Detailed Flow Field Structure in Rapid Valve Closure*

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In this paper the detailed structure of velocity and pressure fields during and subsequent to instantaneous valve closure are presented. The full Navier-Stokes partial differential equations, formulated for a two-dimensional axi-symmetric flow geometry, have been solved to obtain the transient solutions. The Finite Volume Integration Technique together with a procedure involving splitting of operators is employed instead of the Method of Characteristics which is commonly used in the numerical solution of transient flows in pipelines. A solution in two dimensions, viz., in both the pipe axial and radial axis, is obtained for velocity and pressure. The well known oscillations of both the pressure and velocity components have been confirmed. However, it has been revealed, in this work, that the velocity field has a lot of structure in the radial axis which includes a region of re-circulation flow immediately upstream of the closed valve.

Key Words : Fluid Dynamics, Pipe Flow, Rapid Valve Closure, Pressure Transients, Oscillating Flow, Numerical Analysis, Finite Volume Integration

1. Introduction

Transient flow, and associated wave propagation, is a frequent occurrence in pipe systems. In some cases the feature is imposed on a flow by virtue of the characteristics of an appliance, such as a reciprocating pump, connected to the system, and operations such as pump starts and pump trips. In other cases, such as in the water-hammer phenomenon, the feature is a natural response of an entire system to temporal change at a boundary or at a critical section of a flow. Under conditions of transient flows, large pressure surges, may be generated and they may be destructive to the pipe systems. Thus, in design of pipe systems such conditions are avoided or a system is designed to withstand the most adverse pressure conditions that it may experience. The need to accurately predict the magnitude of such pressure surges and the conditions under which they occur has been a major driving force in the study of transient flows.

The governing system of equations which is normally the basis of numerical analysis of transient flow in pipe systems consists of two equations describing the conservation of mass and of linear momentum. The development of these equations is well documented by Wylie et al.⁽¹⁾. In their most general form they are non-linear hyperbolic partial differential equations in which the dependent variables are the pressure and the mean flow velocity at a flow cross section whereas the independent variables are time and the distance along the axis of the flow conduit. The pair of equations has been solved for most applications using the Method of Characteristics which has been the most popular technique with major writers in the field⁽¹⁾⁻⁽³⁾. This one-dimensional approach makes the numerical analysis of long pipe networks tractable and has been successful in providing the key information required in the design of a large variety of engineering systems in which flow transients are important. However, the approach is not adequate in cases where two or three dimensional flow structure is important.

There are cases of hydraulic systems, such as the hydraulic ram pumps where the objective is to harness the pressure surges to perform some positive work

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rather than to minimize or eliminate them. For such purposes, further insight into the pressure surge phenomenon, the way in which the pressure field is coupled to the velocity field, and knowledge of how the flow field interacts with the pipe system devices, for example valves, would be useful. Some of these requirements may be fulfilled through two or three-dimensional analysis of the flow. However, in the two- or three-dimensional analysis a larger number of grid cells is used per unit length of the pipe, and hence a considerably higher computational effort is required than for the one-dimensional analysis. Consequently the analysis is appropriate only to very short pipe sections such as in the immediate neighborhood of the location of an event of interest.

In this paper a general system of governing equations and appropriate boundary conditions are presented for a simple axi-symmetric transient pipe flow. A numerical procedure and a computational algorithm suitable for solution of the system of governing equations are briefly described. The computer code in which the algorithm is implemented is validated using theoretical solutions for oscillations in pipe flows. The code is then used to simulate in detail the sudden valve closure phenomenon, for which results are presented.

2. Governing Equations and Boundary Conditions

The independent variables are the time t and the coordinates x and y of a cartesian coordinate system, x is aligned with the pipe axis. The dependent variables are the x and y velocity components u and v , respectively, pressure p and density ρ . For convenience of presentation accumulative, convective and diffusive terms are grouped together in an operator L defined as

$$L(\phi; \Gamma_\phi) = \frac{\partial(y^c \rho \phi)}{\partial t} + \frac{\partial(y^c \rho u \phi)}{\partial x} + \frac{\partial(y^c \rho v \phi)}{\partial y} - \frac{\partial}{\partial x} \left(y^c \Gamma_\phi \frac{\partial \phi}{\partial x} \right) - \frac{\partial}{\partial y} \left(y^c \Gamma_\phi \frac{\partial \phi}{\partial y} \right), \quad (1)$$

where ϕ is a conserved variable and Γ_ϕ is the respective diffusion coefficient to be specified. The dynamic viscosity is denoted by μ . Parameter c takes a value of 1 or 0 for a physical geometry which is axi-symmetric or rectilinear, respectively. Thus, in general, the conservation equations are of the following standard form

$$L(\phi; \Gamma_\phi) = y^c S_\phi, \quad (2)$$

where S_ϕ represents the source terms of a variable ϕ . Specifically, the conservation equations are

- mass :
- $L(1; 0) = 0;$ (3)
- x -momentum :

$$L(u; \mu) = -y^c \frac{\partial p}{\partial x} + y^c \rho g_x + \frac{\partial}{\partial x} \left[y^c \mu \left(\frac{\partial u}{\partial x} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right) \right] + \frac{\partial}{\partial y} \left(y^c \mu \frac{\partial v}{\partial x} \right); \quad (4)$$

• y -momentum :

$$L(v; \mu) = -y^c \frac{\partial p}{\partial y} + y^c \rho g_y + \frac{\partial}{\partial y} \left[y^c \mu \left(\frac{\partial v}{\partial y} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right) \right] + \frac{\partial}{\partial x} \left(y^c \mu \frac{\partial u}{\partial y} \right) - W \quad (5.a)$$

where

$$W = c \cdot \left[\mu \left(\frac{2v}{y^c} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right) \right]. \quad (5.b)$$

In the above equations $\nabla \cdot \mathbf{v}$ is defined as

$$\nabla \cdot \mathbf{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{c \cdot v}{y^c}. \quad (6)$$

Closure of the system of governing equations is achieved through the compressibility equation, viz.,

$$\frac{\partial p}{\partial \rho} = \frac{K}{\rho} \quad (7)$$

where K is the bulk modulus of a fluid.

Assumptions

The following assumptions have been made

- The flow is laminar.
- The effective bulk modulus, K , embodying the bulk modulus of the fluid, the properties of the pipe material, pipe dimensions and structural support, is constant.
- The change in volume of the computational space due to change of pressure is negligible.
- There is no separation of liquid column or formation of vapor cavities.
- The dynamic viscosity is constant.
- The flow is axi-symmetric.

Boundary Conditions

Solution of the above system of governing equations requires suitable boundary condition to be imposed. These are specified as given below for the pipe flow configuration illustrated in Fig. 1(a).

- Walls: $y = \pm R$, no slip condition is imposed; that is, $u = v = 0$
- Line of axi-symmetry: $y = 0$, $v = 0$, $\partial u / \partial y = 0$, $\partial p / \partial y = 0$
- Inlet: $x = 0$, the velocity component $v = 0$, and the velocity u is prescribed; alternatively the pressure gradient may be specified for pressure field and the condition $\partial u / \partial x = 0$, applied to the velocity field.
- Outlet: $x = l$, it is assumed that the outlet is far enough for the condition $\partial u / \partial x = 0$, $\partial v / \partial x = 0$ to apply. The pressure may be evaluated at the outlet to satisfy the momentum equation.

Fig. 1 (a) An illustration of the flow configuration and the physical geometry; (b) An illustration of the computational domain and the five point finite volume integration stencil

3. Discretization and Numerical Solution of the System of Governing Equations

The system of governing equations in the general form given in Eq. (1) is integrated over an arbitrary cell, following the Finite Volume Technique⁽⁴⁾, to obtain second order accurate discretized equations for the mass, x , and y -momentum components. The Euler implicit scheme is applied for the temporal differencing and the hybrid scheme is used for the spatial derivative terms. The discretization is performed on a collocated grid, instead of a staggered grid. A discretized pressure equation is derived from the discretized continuity equation and the two momentum equations. The standard form of the discretized equation is

$$a_P \phi_P = \sum_{E, W, N, S} a_i \phi_i + S'_{\phi, P} + b_P \phi_P^n \quad (8)$$

where ϕ denotes any of the variables p , u and v ; a and b are coefficients and S' represents the source terms. The superscript n in Eq. (8) is used to identify variables or expressions taken at time level $t \equiv t^n$ at which a solution to the governing equations is known. Variables or expressions not identified by a subscript are taken at time $t \equiv t^n + \Delta t$ and are unknown. Solutions to the governing equations at $t \equiv t^n + \Delta t$ are sought. The letters P , E , W , N , and S used as subscripts denote the nodes as illustrated in Fig. 1(b).

The compressibility equation is discretized using the Euler implicit scheme to give the following equation

$$(\rho_P - \rho_P^n) = (p_P - p_P^n) \frac{\rho_P^n}{K} \quad (9)$$

Equation (9) is used in the approximation of the time derivative of density in the discretized continuity equation and in updating density, ρ , every time a new

pressure field, p , is evaluated.

Solution of the discretized algebraic equations require suitable implementation of the boundary conditions. In the solution of the numerical pressure equation, Eq. (8), either pressure or velocity may be prescribed at the inlet boundary. In the simulation of the valve closure the velocity boundary condition is used. Hence for an interior grid cell whose western cell interface is part of the inlet, the mass flux through the interface is evaluated directly using the value prescribed for the boundary and then it is introduced directly in the discretized mass conservation equation. This results in the elimination of the western pressure p_E from the continuity equation centered in this cell. The cells adjoining the boundary at the axis and the pipe wall are treated in a similar way to eliminate the pressure p_N and p_S at the northern and southern cell interfaces respectively. Either a gradient is imposed across the interfaces at the inlet or a value for the pressure p_W is specified when a pressure boundary condition is used; this procedure is also applied at the outlet.

The discretized system of the governing equations is solved following PISO (Pressure Implicit with Splitting of Operators)⁽⁶⁾, which is a pressure based procedure. The procedure is based on operator splitting concept and involves a predictor step followed by two corrector steps in the evaluation of a solution in each time step. In the predictor step the discretized momentum equations are each solved implicitly using a pressure field solution p^n , determined for the previous time step, to obtain a velocity field u^* and v^* . The superscript $*$ denotes a preliminary solution. The preliminary velocity field solution does not satisfy the continuity equation.

In the first corrector step the preliminary velocity field is utilized in the implicit solution of the discretized pressure equation to obtain the pressure field p^* , and subsequently, the mass fluxes at the cell interfaces, which numerically satisfy the mass conservation equation. Using the new pressure field and the new mass fluxes, an updated velocity field u^{**} and v^{**} is obtained. In the second corrector step the new solutions are used in the determination of final and more accurate solutions for the pressure field, interface mass fluxes, and velocity field following the same scheme described for the first corrector step. The implementation of this algorithm is discussed in detail in Ref. (6).

4. Results and Discussions

4.1 Case 1: Code validation-oscillating flow in a pipe

The problem of oscillating flow in a pipe presents

transient features and computational challenges similar to those associated with pipe flows following sudden valve closure. In addition, analytic solutions, in agreement with experimental results, have been obtained for the problem and this makes it suitable for validation of the computer code. The formulation and theoretical solution of the problem is well documented in Ref. (7) and, therefore only the key aspects of the problem and the solution are highlighted here for ease of reference.

The specific problem is a flow in a very long pipe of circular cross-section with the fluid at the inlet subjected to a periodic pressure difference. For the analytic solution of the problem it is observed that the velocity u does not vary with the axial coordinate x , a condition which holds, upon application of the continuity equation, when the radial velocity component is zero; hence, the convection terms parallel to the axis also vanish. Thus, neglecting the effects of body forces, the Navier-Stokes equation assumes the form

$$\frac{\partial(y\rho\phi)}{\partial t} = -y\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(y\mu\frac{\partial\phi}{\partial y}\right). \quad (10)$$

At the walls, $y = \pm R$, no slip condition is imposed. This problem is solved by assuming that the pressure gradient term is harmonic and that the equation can therefore be written in the form

$$\frac{\partial(y\rho\phi)}{\partial t} = -y[A \cos(nt)] + \frac{\partial}{\partial y}\left(y\mu\frac{\partial\phi}{\partial y}\right) \quad (11)$$

where A is a constant. For very slow oscillations the analytic solution is of the form

$$u(y, t) = \frac{A}{\mu}(R^2 - y^2) \cos(nt) \quad (12)$$

and for very fast oscillations it is of the form

$$u(y, t) = \frac{A}{n} \left\{ \sin(nt) - \sqrt{\frac{R}{y}} \exp\left(\sqrt{\frac{n}{2\mu}}(R-y)\right) \times \sin\left[nt - \sqrt{\frac{n}{2\mu}}(R-y)\right] \right\}. \quad (13)$$

The parameter selected for use in the validation of the computer code is the mean, with respect to time, of the velocity squared, denoted by $\overline{u^2(y)}$, which for the analytic solution is given by

$$\frac{\overline{u^2(y)}}{A^2/2n^2} = 1 - 2\sqrt{\frac{R}{y}} \cos(\eta) \exp(-\eta) + \frac{R}{y} \exp(-2\eta) \quad (14)$$

where, the parameter, η , a dimensionless distance from the pipe wall, is defined as

$$\eta = (R-y)\sqrt{n/(2\mu)}. \quad (15)$$

The term $A^2/(2n^2)$ corresponds to the mean with respect to time of the velocity squared far away from the wall, that is, as $\eta \rightarrow \infty$. The maximum value of $\overline{u^2(y)}$ occurs at $\eta = 2.28$ and this is in agreement with experimental results.

In the numerical solution of the oscillating flow problem a pressure gradient, given as a sine function

Fig. 2 A plot of the parameter $\overline{u^2(y)}/(A^2/(2n^2))$ as a function of η , the non-dimensional distance from the pipe wall, for a theoretical solution and for a solution obtained in the computer modeling

of time, is imposed at the inlet, and the Navier-Stokes equations are solved as explained in the section above.

Shown in Fig. 2 is a plot of the parameter $\overline{u^2(y)}/(A^2/(2n^2))$ as a function of η for both the theoretical solution, given in Eq. (12) and the numerical solution. There is a very good agreement between the two solutions.

4.2 Case 2: Sudden valve closure

In the case of sudden valve closure, the initial condition is a fully developed flow, which is easily obtained, using the computer code, by specifying a parabolic inlet velocity profile such that the fluid moves out of the computational domain at the inlet. During valve closure the inlet velocity is

$$u(y) = \text{erfc}(t)U(y) \quad (16)$$

where $U(y)$ is the inlet velocity value for the fully developed flow, $\text{erfc}(t)$ is a complementary error function of time, and t is the elapsed time from onset of closure. Consequently, the parabolic profile is maintained throughout the closure period and hence a zero velocity is simultaneously attained throughout the inlet at full closure. In this work the closure time t_c is 0.2 of a millisecond which is short enough to meet the requirements for a rapid valve closure case. The magnitude of initial maximum velocity at inlet is -1.0 ms^{-1} (which gives a mean velocity, $\bar{u}_0$, of -0.667 ms^{-1}). At steady state pressure the density is taken to be 1000 kgm^{-3} , the viscosity as $0.001 \text{ kgm}^{-1}\text{s}^{-1}$ and the effective bulk modulus K as 2.2 GNm^{-2} . (Note that this value for K is approximately the bulk modulus of elasticity of water, at 20°C ; other values may be used to include the effects of structural support, pipe material properties and pipe wall thickness.) The

Fig. 3 A pressure-time history at the inlet during and after full closure of the valve

pipe diameter, d , is 10 mm. The pipe length, l , is 675 mm which is sufficiently long to allow full development of the pipe flow. The length selection was based on numerical experiments in which different pipe lengths were tested. A time step, Δt , of 5 microseconds is used in the computations.

4.2.1 Pressure Shown in Fig. 3 is the pressure-time history at the pipe axis at the inlet on the upstream side of the valve; the time, $t=0.2$ millisecond, at which the valve fully closes, is highlighted. The oscillation of pressure, which begins with a pressure rise, is initiated by the valve closure. The pressure rises, above the steady state value, to a maximum value of 0.991 MNm^{-2} some time after the valve is fully closed. The pressure oscillations persist after full closure of the valve but with monotonically decreasing amplitudes and changing waveforms. Note that the Joukowsky pressure rise, predicted using the equation

$$\Delta p = \rho a \bar{u}_0, \quad (17)$$

is 0.988 MNm^{-2} . In Eq. (17), a , the theoretical wave speed, is obtained from the relationship

$$a = \sqrt{K/\rho}. \quad (18)$$

Shown in Fig. 4 is the pressure-time history at the pipe axis at a point midway between the inlet and the outlet of the pipe superimposed on the pressure-time history at the inlet which is shown in Fig. 3. In the figure it is seen that the pressure wave initiated at the inlet arrives at the midway point some time later as it travels towards the outlet. Upon reflection at the outlet it is converted into a pressure reduction wave which annihilates the high pressure region as it travels back to the inlet. At the inlet, the reflection of the pressure reduction wave yields a low-pressure wave which travels towards the outlet. The low-pressure wave is reflected at the outlet and transformed into a

Fig. 4 A pressure-time history at a point located midway between the inlet and the outlet of the pipe, that is, at $x=l/2$, superimposed on the pressure-time history at the inlet

pressure augmenting wave which travels towards the inlet thereupon canceling out the low-pressure region. When this wave reaches the outlet a new cycle is started.

The plots of pressure-time history for points at the same axial but different radial positions are visually identical; hence axial pressure changes are of dominating importance. None the less, in the radial direction, there is a very small pressure gradient; this is associated with the velocity structure.

After the first half-period the cycles have the same periodic time of 1.84 millisecond which, for the pipe length specified above is equivalent to a wave speed of 1467 ms^{-1} . The theoretical wave speed, a , from the relationship $a = \sqrt{K/\rho}$, is 1483 ms^{-1} .

4.2.2 Velocity Shown in Fig. 5(a) is a velocity vector plot next to the inlet on the upstream side of the valve at time, $t=0.2$ millisecond, when the valve is fully closed. It is observed that at the axis, the flow impinges on the valve surface and is consequently deflected to the pipe periphery where the flow direction reversal towards the outlet is completed. In comparing Figs. 5(a) and 5(b), where Fig. 5(b) is a vector plot at $t=1.0$ millisecond, it is seen that the general structure of the velocity field, in the region next to the valve, is largely unaffected by the oscillations observed in the pressure field and in the velocity field far away from the inlet. This is further evident in the x -component of velocity-time history shown in Fig. 6 for three points located at the following (x, y) coordinates: $A(0, 0)$, $B(5.4, 0)$ and $C(5.4, 1.6)$ mm. Point A is on the valve surface, hence, the velocity is seen to change from the maximum inlet velocity value to zero upon full valve closure. At point B the

Fig. 5 Velocity vector plots in the immediate vicinity of the valve after full valve closure at: (a) $t=0.2$ and (b) $t=1.0$ millisecond

Fig. 6 A velocity-time history at the inlet and in its immediate vicinity at three points located at the following (x, y) coordinates: $A(0, 0)$, $B(5.4, 0)$ and $C(5.4, 1.6)$ mm during and after full closure of the valve

Fig. 7 A velocity-time history at a section midway between the inlet and the outlet at three points located at the following (x, y) coordinates: $D(322, 0)$, $E(322, 1.6)$ and $F(322, 4.67)$ mm during and after full closure of the valve

velocity rapidly decreases as the valve closes and upon the full closure of the valve changes fairly slowly. A similar behavior is observed at point C except that at this point the flow is in the direction opposite to that at point B . The small wiggles evident in the plots are a response to the large oscillations in the velocity field farther away from the valve. Thus, after valve closure, the region adjoining the valve develops into a steady flow reversal region.

Farther away from the valve, there are large oscillations of velocity field. This is clearly observed in the velocity-time history plots for three points, located midway between the inlet and the outlet of the pipe at the following (x, y) coordinates: $D(322, 0)$, $E(322, 1.6)$ and $F(322, 4.67)$ mm, shown in Fig. 7. Shown in Figs. 8(a) and 8(b) are the velocity vector plots shown, for a pipe section encompassing the three points, at $t=1.0$ and 1.895 millisecond, respectively. The velocity oscillations at the three points, and hence at a pipe cross-section, are in phase. However, for the particular case modeled in this work, the velocity oscillations lag behind the pressure field oscillations. When the mean flow direction is towards the outlet, the velocity magnitude is largest near the pipe wall but when the mean flow direction is towards the inlet, the velocity magnitude is largest at the pipe axis.

The velocity-time history, at the points mid-way between the pipe inlet and the outlet shown in Fig. 7, exhibit, in each cycle, four distinct stages each of which is a response to the pressure wave passing through the section as it travels back and forth between the inlet and the outlet. In the first stage the velocity at point D rapidly increases following the

Fig. 8 Velocity vector plots for a pipe section midway between the inlet and outlet of the pipe after valve closure at: (a) $t=1.0$ and (b) $t=1.895$ millisecond

initial pressure surge towards the outlet. For a short while, marking the wave travel to the outlet and back, the velocity increase is suppressed as evident in the inflexion on the curve. The velocity rapidly increases in the second stage to the maximum value for the cycle in response to the pressure reduction wave traveling to the inlet. As the low pressure wave travels to the outlet it causes a fast decrease of the velocity to some intermediate value where a second inflection of the curve is observed. The inflexion is the effect of the time lapse between the wave travel to the outlet and back to point D and will, therefore, become more pronounced as the distance traveled, and hence the length of the pipe, increases. In the fourth stage, a final rapid decrease in the velocity is observed as the wave travels back to the inlet where a new cycle begins.

Shown in Fig. 9 are the velocity-time history plots for three points located at the following (x, y) coordinates: $G(49, 0)$, $H(49, 1.6)$ and $I(49, 4.67)$ mm. These points are in the transient flow region which lies

Fig. 9 A velocity-time history at a section in the transient flow region at three points located at the following (x, y) coordinates: $G(49, 0)$, $H(49, 1.6)$ and $I(49, 4.67)$ mm during and after full closure of the valve

Fig. 10 Velocity vector plots for a pipe section in the transient flow region after valve closure at: (a) $t=1.0$ and (b) $t=1.895$ millisecond

between the steady flow region adjoining the valve surface and the fully oscillating flow region far away

from the valve. In comparing Figs. 6 and 9 it is seen that the magnitudes of oscillation in the velocity field increase with increasing axial distance from the valve surface. Shown in Figs. 10(a) and 10(b) are the velocity vector plots, for a pipe section encompassing the three points, and corresponding to the times $t=1.0$ and 1.895 millisecond, respectively—note that, in the first cycle, the maximum velocity and the minimum velocity at any point are attained at approximately $t=1.0$ and 1.895 millisecond, respectively. It is seen in Fig. 10(a), that even when the velocity and, hence, the mean flow towards the outlet is at a maximum, there exists a region around the pipe axis in which the flow is towards the inlet.

5. Conclusions

In this work a numerical algorithm based on the Finite Volume Integration Technique and a pressure based procedure with splitting of operators has been successfully applied in numerical solution of the full Navier-Stokes equations for an axi-symmetric transient pipe flow problem. In addition the compressibility equation has been successfully incorporated in the system of governing equations and in the numerical algorithm.

The valve closure phenomenon has been studied in detail using the numerical model. The characteristic oscillations, with respect to the axial direction, of both the velocity and the pressure field have been accurately predicted. In addition, it has been found that after full valve closure, there is a region, in the immediate vicinity of the valve, in which velocity field is fairly steady and the velocity oscillation is insignificant relative to the oscillation farther up-

stream.

The numerical method described in this paper provides solution in two dimensions and may be easily extended to provide solutions in three dimensions. Thus, it is a useful tool in situations where an insight into the multi-dimensional flow field structure in complex geometries is required, such as in design of hydraulic devices, and, therefore, it can be used to augment the solutions of the one-dimensional formulation^{(1)–(3)} of the governing equations for pipelines usually solved using the Method of Characteristics.

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