High Level Noise-Induced Bypass Transition in Compressible Plane Poiseuille Flow*

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A numerical experiment is conducted to study the bypass transition of compressible plane Poiseuille flow. The space and time development of the disturbances into streaks is captured by employing a spatial DNS, which although requires more computational resources than the temporal one, allows having a full picture of the process. The generation and algebraic growth of the streaks, which are lifted away from the walls and oscillate in the spanwise direction, characterize the transition. The numerical results give some answers related to the transition of compressible plane channel flows, initiated by large amplitude random disturbances, and point to the universality of the breakdown mechanism in wall bounded flows (incompressible and subsonic) and to the important role of the near-wall streaks.

Key Words: Spatial DNS, Random Disturbances, Near-wall Streaks, Bypass Transition

1. Introduction

Renewed research efforts in laminar-turbulent transition of compressible wall-bounded flows, which are common in transatmospheric flight engine, have been recently promoted. Intensified inroads have been addressed towards the understanding of near-wall turbulence and transition in incompressible wall shear flows and with the evolution of the computational resources, numerical simulations of these flows have shown that they are powerful tools in revealing the secrets concealed in the numerical solutions. However, there have been even fewer simulations of their counterpart compressible flows because of their more stringent computational demands. Numerically, this is largely due to the added complexity to solve a coupled velocity-temperature field. Furthermore, viscous and thermal transport coefficients depend upon the instantaneous local temperature. This leads to a significant increase in the number of parameters that need to be considered in comparison with incompressible flows.

There are several routes to turbulence in wall-bounded shear flows and the one that has been deeply investigated was the TS mechanism and its modal exponential growth. Morkovin1,2 and Morkovin & Roshko3 have pointed out to the possibility of replacing this classical mechanism by a stronger one that bypasses it and leads to earlier subcritical transition. Actually, Klebanoff et al.4 have used the term 'bypass' in connection with the effect of large amplitude disturbances on the transition in their famous boundary layer experiment.

In recent years much attention was addressed to the investigation of bypass transition numerically and experimentally because of its importance in the understanding of the observed and yet unexplained contradiction between experiments and linear stability theory. Theoretical and experimental studies treated a variety of incompressible/compressible wall-bounded flows and their classical transition (TS mechanism). However, and in contrast to incompressible wall shear flows, compressible bypass transition, which is very important in many engineering applications (i.e. prediction of the aerodynamic characteristics of commercial transport and high-speed flight, effect of high level noise in wind tunnels and in internal flows etc.), are less common (sometimes non-
existential). Hereunder, a selection of incompressible bypass transition studies is reviewed.

Klingmann\(^{(3)}\) has studied, experimentally, the development of localized disturbances (triggered at the lower wall of an airflow channel) in plane Poiseuille flow at subcritical Reynolds number of 1 600. The disturbances evolved into streaks, which grew in amplitude and streamwise extension and thereafter either decayed or gave rise to a turbulent spot. Alfredsson and co-workers\(^{(6),(7)}\) used the same apparatus, after modifications, in other plane Poiseuille flow studies; the first one dealt with oblique transition at \(Re = 1 600\) and 2 000 and the second with stability of the streaks at \(Re = 2 000\) and 2 500. A result of the oblique experiment was a transition scenario consisting of the formation of high/low speed streaks which initially grew in amplitude and later exhibited decay or growth depending on the amplitude of the initial disturbances. In the second experiment continuous suction was applied at the upper wall of the channel in order to create spanwise alternating high/low speed streaks. The streak amplitude first grew algebraically and when it exceeded a certain threshold secondary instability of the streaks was observed.

Numerical simulations of bypass transitions in incompressible wall–bounded flows were also performed and brought much light to these mechanisms. The spatial evolution of a pair of oblique waves in plane Poiseuille and zero pressure gradient boundary layer flows was simulated by Lundbladh et al.\(^{(9)}\), who used a fringe method to handle the spatial development of the disturbances. An oblique transition scenario was proposed and consisted of the following three stages: Firstly, initial non-linear generation of a streamwise vortex by two oblique waves. Secondly, generation of streaks from interaction of the streamwise vortex with the mean shear, and finally, breakdown of the flow due to a secondary instability of the streaks when they exceed a certain threshold. Similar results were reported in the temporal DNS of Schmid & Henningson\(^{(9)}\). The stability of streamwise streaks and transition thresholds in plane channel flows were investigated by Reddy et al.\(^{(10)}\). They compared threshold energies for transitions initiated by a pair of counter rotating streamwise vortices and a pair of oblique waves with those for 2D TS waves and 2D optimal waves (Butler & Farrell\(^{(11)}\)). Which scenario is most likely to take place at subcritical Reynolds numbers and what mechanism describes transition initiated by random noise were unanswered open questions in the end of their paper.

In the attempt of answering some of these questions in connection to compressible wall–bounded flows, we numerically investigated, in a previous work\(^{(12)}\), the spatial development of random disturbances in subsonic plane Poiseuille flows undergoing bypass transition. As conclusions, we firstly found that the numerical tests showed a good agreement with the theoretical results of laminar compressible flows. Secondly, we used Linear Stability Theory calculations to determine the critical Reynolds number and its dependence on the Mach number. Finally a preliminary transition scenario was deduced from the simulations; the random disturbances evolved into low/high speed streaks with high normal (i.e. \(\omega_y\)) and spanwise shear (i.e. \(\omega_z\)). The vortical and streaky structures were elongated downstream with a quasi-periodicity in the spanwise direction. The transition depended on many factors such as the disturbance amplitude (threshold). As for the Mach number effect, the compressibility did not affect the bypass transition itself, i.e. generation of near-wall streaks that grow downstream, but the details were sensitive to the change in the Mach number, i.e. streak magnitude, due to the changes in the mean flow thermodynamic properties. In other words, the scenario was, qualitatively, the same but the quantities involved might change with Mach number.

In continuation of this work, we simulate a relatively longer channel (the computational box was four times longer than the one used in the previous simulations) to be able to see the subsequent stages of the transition.

Section 2 of this paper will review the numerical methods used in the simulations. Linear stability and simulations results, containing the flow fields (i.e. streaks and vortices) and the spectra, will be presented in sections 3 and 4, respectively. Finally, the conclusions of this work will be given in section 5.

**Nomenclature**

\[ A : \text{disturbance amplitude} \]
\[ c : \text{sound speed} \]
\[ c_i : \text{imaginary part of the phase speed of instability wave} \]
\[ c_r : \text{real part of the phase speed of instability wave} \]
\[ D_x, D_\alpha : \text{first and second derivatives in the \(x\)-direction} \]
\[ E : \text{disturbance energy} \]
\[ f : \text{frequency} \]
\[ h : \text{grid size in compact finite difference schemes} \]
\[ k : \text{wavenumber} \]
\[ k_z : \text{spanwise wavenumber} \]
\[ L : \text{half channel height} \]
\[ M : \text{Mach number} \]
\[ N_s : \text{number of Fourier modes} \]
\[ p : \text{pressure} \]
2. Numerical Methods

The compressible governing equations, the perturbation field and the numerical schemes used in the simulations are reviewed below. Note that in the set of the N-S equations, all distances are non-dimensionalized by the half channel height $L$ and all velocities by the centerline streamwise velocity $U$. The summation convention is used with $x_1=x$, $x_2=y$ and $x_3=z$.

### 2.1 Governing equations

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \mathbf{u} \cdot \nabla \mathbf{u} &= 0 \\
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho} \nabla p &= \frac{1}{\rho} \frac{\partial \mathbf{u}}{\partial x_j} \\
\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \frac{1}{\rho} \nabla p &= \frac{1}{\rho} \frac{\partial \mathbf{u}}{\partial x_j} + \gamma p \frac{\partial \mathbf{u}}{\partial x_j} 
\end{align*}
\]

\[
\tau_0 = -\left(\gamma - 1\right) \frac{\partial q_1}{\partial x_1} + \left(\gamma - 1\right) \tau_0 \frac{\partial u_1}{\partial x_1} \\
q_1 = -\lambda \frac{\partial T}{\partial x_1} \quad \text{with} \quad \lambda = \frac{(\gamma - 1) M^3}{\gamma M^2} \\
p = \rho T / \gamma M^2
\]

\[Re = \rho UL / \mu\] is the Reynolds number; $M = U/c$ is the Mach number, based on $U$ and the wall sound speed, $c$; $\mu = T \nu$ is the dynamic viscosity; $Pr$ is the Prandtl number; $\gamma$ is the ratio of the specific heats $c_p$ and $c_v$.

At the inlet, random disturbances are added to the undisturbed laminar Poiseuille profile of the streamwise velocity component at each time step of the time integration.

\[
u(0, x_3, s, t) = \psi(\psi, x_3, s) \]

This choice allows for normal (with the respect of the non-slip wall) and spanwise random vibration of the disturbances with angular frequencies in the range of $\omega_{nm} = 0.02 \pi \leq \omega \leq 4 \pi$. The computational box and the initial conditions are illustrated in Fig.1 and the disturbance spectra plotted in $(y, k)$ wall/spanwise wave number space and in $(f, k_x)$ frequency/spanwise wave number space are presented in Fig. 2.

### 2.2 Numerical schemes

High-order compact finite difference schemes are used in both the streamwise $(x)$ and normal $(y)$ directions.

The compact 6th-order approximations for the first and second derivatives are expressed as in the following linear combinations.

\[
\begin{align*}
\frac{1}{3} f_{i-1} + f_i + \frac{1}{3} f_{i+1} &= \frac{1}{9} f_{i+2} - f_{i+1} + \frac{14}{9} f_{i+1} - f_{i-1} \\
\frac{2}{11} f_{i+1} + f_i + \frac{2}{11} f_{i-1} &= \frac{1}{4} f_{i+2} - f_{i+1} + \frac{1}{2} f_{i+1} - f_{i-1}
\end{align*}
\]
\[ \tilde{u} = g(y, k) \text{ for } -L \leq y \leq 0 \]
\[ \tilde{u} = g(f, k) \text{ for } y = -L/3 \]

Fig. 2. \( \tilde{u} \)-spectra at the inlet

\[ c_1 = f(M, Re) \]
\[ c_2 = f(M, Re) \]

Fig. 3. Effects of compressibility and wall surface temperature on the stability of subsonic plane Poiseuille flow \((\text{for } a = 1.0)\)

\[ -3 \frac{f_{i+2} - 2f_{i+1} + f_i}{4h^2} + \frac{12}{11} \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2} = 0 \]  
(9)

The 4th-order derivatives at the boundary \(i = 0\) are given by

\[ f_{0} + 3f'_{0} = \frac{1}{h^2} \left( -\frac{17}{6} f_{0} + \frac{3}{2} f_{1} + \frac{3}{2} f_{2} - \frac{1}{6} f_{3} \right) \]  
(10)

\[ f''_{0} + 10f''_{0} = \frac{1}{h^2} \left( \frac{145}{12} f_{0} - \frac{76}{3} f_{1} + \frac{29}{2} f_{2} - \frac{4}{3} f_{3} + \frac{1}{12} f_{4} \right) \]  
(11)

The derivatives and their expressions at the boundaries are written in the following matrix formulations, where \(A_0, A_1, B_0, B_1\) are \(N \times N\) matrices and \(f, \tilde{f}, \tilde{f}'\) are \(N\) vectors, representing the function \((u, v, w, \rho, \mu, T\) etc.) and its first and second derivatives, respectively.

\[ A_0 \tilde{f}' = \frac{1}{h^2} B_0 \tilde{f} \]  
(12.a)

\[ A_0 \tilde{f}'' = \frac{1}{h^2} B_0 \tilde{f} \]  
(12.b)

As for the periodic spanwise \((z)\) direction, a classical Fourier method is employed and approximations for the first and second derivatives are obtained. In this paper, sixteen Fourier modes are used:

\[ \phi(z) = \frac{N_e}{N_e} \sum_{k_{e}} \hat{\phi}(k_{e}, l) e^{i2\pi k_{e}z/2L_{z}} \]  
(13)

\[ \hat{\phi}_{k_{e}} = \frac{1}{N_e} \sum_{l=0}^{N_e-1} \phi(z_{l}) e^{-i2\pi k_{e}z_{l}/2L_{z}} \]  
(14)

with \(z_{l} = l\Delta z\) and \(\Delta z = L_{z}/N_{e}\)

\[ D_{\mu}[\phi_{l}l] = -i2\pi L_{z} \sum_{k_{e}} h_{e} \hat{\phi}_{k_{e}} e^{i2\pi k_{e}z_{l}/2L_{z}} \]  
(15)

\[ D_{\mu}[\phi_{l}l] = -\left( \frac{2\pi}{L_{z}} \right)^{2} \sum_{k_{e}} \frac{h_{e}}{h_{e}} \hat{\phi}_{k_{e}} e^{i2\pi k_{e}z_{l}/2L_{z}} \]  
(16)
Table 1 Comparison of present quasi-incompressible LST results with published incompressible results of Orszag (1971)

<table>
<thead>
<tr>
<th>Orszag (1971) incompressible, ( \alpha = 10, \text{Re} = 10000 )</th>
<th>Current subsonic ( M = 0.61 ), ( \alpha = 10, \text{Re} = 10000 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>isothermal walls</td>
<td>adiabatic walls</td>
</tr>
<tr>
<td>( c_l )</td>
<td>0.00374</td>
</tr>
<tr>
<td>( c_v )</td>
<td>0.23753</td>
</tr>
</tbody>
</table>

(a) Isothermal walls

![Figure 4](image)

(b) Adiabatic walls

Fig. 4 \( c_l = f(\alpha, \text{Re}) \) for \( M = 0.5 \)

3. Linear Stability

The stability of compressible flows is an important issue in fluid mechanics but is rather more complicated than the incompressible one. Its complexity comes from its dependence on many factors such as compressibility, surface temperature, surface roughness, background turbulence etc. We used linear stability theory to see the effects of some of these variables on the stability of subsonic plane Poiseuille flow. An example of the effects of Mach number and wall temperature on the critical Reynolds number is illustrated in Fig. 3, showing \( c_l, c_v = f(M, \text{Re}) \), and Fig. 4, showing \( c_l = f(\alpha, \text{Re}) \), where \( \alpha \) is the streamwise wavenumber and \( c_l, c_v \) are the imaginary and real parts of the phase velocity, \( c = \omega / \alpha \), respectively. When \( c_l < 0 \), the wave is linearly stable and when \( c_l > 0 \), instability occurs. For isothermal walls, the compressibility effect is found to be destabilizing i.e. the critical Reynolds number, \( \text{Re}_{cr} \), decreases with increasing the Mach number, whilst with adiabatic walls it is the contrary. In the isothermal case, stable and unstable regions are both present, whilst in the adiabatic case, there is almost no unstable region for the same range of Reynolds and Mach numbers.

It is noteworthy that comparing very low Mach number results, for isothermal and adiabatic walls with published incompressible results of Orszag (1971) validated the stability analysis (Table 1). For more details about the linear disturbance equations and the numerical methods used in this study, the reader is referred to our previous JSME paper (12).

4. Simulation Results

The aim of this numerical experiment is to study the transition of subsonic, \( M = 0.5 \), plane Poiseuille flow at a subcritical (from the aforementioned linear stability point of view) Reynolds number of 2500. The laminar flow is perturbed at the inlet with random disturbances of large amplitude, equal to 7.5% of the streamwise velocity component, in order to bypass the secondary instability of the classical TS waves. Note that the threshold amplitude for streak generation and initial growth was found, in our previous simulations, to be around 5% of \( U \).

4.1 Vortices and streaks

In this section we shall describe the development of the streaky and vortical structures which are the sign of the onset of the transition. The numerical simulations are carried out for a relatively long channel \( (L_x = 40 L) \). Figure 5(a), which is a top view of the streaks in the \((x-z)\) plane, shows the well-defined high/low speed streaks (arrows of velocity excess/defect). Three features of the streaks can be drawn from this illustration; their spanwise spacing and periodicity, their downstream development (elonga-
Fig. 7(a) Side view of the streaks at $t=1.085$; high-speed, $u_\gamma^+ = 0.02$, (dark); low-speed, $u_\gamma^+ = -0.02$, (light)

Fig. 7(b) Side view of the streaks at $t=1.085$; high-speed, $u_\gamma^+ = 0.015$, (dark); low-speed, $u_\gamma^+ = -0.015$, (light)

Fig. 8 Side view of the vertical vorticity component at $t=1.085$; positive iso-surface, $\omega_y = 0.15$, (dark); negative, $\omega_y = -0.15$, (light)

Fig. 9(a) Top view of the streamwise vorticity component at $t=1.085$ (plane $y = -2L/3$)

Fig. 9(b) Contours of the streamwise vorticity component (plane $y = -2L/3$); a) $t = 1.040$; b) $t = 1.085$

Fig. 10 Top view of the spanwise vorticity component at $t=1.085$ (plane $y = -2L/3$)

...ision) and importantly their spanwise oscillations (Fig. 5(b)). Due to the correlation between the streaks and the vertical vorticity, as mentioned in our previous work, Fig. 6 exhibits the same features. The side views of the streaks and vertical vorticity are illustrated in Figs. 7 and 8, which show the inclination of the structures that the slowly moving fluid particles are being moved away from the walls (Fig. 7(b)).

Before continuing, we feel it is important to mention that the streamwise component of the vorticity, shown in Fig. 9, is initially very weak in the near-wall region in contrast to the other two components. The streamwise vorticity has been often related to the streak generation in several bypass transition studies, however our observations along with those of Asai & Nishioka, Gustavsson, Elofsson & Alfredsson (transitional Poiseuille flow numerical and experimental investigations) and Kim et al. (turbulent channel) don't support this conjecture. For the $M=0.5$ case discussed in this paper, the streamwise vorticity has a maximum at the wall (Fig. 9(a)) and is weak (minimum) in the near-wall region but it becomes stronger when the streaks start...
oscillating in the spanwise direction (Fig. 9(b)).

The next figures, Figs. 10 and 11, display the spanwise vorticity component (≈ du/du in the near-wall region), which is located under the streaks. That the compressed vortices are connected to the low-speed streaks while the stretched vortices correlate with the high-speed ones (see Kline et al. for more details regarding this model). In the phase of streak’s spanwise oscillation, these vortices (and the shear layers) are having a sort of spanwise wavy/wrapping motion that was observed in the simulations of Jimenez & Moin.

The spanwise oscillation could also be seen in the spanwise velocity component and the pressure-fluctuation field (Figs. 12 and 13).

We finish this section by plotting the vorticity vector in Fig. 14, which shows the wavy character (Fig. 14(a)) and the $V$-structure (Fig. 14(b)). This backward-pointing structure was also observed by Klinkmann and was associated with the spikes that preceded the appearance of turbulent spots.

### 4.2 Spectra analysis

In the purpose of understanding the mechanisms behind the observed streak generation, elongation and oscillation, the spectra of the velocity and vorticity components are obtained. The method consists of a single Fourier transform of the spanwise-averaged time series.

$u$ and $\omega_y$ spectra, which exhibit a downstream sustain of low frequencies, are very correlated and have maximum magnitudes in the near-wall region \( y \approx -\frac{2L}{3} \). This correlation explains the instantaneous flow field visualization of the vertical vorticity,
Fig. 15(a) $u, \rho, \omega_x$ and $\omega_y$ spectra at $x=10L$

Fig. 15(b) $u, \rho, \omega_x$ and $\omega_y$ spectra at $x=20L$

Fig. 15(c) $u, \rho, \omega_x$ and $\omega_y$ spectra at $x=30L$
being the sidewalls of the streaks, in the previous section.

The $\omega_x$ spectra show, again, that the streamwise vorticity is very weak in the near-wall region but has a maximum at the walls, which is correlated to the acoustics of the flow, i.e. the pressure.

In order to check the energy growth of the disturbances, a double Fourier transform is now employed.

$$\tilde{u}(f, \beta) = \int_0^{L_{\max}} \int_0^L u(t, z) e^{-i(2\pi f t + \beta_0 z)} dz dt$$

(17)

an approximation of Eq.(17) gives

$$\tilde{u}(f, \beta_0) = \frac{N}{N_{\max}} \sum_{n=0}^{N_{\max}} u(t_n, z_n) e^{-i(2\pi f t_n + \beta_0 z_n)} \Delta z \Delta t$$

(18)

The perturbation energy per unit length normal to the wall in the frequency/spanwise wave number space is

$$E = \frac{1}{2} \int_0^{L_{\max}} \int_0^L u^2(t, z) dz dt$$

(19)

This expression can be approximated by using the energy densities, $e_\omega$, $e_\beta$.

$$E = \int_0^{L_{\max}} e_\omega(\omega) d\omega = \int_0^{L_{\max}} \omega e_\beta(\beta) d\beta$$

(20)

$$e_\omega(\omega) \approx \frac{1}{2} \sum_{n=0}^{N_{\max}} |\tilde{u}(\omega, \beta_0)|^2 / M \Delta z$$

(21)

$$e_\beta(\beta) \approx \frac{1}{2} \sum_{n=0}^{N_{\max}} |\tilde{u}(\omega_0, \beta)|^2 / N \Delta t$$

(22)

with $\omega = 2\pi f$ being the angular frequency.

The perturbation energy, plotted for different downstream locations, exhibits sustain and amplification of modes with very low frequencies ($f \approx 0$ and $kz \approx 2$) corresponding to the streaky structures in accordance to the results of Gustavsson[18].

4.3 Shear stress

The wall stress, $\tau_w$, the friction velocity, $u_f$, and the transitional Reynolds number, $R_t$, based on the friction velocity, are plotted.

Figure 17 gives an insight on the location of the transition and its beginning characterized by an increase in the wall shear stress and friction velocity (thus the transition Reynolds number) just downstream of the inlet where nonlinear activities take place. Farther downstream the linear behaviour (algebraic growth) can be clearly seen.

5. Concluding Remarks

Direct numerical simulations of bypass transition, induced from high level random noise, in subsonic plane Poiseuille flows were performed and a transition scenario was concluded. This scenario contains three main stages;

- Nonlinear development of the perturbation field and generation of near wall streaky structure with intense shear (i.e. $\omega_x$ and $\omega_\phi$).
- Algebraic growth of the streaks which extended downstream and were lifted away from the walls.
- Secondary instability of the streaks; spanwise oscillation accompanied with generation of streamwise vorticity in the near-wall region.
The final stage can be considered as the trigger of the final breakdown (i.e. emergence of turbulent spots).

This transition scenario points out to the importance of the streaks in the breakdown of wall-bound ed shear flows and to the universality of their bypass transition (regardless of the disturbances, although specific details can differ from a study to another). However, we believe that this work gave some answers related to the transition of compressible plane channel flows initiated by large amplitude random disturbances (such as the selection of the resonance 'of the streaks and the vertical vorticity' in the competition between the different candidate scenarios).

The other conclusions of this work are stated hereunder.

- The importance of the spatial simulation in giving a full picture of the time and space development of the disturbances and the flow fields.

- The feasibility of such simulations. However it would be profitable to optimize the size of the computational domain. For instance, a simulation with a computational box that was doubled in the spanwise direction showed that the results of the present simulations were unaltered but we believe it would have a slight effect on the acoustics of the flow.

- This choice of the computational domain is crucial to the study of wall-bound flows' transition. The streak spacing is believed to be a necessary condition for the final breakdown and for the sustaining process of near-wall turbulence, i.e. 100 wall units (Jimenez & Pinelli19 and Hamilton et al.21). However it is not clear whether the streak length is as important as the streak spacing or not. Jimenez & Moin14 suggested values around 500 wall units, but their study was not concerned with transition and dealt with turbulent channel. In the present study, streak length and spacing were around 90 and 90 wall units, respectively. It was also observed that the streak spacing increases downstream and with the disturbance amplitude, suggesting that the streaks thicken downstream, i.e. streamwise and spanwise extension, and that the forcing level accelerates the transition, i.e. streak generation and growth.

At the end, it is important to notice that although, the present algorithm cannot be used to simulate incompressible plane channels, the linear stability showed that for very low Mach numbers, the results converged to the incompressible ones and that wall temperature effects became negligible. So the strategy of comparing the present results to their counterpart incompressible ones (i.e. the numerical results of Henningson and co-workers13,14,18,19, through M=0.25 results (performed but not presented here) helped in concluding the importance and universality of near-wall structure in bypass transition of both, incompressible and subsonic compressible, flows. However we felt that it was premature to extend these findings to high Mach number, supersonic, plane Poiseuille flows without further studies.

Future work addressing the effects of compressibility, wall temperature and boundary conditions would be very important in the complete understanding of bypass transition of compressible internal flow.

References


(12) Yabaha, A., Maekawa, H. and Yamamoto,


