Numerical Analysis on a Flow Field of Liquid Metals Under a Magnetic Field, Using a Spectral Finite Difference Scheme*

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A steady-state axisymmetric flow field of a liquid metal in a coreless induction furnace under an axisymmetric magnetic field is analyzed numerically, using a spectral finite difference method. Vorticity-stream function formulation is used in conjunction with Maxwell's equations, in a boundary-fitted coordinate system. For boundary conditions, both no-slip on the wall and no shear stress tensor on the free surface are used as dynamic conditions, and a field equivalent to the magnetic field induced by external coils is adopted as an electromagnetic field condition. Presented are streamlines, magnetic streamlines, and radial profiles of the axial velocity component at two Reynolds numbers for various parameters. It is found that the flow field varies remarkably according to the Reynolds number, the dimensionless height of the liquid metal, and the dimensionless height of external coils.

Key Words: Spectral Method, Finite Difference, Liquid Metal, Induction Furnace, Streamlines, Magnetic Streamlines

1. Introduction

An induction furnace is popularly used as an important metal melting tool for both ferrous and nonferrous metals. The situation of flow stirring at the bath in an induction furnace is one of major concerns on the furnace designers and operators. Many analytical and experimental studies have been reported. In 1975 J. Szekely and K. Nakanishi⁽¹⁾ performed a numerical analysis on a flow field in the ASEA-SKF Furnace, based on a finite difference method. Using a finite difference method, both an infinitely long circular cylinder and a truncated circular cylinder had been analyzed and compared with a asymptotic solution by Y. R. Fautrelle⁽²⁾. Ch. Vivès and R. Ricou⁽³⁾ performed an experimental study in order to control a liquid metal flow in a single phase induction furnace. A. J. Meir et al.⁽⁴⁾, using a finite element method, calculated the flow of a molten metal in a cylindrical container rotating steadily and

compared to theoretical predictions.

On the other hand, spectral schemes have been used recently along with finite difference schemes and finite element schemes for a numerical analysis of hydrodynamic and magnetohydrodynamic phenomena. Although a spectral finite difference method (SFDM) is a kind of spectral methods, it has vastly different characteristics from a conventional spectral finite element method, i.e. very high computational speed, high resolution in space⁽⁵⁾. Many SFDM applications for a numerical simulation of heat and fluid flow are reported by Y. Mochimaru⁽⁶⁾. However, there have been very few previous SFDM applications to magnetohydrodynamics. In 1992 Y. Mochimaru^{(7),(8)} analyzed numerically a two-dimensional steady flow and temperature field around a circular cylinder placed normally to a uniform liquid metal flow with a uniform magnetic flux density perpendicular to the axis of the cylinder, using SFDM. The goal of this paper is numerical analysis on a steady-state axisymmetric flow field in a coreless induction furnace formed a truncated circular cylinder shape under an axisymmetric magnetic field, using a SFDM. Figure 1 shows a sketch of a typical coreless induction furnace.

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Fig. 1 Sketch of an induction furnace

2. Analysis

2.1 Basic equations

Steady-state characteristics of the flow of a liquid metal in a coreless induction furnace are sought with a time marching method. The equation of motion and the equation of continuity for a liquid metal can be expressed as

$$\rho \left\{ \frac{\partial}{\partial t} \mathbf{V} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right\} = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g}, \quad (1)$$

$$\nabla \cdot \mathbf{V} = 0, \quad (2)$$

respectively, where ρ : density, $\mathbf{V}$: velocity vector, p : pressure, $\boldsymbol{\tau}$: stress tensor, $\mathbf{g}$: acceleration vector of gravity. The following constitutive equation is adopted:

$$\tau_{ij} = \mu d_{ij} + H_i B_j - \frac{1}{2} g_{ij} H^k B_k, \quad (3)$$

where μ : viscosity, d_{ij} : rate of deformation tensor, H_i : magnetic field, B_i : magnetic flux density, and g_{ij} : metric tensor.

Maxwell's equations governing the electromagnetic field are expressed as

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (4)$$

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}, \quad (5)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (6)$$

respectively, where $\mathbf{E}$: electric field, $\mathbf{j}$: current density, $\mathbf{D}$: electric flux density. Furthermore, it is assumed that

$$\mathbf{B} = \mu_m \mathbf{H}, \quad (7)$$

where μ_m : magnetic permeability. In addition to the above, Ohm's law for a moving medium is used:

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}), \quad (8)$$

where σ is electrical conductivity.

2.2 Dimensionless forms of basic equations in a boundary-fitted conformal coordinate system

The following are assumed:

- (1) The field is laminar as the field is incompressible.

- (2) The gravity term is ignored.
- (3) The displacement current term $\partial \mathbf{D} / \partial t$ is ignored.
- (4) The physical properties (magnetic permeability and electrical conductivity) are constant.
- (5) The magnetic flux density on a boundary is a priori given.

Assuming the upper surface of a liquid metal is horizontal, the half region in the cross section as in Fig. 1 is mapped into a circular region in w using $\exp(\alpha + i\beta) \equiv w = (1 + if)/(1 - if)$ where f is an analytic function of $z + ir$ such that along the closed boundary f is real and varies monotonously from $-\infty$ to $+\infty$ with increasing β . In this case if the said domain is expressed by $-\infty < \alpha \leq 0$, $-\pi < \beta \leq \pi$, then dimensionless governing equations are given by

$$\begin{aligned} & \frac{J}{R^2} \frac{\partial \xi}{\partial t} + \frac{1}{R} \frac{\partial(\xi, \phi/r)}{\partial(R, \beta)} + \frac{1}{R} \frac{\partial}{\partial R} \left(\frac{1}{r} \frac{\partial r}{\partial \beta} \xi \frac{\phi}{r} \right) \\ & - \frac{1}{R} \frac{\partial}{\partial \beta} \left(\frac{1}{r} \frac{\partial r}{\partial R} \xi \frac{\phi}{r} \right) \\ & = \frac{1}{Re} \left\{ \left(\frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{1}{R^2} \frac{\partial^2}{\partial \beta^2} \right) \xi \right. \\ & + \frac{1}{R} \frac{\partial}{\partial R} \left(\frac{R}{r} \frac{\partial r}{\partial R} \xi \right) + \frac{1}{R^2} \frac{\partial}{\partial \beta} \left(\frac{1}{r} \frac{\partial r}{\partial \beta} \xi \right) \} \\ & + \frac{1}{R} \frac{\partial(\xi_B, \phi_B/r)}{\partial(R, \beta)} + \frac{1}{R} \frac{\partial}{\partial R} \left(\frac{1}{r} \frac{\partial r}{\partial \beta} \xi_B \frac{\phi_B}{r} \right) \\ & - \frac{1}{R} \frac{\partial}{\partial \beta} \left(\frac{1}{r} \frac{\partial r}{\partial R} \xi_B \frac{\phi_B}{r} \right), \end{aligned} \quad (9)$$

$$\begin{aligned} & - \frac{J}{R^2} \xi = \left(\frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{1}{R^2} \frac{\partial^2}{\partial \beta^2} \right) \left(\frac{\phi}{r} \right) \\ & + \frac{1}{R} \frac{\partial}{\partial R} \left(\frac{R}{r} \frac{\partial r}{\partial R} \frac{\phi}{r} \right) + \frac{1}{R^2} \frac{\partial}{\partial \beta} \left(\frac{1}{r} \frac{\partial r}{\partial \beta} \frac{\phi}{r} \right), \end{aligned} \quad (10)$$

$$\begin{aligned} & \frac{J}{R^2} \frac{\partial \xi_B}{\partial t} + \frac{1}{R} \frac{\partial(\xi_B, \phi/r)}{\partial(R, \beta)} + \frac{1}{R} \frac{\phi}{r} \frac{1}{r} \frac{\partial(\xi_B, r)}{\partial(R, \beta)} \\ & - \left\{ \frac{1}{R} \frac{\partial(\xi, \phi_B/r)}{\partial(R, \beta)} + \frac{1}{R} \frac{\phi_B}{r} \frac{1}{r} \frac{\partial(\xi, r)}{\partial(R, \beta)} \right\} \\ & + \frac{J}{R^2} \left\{ \left(R \frac{\partial v_a / \sqrt{J}}{\partial R} - \frac{\partial v_B / \sqrt{J}}{\partial \beta} \right) \right. \\ & \times \left(R \frac{\partial B_\beta / \sqrt{J}}{\partial R} + \frac{\partial B_a / \sqrt{J}}{\partial \beta} \right) \\ & - \left(R \frac{\partial v_B / \sqrt{J}}{\partial R} + \frac{\partial v_a / \sqrt{J}}{\partial \beta} \right) \\ & \times \left. \left(R \frac{\partial B_a / \sqrt{J}}{\partial R} - \frac{\partial B_\beta / \sqrt{J}}{\partial \beta} \right) \right\} \\ & = \frac{1}{Re \cdot P_m} \left\{ \left(\frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{1}{R^2} \frac{\partial^2}{\partial \beta^2} \right) \xi_B \right. \\ & + \frac{1}{R} \frac{\partial}{\partial R} \left(\frac{R}{r} \frac{\partial r}{\partial R} \xi_B \right) + \frac{1}{R^2} \frac{\partial}{\partial \beta} \left(\frac{1}{r} \frac{\partial r}{\partial \beta} \xi_B \right) \}, \end{aligned} \quad (11)$$

$$\begin{aligned} & - \frac{J}{R^2} \xi_B = \left(\frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{1}{R^2} \frac{\partial^2}{\partial \beta^2} \right) \left(\frac{\phi_B}{r} \right) \\ & + \frac{1}{R} \frac{\partial}{\partial R} \left(\frac{R}{r} \frac{\partial r}{\partial R} \frac{\phi_B}{r} \right) + \frac{1}{R^2} \frac{\partial}{\partial \beta} \left(\frac{1}{r} \frac{\partial r}{\partial \beta} \frac{\phi_B}{r} \right), \end{aligned} \quad (12)$$

$$J \equiv \frac{\partial(z, r)}{\partial(\alpha, \beta)}, \quad \frac{\sqrt{J}}{R} = \frac{2}{|f'| |1+w|^2}, \quad (13)$$

$$R \equiv \exp(\alpha), \quad (14)$$

where b : radius of the furnace, B_0 : reference magnetic flux density, $U \equiv B_0/\sqrt{\mu_m \rho}$, t : dimensionless time based on b/U , ξ : dimensionless vorticity based on U/b , ψ : dimensionless stream function based on bU , ζ_B : dimensionless magnetic vorticity based on B_0/b , ψ_B : dimensionless magnetic stream function based on bB_0 , v_α : dimensionless velocity component in the α -direction based on U , v_β : dimensionless velocity component in the β -direction based on U , B_α : dimensionless magnetic flux density component in the α -direction based on B_0 , B_β : dimensionless magnetic flux density component in the β -direction based on B_0 , $Re \equiv (B_0 b/\mu)\sqrt{\rho/\mu_m}$, $P_m \equiv \mu\sigma\mu_m/\rho$. In this case the magnetic stream function is defined as:

$$B_z = \frac{1}{r} \frac{\partial \psi_B}{\partial r}, \quad (15)$$

$$B_r = -\frac{1}{r} \frac{\partial \psi_B}{\partial z}, \quad (16)$$

where B_z : dimensionless magnetic flux density component in the z -direction based on B_0 , B_r : dimensionless magnetic flux density component in the r -direction based on B_0 . In general, for a given simply-connected cavity domain, f is not determined uniquely. For the current case, one possible concrete form is given by

$$f = \text{sn}\left\{-\frac{i}{h}(\tilde{z}-h)K'(k), k\right\}; \frac{1}{h} \frac{K'(k)}{K(k)} = 1; \quad (17)$$

$$\tilde{z} = z + ir,$$

where sn : Jacobian elliptic function, $K(k)$: complete elliptic integral of the first kind, $K'(k) \equiv K(\sqrt{1-k^2})$, h : dimensionless height of the liquid metal based on b .

2.3 Formulation by a spectral finite difference scheme

The following form of a solution is considered;

$$\xi = \sum_{n=1}^{\infty} \xi_n(R, t) \sin n\beta \quad (18)$$

$$\frac{\phi}{r} = \sum_{n=1}^{\infty} \phi_n(R, t) \sin n\beta \quad (19)$$

$$\zeta_B = \sum_{n=1}^{\infty} \zeta_{Bn}(R, t) \sin n\beta \quad (20)$$

$$\frac{\psi_B}{r} = \sum_{n=1}^{\infty} \psi_{Bn}(R, t) \sin n\beta \quad (21)$$

$$\frac{v_\alpha}{\sqrt{J}} = \sum_{n=0}^{\infty} v_{\alpha n}(R, t) \cos n\beta \quad (22)$$

$$\frac{v_\beta}{\sqrt{J}} = \sum_{n=1}^{\infty} v_{\beta n}(R, t) \sin n\beta \quad (23)$$

$$\frac{B_\alpha}{\sqrt{J}} = \sum_{n=0}^{\infty} B_{\alpha n}(R, t) \cos n\beta \quad (24)$$

$$\frac{B_\beta}{\sqrt{J}} = \sum_{n=1}^{\infty} B_{\beta n}(R, t) \sin n\beta \quad (25)$$

2.4 Boundary conditions

Both no-slip on the wall and no shear stress tensor on the free surface are considered as the velocity field condition. That is,

$$\frac{\phi}{r}(R=1, \beta, t) = 0 \quad (26)$$

$$\left(\frac{\xi}{Re} + B_\alpha B_\beta\right)_{(R=1, |\beta| < \frac{\pi}{2}, t)} = 0$$

$$\text{and } \frac{\partial}{\partial R}\left(\frac{\phi}{r}\right)_{(R=1, |\beta| \geq \frac{\pi}{2}, t)} = 0 \quad (27)$$

Equation (26) leads to

$$\phi_n(1, t) = 0 \quad (n \geq 1). \quad (28)$$

Treatment of a mixed type of boundary conditions is the same as in Ref. (9), so that Eq. (27) leads to

$$\pi \xi_n(R_1, t) + \frac{4}{(1-R_1)^2} \int_{\pi/2}^{\pi} \sin n\beta \left(\frac{1}{J} \frac{\phi}{r}\right)_{R=R_1} d\beta \\ + 2Re \int_0^{\pi/2} \sin n\beta (B_\alpha B_\beta)_{R=1} d\beta = 0, \quad (29)$$

where R_1 is the R -value at the adjacent grid point to the wall.

Magnetic field conditions are

$$B_\alpha(R=1, \beta, t) = \left(\frac{1}{\sqrt{J}} \frac{1}{r} \frac{\partial \psi_B}{\partial \beta}\right)_{R=1} \quad (30)$$

$$B_\beta(R=1, \beta, t) = -\left(\frac{1}{\sqrt{J}} \frac{1}{r} \frac{\partial \psi_B}{\partial \alpha}\right)_{R=1} \quad (31)$$

Equation (30) leads to

$$\pi n \phi_{Bn}(R=1, t) + 2 \int_0^{\pi} \cos n\beta \left(\frac{\phi}{r} \frac{1}{r} \frac{\partial r}{\partial \beta}\right)_{R=1} d\beta \\ = 2 \int_0^{\pi} \cos n\beta (\sqrt{J} B_\alpha)_{R=1} d\beta \quad (32)$$

Equation (31) can be replaced through Eq. (12) to get higher numerical stability by

$$\int_0^{\pi} \sin n\beta \left(\frac{J}{R^2} \zeta_B\right)_{R=R_1} d\beta + \frac{\pi}{h_0(h_0+h_1)} \phi_{Bn}(1, t) \\ - \frac{\pi}{h_0 h_1} \phi_{Bn}(R_1, t) + \frac{\pi}{h_1(h_0+h_1)} \phi_{Bn}(R_2, t) \\ + \int_0^{\pi} \sin n\beta \left\{ -\frac{2h_0+h_1}{h_0(h_0+h_1)} \left(\frac{\phi_B}{r}\right)_{R=1} \right. \\ \left. + \frac{h_0+h_1}{h_0 h_1} \left(\frac{\phi_B}{r}\right)_{R=R_1} - \frac{h_0}{h_1(h_0+h_1)} \left(\frac{\phi_B}{r}\right)_{R=R_2} \right\} \left(\frac{1}{r} \frac{\partial r}{\partial R}\right)_{R=1} d\beta \\ + \int_0^{\pi} \sin n\beta \left\{ \frac{\phi_B}{r} \frac{\partial}{\partial R} \left(\frac{1}{r} \frac{\partial r}{\partial R}\right) \right\}_{R=1} d\beta \\ = \int_0^{\pi} \sin n\beta (\sqrt{J} B_\beta)_{R=1} d\beta \\ + n \int_0^{\pi} \cos n\beta (\sqrt{J} B_\alpha)_{R=1} d\beta, \quad (33)$$

where R_2 is the R -value at the second adjacent grid point to the wall, $h_0 = R_1 - 1$, $h_1 = R_2 - R_1$. Both $(B_\alpha)_{R=1}$ and $(B_\beta)_{R=1}$ are a priori given.

2.5 Auxiliary conditions at $R=0$

Since the point $R=0$ is by no means a singular point, as a necessary condition the following hold:

$$\xi_n(0, t) = \phi_n(0, t) = \zeta_{Bn}(0, t) = \psi_{Bn}(0, t) = 0 \quad (n \geq 1) \quad (34)$$

$$v_{\beta n}(0, t) = B_{\beta n}(0, t) = v_{\alpha n}(0, t) = B_{\alpha n}(0, t) = 0 \quad (n \geq 1) \quad (35)$$

$$\frac{\partial}{\partial R} v_{\alpha 0}(0, t) = \frac{\partial}{\partial R} B_{\alpha 0}(0, t) = 0 \quad (36)$$

2.6 Numerical integration schemes

The Fourier series (18) - (25) can be truncated up to a suitable order, and Eqs. (9) - (12) can be split

into corresponding Fourier components to constitute a simultaneous partial differential equations in R and t , which can be integrated semi-implicitly with respect to time by starting from at rest with a uniform magnetic field [$\xi(R, \beta, 0) = \phi/r(R, \beta, 0) = \zeta_B(R, \beta, 0) = \phi_B/r(R, \beta, 0) = 0$ ($0 \leq R < 1$)] to get a steady-state solution.

3. Numerical Results

Mean magnetic vorticity ζ_{Bm} along the boundary is given by

$$\begin{aligned} \zeta_{Bm} &= \frac{1}{L_0} \int_0^\pi (\zeta_B \sqrt{J})_{R=1} d\beta \\ &\simeq \sum_{n=1}^{\infty} \zeta_{Bn}(R_1, t) \int_0^\pi \sin n\beta (\sqrt{J})_{R=R_1} d\beta, \end{aligned} \quad (37)$$

where

$$L_0 = \int_0^\pi (\sqrt{J})_{R=1} d\beta = 2 + h. \quad (38)$$

If a magnetic flux density on a boundary is uniform, a magnetic flux density in a liquid metal must be uniform using Maxwell's equations. In this case both the gradient of the magnetic flux density and the external force term in the steady-static equation of motion are all zero, so that a flow of a liquid metal don't occur. When a magnetic flux density changes on a boundary, a liquid metal flows in a furnace. Assuming that the influence of a liquid metal on an external magnetic flux density is ignored, variation of the magnetic flux density on a boundary is the most great at the edge of the furnace when the height of the liquid metal is equal to the height of coils that the wall of the crucible is surrounded by. Coils position are arranged so that boundary conditions of the magnetic flux density are given by

$$(B_r)_{R=1} = 0, \quad (39)$$

Fig. 2 Steady-state streamlines at $Re=500$, $P_m=4.58 \times 10^{-5}$, $h=1.5$; $\delta\phi=0.01$

$$(B_z)_{R=1} = \begin{cases} 0.5 & \text{at } 0 \leq \beta < \pi/2, \beta_0 < \beta \leq \pi \\ 1 & \text{at } \pi/2 \leq \beta \leq \beta_0, \end{cases} \quad (40)$$

where β_0 is the β -value of the point at $z=0$ and $r=1$.

In this study, the numerical calculation is performed at $M=30$ and $N=20$, where M : the number of grid points of R -direction and N : the number of Fourier coefficients. Figures 2, 3, and 4 show steady-state streamlines, magnetic streamlines, and a radial profile of the axial velocity component at $Re=500$, $P_m=4.58 \times 10^{-5}$ (molten aluminum), $h=1.5$ respectively. For a moderate Reynolds number case ($Re=10$, $P_m=4.58 \times 10^{-5}$, $h=1.5$), steady-state streamlines, magnetic streamlines, and a radial profile of the axial velocity component are shown in Figs. 5, 6, and 7

Fig. 3 Steady-state magnetic streamlines at $Re=500$, $P_m=4.58 \times 10^{-5}$, $h=1.5$; $\delta\phi_B=0.04$, $\zeta_{Bm}=-0.734$

Fig. 4 A radial profile of the axial velocity component at $Re=500$, $P_m=4.58 \times 10^{-5}$, $h=1.5$, $z=1.125$

Fig. 5 Steady-state streamlines at $Re=10$, $P_m=4.58 \times 10^{-5}$, $h=1.5$; $\delta\psi=0.001$

Fig. 6 Steady-state magnetic streamlines at $Re=10$, $P_m=4.58 \times 10^{-5}$, $h=1.5$; $\delta\psi_B=0.04$, $\zeta_{Bm}=-0.732$

respectively. The difference of the mean magnetic vorticity between Fig. 3 and 6 is very small because of the low magnetic Prandtl number. Therefore the magnetic field is not nearly changed as variation of the Reynolds number. In Figs. 2 and 5, the value of $\delta\psi$ increases as the Reynolds number increases, so that the high velocity is shown at the high Reynolds number. Configuration of the flow is changed as variation of Reynolds number. At the high Reynolds number, a rapid velocity variation near the furnace wall is shown through comparison of Figs. 4 and 7.

At a dimensionless liquid metal height h of 2 and

Fig. 7 A radial profile of the axial velocity component at $Re=10$, $P_m=4.58 \times 10^{-5}$, $h=1.5$, $z=1.125$

Fig. 8 Steady-state streamlines at $Re=500$, $P_m=4.58 \times 10^{-5}$, $h=2$; $\delta\psi=0.01$

a Reynolds number of 500 for molten aluminum, steady-state streamlines, magnetic streamlines, and a radial profile of the axial velocity component are shown in Figs. 8, 9, and 10 respectively. Figures 2, 4, 8, and 10 show that configuration of the flow field is changed as variation of h .

Figures 2 through 10 are calculated when the height of the liquid metal is equal to the height of coils that the wall of the crucible is surrounded by, while Figs. 11 through 13 are considered when the liquid metal height is the half of the coil height from the

Fig. 9 Steady-state magnetic streamlines at $Re=500$, $P_m=4.58 \times 10^{-5}$, $h=2$; $\delta\phi_B=0.04$, $\zeta_{Bm}=-0.931$

Fig. 10 A radial profile of the axial velocity component at $Re=500$, $P_m=4.58 \times 10^{-5}$, $h=2$, $z=1.5$

same bottom. In this case the most great variation of the magnetic flux density on a boundary occurs at the point of $z=0$ and $r=1$, so that boundary conditions of the magnetic flux density are considered:

$$(B_r)_{R=1}=0, \quad (41)$$

$$(B_z)_{R=1}=\begin{cases} 1 & \text{at } 0 \leq \beta < \beta_0 \\ 0.5 & \text{at } \beta_0 \leq \beta \leq \pi. \end{cases} \quad (42)$$

Figures 11, 12, and 13 show steady-state streamlines, magnetic streamlines, and a radial profile of the axial velocity component at $Re=500$ (above a critical Reynolds number), $P_m=4.58 \times 10^{-5}$, $h=1.5$ respective-

Fig. 11 Steady-state streamlines at $Re=500$, $P_m=4.58 \times 10^{-5}$, $h=1.5$; $\delta\phi=0.01$

Fig. 12 Steady-state magnetic streamlines at $Re=500$, $P_m=4.58 \times 10^{-5}$, $h=1.5$; $\delta\phi_B=0.04$, $\zeta_{Bm}=-0.399$

ly. Figures 2, 3, 4, 11, 12, and 13 show that both the flow field and the magnetic field are changed as variation of the dimensionless height of external coils.

Figure 14 shows variation of the axial velocity component with the Reynolds number at $z=3h/4$, $r=0.9$, $P_m=4.58 \times 10^{-5}$, the first magnetic boundary condition for a couple of dimensionless heights of liquid metal as a parameter.

4. Conclusions

It was demonstrated that, with proper transformation of governing equations and boundary condi-

Fig. 13 A radial profile of the axial velocity component at $Re=500$, $P_m=4.58 \times 10^{-5}$, $h=1.5$, $z=1.125$

Fig. 14 Relation between axial velocity components and Reynolds numbers at $z=3h/4$, $r=0.9$, $P_m=4.58 \times 10^{-5}$, the first boundary condition of the magnetic flux density

tions, it is possible to obtain steady-state flows of liquid metals in a vertical axisymmetric induction furnace, using a spectral finite difference method. Flow fields, magnetic fields, and radial profiles of the axial velocity component were presented in terms of

various parameters. It is found that the Reynolds number, the dimensionless height of the liquid metal, and the dimensionless height of external coils affect strongly the flow field, while the influence of the Reynolds number on the induced magnetic field is small due to the very low magnetic Prandtl number of the liquid metal.

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