Relationship between gap-to-diameter ratio and substantial fluctuating lift coefficient ($Re=14$ 000, rigid system)

Fig. 7 Relationship between diameter ratio and R_T/d_1 and R_N/d_1 ($d_1=26$ mm, rigid system)

the trailing and the necklace vortex excitation Z_{rms}/d_1 as a function of s/d_2 . In this case, the upstream cylinder is elastically supported and can oscillate in the cross flow direction. The necklace vortex excitation occurs in the region $s/d_2 > 0.25$. The term “over range” in Figs. 8(c), (d) and (e) indicates that the oscillation amplitude was beyond the measurement limitation. When $d_2/d_1=0.69$, the oscillation amplitude

Fig. 8 Relationship between s/d_2 and Z_{rms}/d_1 at various d_2/d_1 ($\delta=0.01$, $d_1=26$ mm, $f_n=14.8$ Hz, elastically supported system) T: Trailing vortex, N: Neck-lace vortex

Fig. 9 Relationship between s/d and f_{v0} at various Re ($d=26$ mm, rigid system) T: Trailing vortex, N: Necklace vortex

due to the necklace vortex excitation is smaller than that in the case of the trailing vortex excitation, as seen in Fig. 8(a). In the case of $d_2/d_1=0.84$, shown in Fig. 8(b), Z_{rms}/d_1 caused by the necklace vortex excitation has a peak value and the peak value becomes higher with increasing U . Moreover, the position of the peak moves to higher values of s/d_2 with increasing U . At $d_2/d_1=1.00$, a tendency similar to the case of $d_2/d_1=0.84$ is observed, but Z_{rms}/d_1 becomes larger.

When f_{v0} is close to the natural frequency f_n , the oscillation of the vortex excitation occurs. Figure 9 shows the f_{v0} as a function of s/d in the rigid system. In the necklace vortex regime, f_{v0} decreases with increasing s/d . On the other hand, increasing U causes f_{v0} to become higher, as seen in Fig. 4. For this reason, the peak of the necklace vortex excitation moves to a larger value of s/d_2 with increasing U .

The limits of the necklace vortex excitation region are given by the data in Fig. 8 and are shown in Fig. 10, where the ordinate is the reduced velocity, Ur_2 ($=U/f_n d_2$). The solid line in Fig. 10 indicates the reduced velocity Ur'_2 ($=U/f_{v0} d_2$) at $f_n=f_{v0}$. At $d_2/d_1=0.69$, the excitation occurs in the narrow regime of Ur_2 . When $d_2/d_1>0.69$, the excitation regime abruptly extends over the large-value area of Ur_2 . In this region, the upper limit of Ur_2 for the excitation could not be determined since it was out of the measurement range. On the other hand, the lower limit of the excitation is almost constant at $Ur'_2=20$ over the entire range of d_2/d_1 .

Since the oscillation frequency of the upstream cylinder is fixed at f_n during the vortex excitation occurs, the oscillation is regarded as the resonance of a linear elastic system with small damping. Hence, the substantial fluctuating lift coefficient $(C_{LR})'_{rms}$ for the necklace vortex is defined by Eq. (2).

$$(C_{LR})'_{rms} = \frac{8\pi St_c^2 m \delta}{\rho d_1^2 \times R_N} \times \sqrt{2} \left(\frac{Z_{rms}}{d_1} \right) \quad (2)$$

Fig. 10 Necklace vortex excitation regime ($\delta=0.01$, $d_1=26$ mm, $f_n=14.8$ Hz, elastically supported system)

Fig. 11 Relationship between s/d_2 and $(C_{LR})'_{rms}$ at various d_2/d_1 ($\delta=0.06$, $5\,000 < Re < 30\,000$)

Here, St_c is the nondimensional oscillation frequency ($=f_n d_1/U$), m is the equivalent mass of the upstream cylinder and δ is the logarithmic damping factor. In the case of the trailing vortex, R_N in Eq.(2) is replaced by R_T . The uncertainty of $(C_{LR})'_{rms}$ is estimated to be about 10%. Figure 11 shows the relationship between s/d_2 and $(C_{LR})'_{rms}$ calculated by Eq.(2). In Fig. 11, the region between the solid curves shows the result for $(C_L)'_{rms}$ shown in Fig. 6. The broken line indicates a ridge where the maximum value of $(C_{LR})'_{rms}$ obtained by the change in U is connected over the entire range of $s/d_2 > 0.25$. Since Z_{rms} is not proportional to U^2 , $(C_{LR})'_{rms}$ is not constant over the range of U at which the longitudinal vortex excitation occurs, as shown in Fig. 12. Therefore, $(C_{LR})'_{rms}$ is widely scattered over the entire range of s/d_2 , as shown in Fig. 11. However, the upper limit of the $(C_{LR})'_{rms}$ values almost fits the curve of $(C_L)'_{rms}$ when $s/d_2 > 0.25$, and it is unaffected by the diameter ratio of the two cylinders. This result shows that the maximum necklace vortex excitation can be predicted well from the measurement of fluctuating lift force on the rigid system.

4.3 Influence of logarithmic damping factor

The influence of the logarithmic damping factor δ

Fig. 12 Relationship between U and $(C_{LR})'_{rms}$ at various δ ($d=26$ mm, $f_n=14.8$ Hz, elastically supported system)

on the necklace vortex excitation is shown in Fig. 13. In this case, d_2 is equal to d_1 and the upstream cylinder is supported elastically. With increasing δ , the peak value of Z_{rms}/d decreases and the region of s/d for the necklace vortex excitation becomes narrower. However, the s/d value for the maximum peak of Z_{rms}/d is not affected by δ at the same free stream velocity.

Figure 14 shows the regime of the necklace vortex excitation obtained from the result shown in Fig. 13. Figures 14(a), (b) and (c) show the results at $\delta=0.009\,2$, $0.021\,5$ and $0.060\,4$, respectively. In these figures, $Sc (=2m\delta/\rho d^2 L)$ is the Scruton number and the ordinate $Ur (=U/f_n d)$ is the reduced velocity. The necklace vortex excitation region on the $s/d - Ur$ plot becomes narrower with increasing δ , as seen in Fig. 14. Here, the upper limit values of s/d for the necklace vortex excitation are not markedly affected by δ . On the other hand, the lower limit values become larger with increasing δ .

Figure 15 shows the influence of δ on the relationship between s/d and $(C_{LR})'_{rms}$ defined by Eq.(2). The region between the solid curves in Fig. 15 represents $(C_L)'_{rms}$ measured for the rigid system shown in Fig. 6. The broken line indicates the ridge where the maximum value of $(C_{LR})'_{rms}$ obtained by the change in U was connected over the range of $s/d_2 > 0.25$. In the case of $\delta=0.060\,4$, $(C_{LR})'_{rms}$ shows good agreement with $(C_L)'_{rms}$, and it becomes lower with decreasing δ .

As a result of these experiments, it is determined that the substantial fluctuating lift force caused by the necklace vortex shedding is strongly related to the diameter ratio of the cylinders, the gap-to-diameter ratio and the logarithmic damping factor. Therefore, careful adjustment of these conditions would enable the prevention of danger due to the necklace vortex

Fig. 13 Relationship between s/d and Z_{rms}/d at various U ($d=26$ mm, $f_n=15$ Hz, elastically supported system)

excitation.

5. Conclusions

Comprehensive experiments were carried out to clarify the behavior of the longitudinal vortex excitations of the upstream cylinder in a crisscross circular cylinder system in a uniform flow. Influences of the gap-to-diameter ratio and the diameter ratio of the two cylinders on the vortex shedding and the oscillation of the upstream cylinder were investigated experimentally. Based on the experimental results, the criteria for the necklace vortex excitation and the

Fig. 14 Necklace vortex excitation regime at various logarithmic damping factors ($d=26$ mm, $f_n=15$ Hz)

Fig. 15 Relationship between s/d and $(C_{LR})'_{rms}$ at various logarithmic damping factors ($d=26$ mm, $f_n=15$ Hz)

substantial fluctuating lift force coefficient are presented. The conclusions deduced from this work are summarized as follows.

(1) In the case of a rigid system, the fluctuating lift coefficient due to the necklace vortex is nearly constant over the Reynolds number range of 5 000~30 000. The fluctuating lift force acting on the upstream cylinder is proportional to the diameter of the downstream cylinder. The substantial fluctuating lift coefficient introduced in this work is independent of the diameter ratio of the two cylinders and it decreases with increasing gap-to-diameter ratio s/d .

(2) The necklace vortex excitation is not marked when the diameter ratio of the two cylinders is less than 0.84. With increasing diameter ratio of the two cylinders, the amplitude of the excitation becomes larger and the s/d value of the maximum oscillation amplitude becomes smaller, approaching the minimum value of 0.25.

(3) With increasing logarithmic damping factor, the oscillation amplitude of the necklace vortex excitation decreases but the peak value of the oscillation amplitude appears at the same gap. The necklace vortex excitation region on the $s/d - Ur$ plane becomes narrower with increasing logarithmic damping factor.

(4) The substantial fluctuating lift coefficient of the necklace vortex estimated from the maximum

oscillation amplitude agrees well with those measured on the rigid system.

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