Comparison Between Triangular Cylinder and Screen Near-Wakes in the Orthogonal Wavelet Representation*

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The near wake generated by a triangular cylinder and a screen, respectively, has been investigated with a view to provide information on the effect of initial conditions on turbulence structures of various scales based on the wavelet multi-resolution analysis. The velocity data were simultaneously obtained at x/h (x is the streamwise distance downstream of the wake generator and h is the height of the wake generator) $=20$ using a rake of eight X-wires aligned in the plane of mean shear. A vector wavelet multi-resolution technique was applied to decompose the velocity data into a number of wavelet components based on the central frequencies. The instantaneous sectional streamlines and vorticity field were used to reveal the turbulent structures at each wavelet component/central frequency. It was found that the behavior of large- and intermediate-scale structures depend on the initial conditions and the small-scale structures are independent of the initial conditions. The contributions from the wavelet components to the time-averaged Reynolds stresses and vorticity variance were estimated. Both the large-scale and intermediate longitudinal structures make the most significant contributions to Reynolds stresses in the triangular cylinder wake, but the contribution from the large-scale structures appears dominating in the screen wake. The relatively small-scale structures of the triangular cylinder wake contribute most to the total spanwise vorticity variance.

Key Words: Flow Measurements, Initial Conditions, Multi-Scale Flow Structures, Orthogonal Wavelet Transform, Turbulence, Near Wake, Vector Wavelet Multi-Resolution Analysis, Vortex

1. Introduction

It is now well established that the topology and transport characteristics of the turbulent structure in the wake depend on the wake-generating bodies or initial conditions. A number of investigators have studied the effect of the initial conditions on the behavior of a near turbulent wake. Screenivasan⁽¹⁾ found substantial differences in the manner wakes produced by different wake-generating bodies, even if the mean velocity profile exhibited the same shape. A

dependence of the normalized distributions of the longitudinal turbulence intensity on the initial conditions was examined by Wygnanski et al.⁽²⁾ Although Louchez et al.⁽³⁾ and Matsumura et al.⁽⁴⁾ have confirmed that the vortical behavior in the wake differs as the initial conditions differ; this difference has not been well documented. Zhou and Antonia⁽⁵⁾ studied turbulent near-wake vortices behind various wake-generating bodies and observed a significant variation in the flow structure and momentum transport characteristics of turbulent vortices with initial conditions. However, due to the limitation of their detection scheme, they could not provide any information on the dependence of organized structures other than the large-scale ones on the initial conditions. The question how the initial conditions produce an effect on the turbulent structure of various scales in a near turbulent wake is still open. It is suspected that the organized structures of various scales all contribute to the dependence of the turbulent topology and

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transport characteristics on the initial conditions.

In the past decade, there has been a growing interest in the use of wavelet analysis for turbulent structures^{(6),(7)}. This technique can track turbulent structures in terms of time and scale, and extracts new information on turbulence structures of various scales. The continuous wavelet transform has been applied to analyzing turbulent structures in terms of time and scale by Li^{(7),(8)}. This technique is capable of extracting continuously the characterization of local regularity, but it fails to reconstruct the original function from wavelet composition because of the non-orthogonal mother wavelet function. On the other hand, the discrete wavelet transform allows an orthogonal projection on a minimal number of independent modes and is invertible. Such analysis can produce a multi-resolution representation and might be also used to analyze turbulent structures of various scales. Meneveau⁽⁹⁾ first used the one-dimensional discrete wavelet transform to obtain local energy spectra and the flux of kinetic energy from experimental and direct numerical simulation data. Staszewski et al.⁽¹⁰⁾ identified the turbulent structures of the atmospheric boundary-layer using the discrete wavelet transform. Li et al.^{(11),(12)} employed discrete wavelet transforms to evaluate eddy structures of a jet in terms of time and scale. Li et al.^{(13),(14)} also applied the two-dimensional orthogonal wavelets to turbulent images, and extracted the multi-resolution turbulent structures. Farge et al.⁽¹⁵⁾ proposed a coherent vortex simulation method to decompose turbulent flows into coherent and incoherent structures based on orthogonal wavelets. To detect the spatial structures of various scales in unsteady flows, Li et al.^{(16),(17)} developed wavelet multi-resolution technique to analyzing the two-dimensional and stereoscopic PIV measurement results.

The present work aims to study the effect of initial conditions on the turbulent structures of various scales. Measurements were conducted in the near-wake generated by a triangular cylinder and a screen of 50% solidity, respectively. The vector wavelet multi-resolution analysis is used to decompose the measured velocity data into a number of wavelet components based on their central frequencies, which are representative of the turbulent structures of different scales. The flow structure of various scales is characterized by the sectional streamlines and vorticity contours of the decomposed wavelet components. The contributions to the time-averaged Reynolds stress and the vorticity variance from the motions of various scales are quantified and compared between the two wake generators.

2. Experimental Details

Experiments were conducted in an open-return low-turbulence wind tunnel with a 2.4 m-long working section (0.35 m × 0.35 m). The bottom wall was tilted to achieve a zero streamwise pressure gradient. Figure 1 shows the experimental arrangement. A triangular cylinder and a screen of 50% solidity with the same height ($h=12.5$ mm) were used to generate the wake, respectively; each was installed in the mid-plane and spanned the full width of the working section, 0.20 m from the exit plane of the contraction. This resulted in a blockage of about 3.6%. The stream-wise length of screen is about 0.5 mm. Measurements were made at $x/h=20$ (x is the streamwise distance downstream of the cylinder) and a constant free stream velocity ($U_\infty=6.7$ m/s). The corresponding Reynolds number $Re(=U_\infty h/\nu)$ was 5 600. The Kolmogorov length scale was estimated to be about 0.16 mm. The characteristic flow parameters for the two wake generators is summarized in Table 1, where Re_θ is the Reynolds number based on the momentum thickness, θ , L is the mean velocity half width, and U_1 is the centerline velocity defect.

A rake of eight X-wires, which was aligned in the (x, y) -plane, i.e. the plane of mean shear, was used to measure the velocity fluctuation u and v at probe locations. The spacing between adjacent X-wires was

Table 1 Characteristic wake parameters

Wake generator	h (mm)	θ (mm)	Re_h	Re_θ	L (mm)	U_1/U_∞ (%)	x/h	x/θ
Triangular	12.5	7.5	5,600	3,300	18	0.22	20	33
Screen	12.5	4.1	5,600	1,800	10	0.33	20	61

Fig. 1 Experimental arrangement

about 5 mm. Wollaston (Pt-10% Rh) wires, 5 μ m in diameter and about 1 mm in working length, were operated with constant temperature circuits. Signals from the circuits were offset, amplified and then digitized using two 16 channel (12 bit) A/D boards and two personal computers at a sampling frequency of $f_s=3.5$ kHz per channel (the cut-off frequency was 1 600 Hz). Using velocity and yaw calibrations, signals proportional to u and v , together with the local mean velocities $\bar{U}$ and $\bar{V}$ (≈ 0), were formed on a digital tape. The duration of each record was about 38 sec.

3. Vector Wavelet Multi-Resolution Analysis

The most basic time-domain description of a signal is the signal itself, but it gives no frequency or scale information. The wavelet transform can decompose the signal and provide a useful description of the signal in both time and frequency or scale. Scalar orthogonal wavelet transform and scalar multi-resolution analysis are introduced in detail in many references, such as Daubechies⁽¹⁸⁾. Here, we limit our discussion to the vector wavelet transform and vector multi-resolution analysis, which is used in the present work.

3.1 Definition of two-dimensional orthogonal vector wavelet transform

For a two-dimensional vector function $\vec{v}(x_1, x_2)$ and a wavelet basis $\Psi_{m;n_1,n_2}(x_1, x_2)$, the two-dimensional discrete vector wavelet transform is defined by

$$\vec{Dv}_{m;n_1,n_2} = \sum_i \sum_j \vec{v}(x_1^i, x_2^j) \Psi_{m;n_1,n_2}(x_1^i, x_2^j). \quad (1)$$

Here m is wavelet level, n_1 and n_2 the locations and $\vec{Dv}_{m;n_1,n_2}$ is called discrete vector wavelet transform coefficients. The two-dimensional orthogonal wavelet basis, $\Psi_{m;n_1,n_2}(x_1, x_2)$, is simply to take the tensor product functions generated by two one-dimensional orthogonal bases as

$$\Psi_{m;n_1,n_2}(x_1, x_2) = \frac{1}{2^m} \phi\left(\frac{x_1}{2^m} - n_1\right) \phi\left(\frac{x_2}{2^m} - n_2\right). \quad (2)$$

In the basis the two variables x_1 and x_2 are dilated separately. In the past ten years, various orthogonal wavelet bases have been constructed, for example, Meyer, Daubechies, Coifman, Battle-Lemarie, Baylkin, and spline. Different wavelet bases will preferentially move, between scales, different characteristics of the target data sets. Examples like the use of Harr basis or low order wavelet basis may emphasize discontinuous in the target data sets since they have good localization property in physical space. However, the high order wavelet bases may emphasize the smoothness of the analyzed data and wavelet coefficients of each level contain more information in physical space, because they have good localization

properties in frequency spaces.

Since the discrete vector wavelet transform is an orthogonal transform, it has the inverse transform and the original vector function can be reconstructed from the inversion of $\vec{Dv}_{m;n_1,n_2}$ as following:

$$\vec{v}(x_1, x_2) = \sum_m \sum_{n_1} \sum_{n_2} \vec{Dv}_{m;n_1,n_2} \Psi_{m;n_1,n_2}(x_1, x_2). \quad (3)$$

The discrete wavelet transform is linear operation that operates on the vector data whose length is an integer power of 2. The high level corresponds to small-scale or high-frequency. In general, the discrete vector wavelet transform can give a parsimonious representation of a vector function and can express the vector function in terms of a relatively small set of coefficients, which preserves the all features of the original vector function.

3.2 Definition of vector wavelet multi-resolution analysis

Since Mallat⁽¹⁹⁾ first applied the wavelet multi-resolution analysis to the field of image processing, researchers have been making widespread use of the wavelet multi-resolution analysis in various areas. The aim of the wavelet multi-resolution analysis is to get a representation of a signal or an image that is written in a parsimonious manner as a sum of its essential components.

As described in previous section, the discrete wavelet transform is an orthogonal and linear transform. Therefore, using Eq.(3) a non-linear vector function can be convert to an infinite summation of vector wavelet components at different central frequencies (from low frequency to high frequency), i.e.,

$$\vec{v} = \sum_{i=1}^N \vec{v}_i, \quad (4)$$

here

$$\vec{v}_i = \sum_{n_1} \sum_{n_2} \vec{Dv}_{i;n_1,n_2} \Psi_{i;n_1,n_2}(x_1, x_2) \quad (5)$$

where N represents the total number of wavelet levels. This method is called the vector wavelet multi-resolution analysis. In Eq.(4), $\vec{v}_1$ and $\vec{v}_N$ represent the wavelet components at level 1 (the lowest frequency) and level N (the highest frequency). The vector wavelet multi-resolution analysis is an orthogonal decomposition processing.

The vector wavelet multi-resolution analysis can be summarized in two steps:

(1) Wavelet coefficients are computed based on the discrete wavelet transform of Eq.(1).

(2) Inverse wavelet transform of Eq.(5) is applied to wavelet coefficients at each wavelet level, and wavelet components are obtained at each level or scale.

The sum of all wavelet components can reconstruct the original vector function. Therefore, the wavelet multi-resolution analysis may process fewer

Fig. 2 Daubechies wavelet with an index of 20

data by selecting the relevant details that are essential to perform an extraction of the multi-scale flow structures, and decompose the vector data in both Fourier and physical spaces. In this study, the Daubechies basis with an index of 20, as shown in Fig. 2, is used, which is not only orthogonal, but also has smoothness and compact support (i.e., be nonzero only over a finite region), as the orthogonal wavelet basis.

3.3 Wavelet decomposition of velocity and vorticity approximation

An instantaneous velocity component V_k can be decomposed as the sum of a time-averaging component $\bar{V}_k$ and a fluctuation component v_k , viz.

$$V_k = \bar{V}_k + v_k \quad (6)$$

where the subscript $k=1, 2$ and 3 represents the velocity component in the x -, y - and z -direction, respectively.

In order to gain insight into the turbulent structures of various scales, the wavelet multi-resolution technique is used to decompose the velocity fluctuation component v_k into a number of orthogonal wavelet components based on the wavelet levels or central frequencies, which are directly linked to the turbulent structure scales. Each wavelet component represents the turbulent structures of a certain range of frequencies (i.e. a non-zero frequency bandwidth). By using Eq.(4), v_k can be expressed as

$$v_k = \sum_{i=1}^N v_{i,k}, \quad (7)$$

where $v_{i,k}$ is the wavelet component of v_k at the i th wavelet level. Accordingly, the instantaneous wavelet component of velocity, Eq.(6), can be rewritten as

$$V_k = \bar{V}_k + \sum_{i=1}^N v_{i,k}. \quad (8)$$

The wavelet component of spanwise vorticity at the i th wavelet level may be approximated by

$$\omega_{zi} = \frac{\partial v_i}{\partial x} - \frac{\partial u_i}{\partial y} \approx \frac{\Delta v_i}{\Delta x} - \frac{\Delta u_i}{\Delta y}, \quad (9)$$

Apparently, velocity data at eight points in the (x, y) -plane may produce spanwise vorticity at each of the seven midpoints between adjacent X-wires. In

Table 2 Frequency characters of wavelet components

	Triangular cylinder			Screen		
Wavelet frequency	Wavelet level	Frequency bandwidth (Hz)	Central frequency (Hz)	Wavelet level	Frequency bandwidth (Hz)	Central frequency (Hz)
f_0	9	45-135	109	10	70-280	145
$2f_0$	10	80-270	218	11	150-550	290
$4f_0$	11	160-500	436	12	300-1150	580
$8f_0$	12	300-1100	872	13	600-1750	1160

Fig. 3 The v -spectrum at $y/h \approx 0.7$

Eq.(9), Δy (≈ 5.0 mm) is spacing between two X-wires in the (x, y) -plane; $\Delta x = -U_c \Delta t$. For simplicity, the average convection velocity $U_c = 0.87 U_\infty$ of large-scale vortical structures on the vortex path⁽²⁰⁾ is used to calculate Δx . Vorticity contours and rms values thus obtained showed no appreciable difference from those obtained using the local mean velocity.

3.4 Central frequencies and their bandwidth of wavelet components

Before the wavelet multi-resolution analysis is applied, a fundamental or dominant central frequency of turbulent structure has to be determined. This frequency is selected at the averaging frequency of Kármán vortices in this study. The power spectral density functions F_v of the v signals at $y/h=0.7$, which is approximately on the vortex path, are shown in Fig. 3. There is a significant difference in spectra between the two wake generators. It is pertinent to comment that F_v shows a pronounced peak at the frequency $f_0=109$ Hz for the triangular cylinder and 145 Hz for the screen wake, indicating the occurrence of Kármán vortices. Therefore, the well-known averaging vortex shedding frequency, f_0 , is specified as the fundamental central frequency.

For determining the frequency characters of each wavelet component, the v signals at $y/h=0.7$ are decomposed to thirteen wavelet levels or components based on the wavelet multi-resolution technique. The spectrum obtained from the Fourier analysis of each wavelet component displays a peak and spread over a range of frequencies. Figure 4 shows the spectra of wavelet level 9 for the triangular cylinder and wavelet level 10 for the screen, whose central frequency, identified with the most pronounced peak, corresponds

Fig. 4 Spectra of the wavelet components of v -signals at $y/h \approx 0.7$; (a) triangular, (b) screen

to the fundamental central frequency f_0 . Subsequently, the central frequency of wavelet level 10, 11 and 12 for the triangular cylinder and 11, 12 and 13 for the screen are given by $2f_0$, $4f_0$ and $8f_0$, respectively. The wavelet components from f_0 to $8f_0$ cover the range of frequencies that are of major concern in the present investigation. These central frequencies and their bandwidth are given in Table 2. Evidently, these central frequencies correspond to the scales of turbulent structures, and comparison between the turbulence structures of the comparable scale between the two wakes may be reduced to that between the wavelet components of the same multiple of f_0 .

4. Results and Discussion

4.1 Instantaneous turbulent structures of various scales

In this study, the sectional streamlines⁽²¹⁾, constructing for each wavelet component of velocity as well as for measured velocities, were used to reveal the instantaneous turbulent structures of various scales. Figure 5 shows the measured instantaneous sectional streamlines and the contours of normalized vorticity; $\omega_z h/U_\infty$, in the triangular cylinder and screen wakes in the (x, y) -plane. The same spatial scales have been used for the two flows to facilitate the comparison. The vertical axis is the spatial coordinate plotted in cylinder height units. The horizontal axis is the temporal coordinate plotted also in cylinder height units after converting the time displacements into spatial ones by using the average convection velocity of vortical structures. The same

Fig. 5 Measured instantaneous sectional streamlines: (a) triangular cylinder; (b) screen. Vorticity contours, $\omega_z h/U_\infty$: (c) triangular cylinder (Max: 2, Min: -1.6, increment: 0.4); (d) screen (Max: 1.2, Min: -1.2, increment: 0.4). The contour levels of $|\omega_z| h/U_\infty \leq 0.4$ have been removed. Solid and broken lines in Figs. 5(c) and 5(d) represent positive and negative contours, respectively.

scales have been also used in the x - and y -direction for avoiding the distortion of flow patterns. The timescale is increasing from left to right in such a way that the flow is observed moving from right to left. The positive and negative vorticities denote clockwise and counterclockwise vortices, respectively. The vortical structures exhibit striking difference in geometrical shape, size, and spacing between the two wake generators. The maximum strength of spanwise vorticity in the triangular cylinder wake is larger than that in the screen wake. However, only the large-scale structures are evident. It would be difficult to study the behaviors of the structures other than the large-scale ones for either flow.

Figures 6–9 present sectional streamlines and vorticity contours of different scales in the (x, y) -plane for the triangular cylinder and screen wakes, calculated from the wavelet components of velocity at the central frequencies of f_0 , $2f_0$, $4f_0$ and $8f_0$. Perry and Chong⁽²¹⁾ proposed to use the theory of critical points to describe eddy motions and flow patterns. Zhou and Antonia⁽²²⁾ successfully applied this theory to studying the topological details and three-dimensional aspects of a turbulent wake. Critical points, in particular, foci and saddle points, represent the major topological characteristics of the flow. In general, the foci of the streamlines coincide with the local vorticity peaks, suggesting that information about the flow structure can be obtained by examining either streamlines or

vorticity.

Figure 6(a) displays six vortical structures in the triangular wake, corresponding quite well to the large-scale structures in Fig. 5(a). The foci and saddle points, as denoted by F and S in Fig. 6, respectively, coincide approximately with those in Fig. 5 associated with large-scale structures. But the structures in Fig. 6 appear better organized and exhibit a much stronger periodicity. The observation indicates a correspondence between the wavelet component of the central frequency f_0 and the large-scale Kármán vortices. Figure 6(b) exhibits eight pairs of vortices in the screen wake. These structures correspond quite well to the large-scale structures in Fig. 5(b), and their occurrence is rather periodical. They are apparently the uppermost and energy-containing structures. Again, we see a great difference in vortical structures between the two wakes. The structures in the triangular cylinder wake are larger than those in the screen wake. Furthermore, the vortices in the screen wake are elliptical in shape. The maximum strength of spanwise vorticity in the triangular wake, however, is smaller than that in the screen wake. These differences reflect basic differences in the vortex formation mechanisms. The vortices in the triangular wake originate from the boundary layer separation from the triangular cylinder. On the other hand, those in the screen wake are likely to arise from the shear layer instability in the developing wake. While the

Fig. 6 Sectional streamlines of the wavelet component at f_0 : (a) triangular cylinder; (b) screen. Vorticity contours, $(\omega_z)_{f_0}h/U_\infty$: (c) triangular cylinder (Max: 0.45, Min: -0.4, increment: 0.05); (d) screen (Max: 0.65, Min: -0.65, increment: 0.1). The contour levels of $|(\omega_z)_{f_0}|h/U_\infty \leq 0.05$ have been removed. Solid and broken lines in Figs. 6(c) and 6(d) represent positive and negative contours, respectively.

former, when advected downstream, are characterized by a decaying strength, the latter may have an increasing strength in the near wake⁽²³⁾.

As the central frequency increases to $2f_0$, as shown in Fig. 7, we see the structures of higher frequency but smaller size than those in Fig. 6. Some of them are apparently associated with the large-scale vortices of f_0 (Fig. 6), for example, at $tU_c/h \approx -6$ and 1.8 in the triangular wake and at $tU_c/h \approx 8$ and 10.2 in the screen wake. Others correspond to the saddle

region between the large-scale vortices, such as at $tU_c/h \approx 4.6$ and 10 in the triangular wake and at $tU_c/h \approx -0.6$ and 11.6 in the screen wake. The latter structures are consistent with the occurrence of longitudinal or rib structures, which occur between successive spanwise structures^{(22),(24),(25)}. The size of the structure in the triangular cylinder wake appears larger than that in the screen wake.

Once the central frequency reaches $4f_0$ (Fig. 8), the structures of smaller scale appear all over the

Fig. 7 Sectional streamlines of the wavelet component at $2f_0$: (a) triangular cylinder; (b) screen. Vorticity contours, $(\omega_z)_{2f_0} h / U_\infty$: (c) triangular cylinder (Max: 0.5, Min: -0.7, increment: 0.1); (d) screen (Max: 0.5, Min: -0.5, increment: 0.1). The contour levels of $|(\omega_z)_{2f_0} h / U_\infty| \leq 0.1$ have been removed. Solid and broken lines in Figs. 7(c) and 7(d) represent positive and negative contours, respectively.

Fig. 8 Sectional streamlines of the wavelet component at $4f_0$: (a) triangular cylinder; (b) screen. Vorticity contours, $(\omega_z)_{4f_0} h / U_\infty$: (c) triangular cylinder (Max: 0.7, Min: -0.8, increment: 0.1); (d) screen (Max: 0.5, Min: -0.5, increment: 0.1). The contour levels of $|(\omega_z)_{4f_0} h / U_\infty| \leq 0.1$ have been removed. Solid and broken lines in Figs. 8(c) and 8(d) represent positive and negative contours, respectively.

Fig. 9 Sectional streamlines of the wavelet component at $8f_0$: (a) triangular cylinder; (b) screen. Vorticity contours, $(\omega_z)_{8f_0}h/U_\infty$: (c) triangular cylinder (Max: 0.9, Min: -0.9, increment: 0.2); (d) screen (Max: 0.6, Min: -0.7, increment: 0.1). The contour levels of $|(\omega_z)_{8f_0}h/U_\infty| \leq 0.1$ have been removed. Solid and broken lines in Figs. 9(c) and 9(d) represent positive and negative contours, respectively.

flow. The structures in the triangular wake appear considerably larger than those in the screen wake. The structures appear to have positive and negative vorticity concentrations aligned periodically along the flow direction. This periodicity appears to be stronger in the screen wake than in the cylinder wake.

At the central frequency of $8f_0$, the vorticity concentrations (Fig. 9) appear in strips, running spanwise. We see the wavy patterns of structures. However, the difference in the structure appears diminishing between the two wakes. This is consistent with the perception that small-scale structures may be less dependent on the initial conditions.

4.2 Contributions to the Reynolds stresses and vorticity variance from different wavelet components

The wavelet components of the time-averaged Reynolds stress and vorticity variance (or mean square vorticity), $\overline{u_i^2}$, $\overline{v_i^2}$, $\overline{u_i v_i}$ and $\overline{\omega_i^2}$, represent the contributions from the turbulent motions of various scales to the Reynolds stresses and vorticity. They are calculated from the wavelet components of instantaneous velocity and vorticity and compared between different central frequencies.

4.2.1 Reynolds stresses Figure 10 presents the lateral distributions of $\overline{u_i^2}$, $\overline{v_i^2}$ and $\overline{u_i v_i}$, as compared with the measured values for the triangular cylinder and screen wakes. The results are normalized by the maximum value of the measured $(\overline{fg})_{\max}$, where f or g each represents u or v , so as to indicate the contribution from each wavelet level or central frequency to the Reynolds stresses. The negative $\overline{u_i v_i}$ /

$(\overline{uv})_{\max}$ near $y/h=0$ and at relatively high central frequencies has been removed in Fig. 10(c) to allow the log-scale presentation. The $(\overline{fg})_{\max}$ values (Table 3) show discernible difference between the triangular cylinder and screen wakes, in particular for $(\overline{u^2})_{\max}$ and $(\overline{v^2})_{\max}$. The observation is in consistency with the results of Zhou & Antonia⁽⁵⁾, suggesting the effect of different generators. The distributions of $\overline{f_i g_i}/(\overline{fg})_{\max}$ are similar to that of the measured data for both triangular cylinder and screen wakes, varying greatly with the central frequency. For $y/h > 1.2$, $\overline{f_i g_i}/(\overline{fg})_{\max}$ of the screen wake is smaller than that of the triangular cylinder wake and tends to fall off quickly, approaching zero. The $\overline{f_i g_i}/(\overline{fg})_{\max}$ value of f_0 in the screen wake is larger than that in the triangular cylinder wake for $y/h < 1.0$. The observation suggests that the coherent contribution from the large-scale vortical structures to the Reynolds stresses is larger in the porous body near-wake than in the solid body near-wake. The result again confirms the conception that the large-scale vortical structures contribute to the effect of initial conditions on the near wake. Furthermore, the difference between the two wakes is appreciable down to the wavelet components of $4f_0$, suggesting that the intermediate-scale structures also play a role in the persistence of the initial conditions. In general, the value of $\overline{f_i g_i}/(\overline{fg})_{\max}$ decreases as the central frequency increases, being consistent with the perception that the lower frequency eddies are energy-containing. For the central frequency greater than $4f_0$, $\overline{f_i g_i}/(\overline{fg})_{\max}$ tends to fall off quickly, approaching zero.

Table 3 Maximum values of $\overline{u^2}/U_\infty^2$, $\overline{v^2}/U_\infty^2$, $\overline{uv}/U_\infty^2$ and $\overline{\omega^2 h^2}/U_\infty^2$

Wake generator	$\overline{u^2}/U_\infty^2$	$\overline{v^2}/U_\infty^2$	$\overline{uv}/U_\infty^2$	$\overline{\omega^2 h^2}/U_\infty^2$
Triangular	1.21×10^{-2}	1.29×10^{-2}	-3.31×10^{-3}	2.37×10^{-7}
Screen	4.9×10^{-3}	3.92×10^{-3}	-2.18×10^{-3}	1.03×10^{-7}

Fig. 10 Velocity variance and shear stress of the measured and the wavelet components at various central frequencies: (a) $\overline{u^2}/(\overline{u^2})_{\max}$ and $\overline{u_i^2}/(\overline{u^2})_{\max}$; (b) $\overline{v^2}/(\overline{v^2})_{\max}$ and $\overline{v_i^2}/(\overline{v^2})_{\max}$; (c) $\overline{uv}/(\overline{uv})_{\max}$ and $\overline{u_i v_i}/(\overline{uv})_{\max}$.

In the case of the triangular cylinder wake, $\overline{u_i^2}/(\overline{u^2})_{\max}$ and $\overline{v_i^2}/(\overline{v^2})_{\max}$ of $2f_0$ are very close to that of f_0 , implying that energies associated with the two central frequencies are quite comparable. As a matter of fact, the former even exceeds, though slightly, the latter near the centreline. This is because the lateral spacing between the two rows of vortices is small in the near-wake and thus the most prominent peak in the fluctuating velocity spectra occurs at the second harmonic of the vortex shedding frequency near the centreline. Furthermore, the wavelet components at f_0 and $2f_0$ are most energy-containing; they account for almost 38%, 58% and 44% of $(\overline{u^2})_{\max}$, $(\overline{v^2})_{\max}$ and $(\overline{uv})_{\max}$, respectively. This indicates a significant contribution from the intermediate-scale structures to the total Reynolds stresses. The value of the wavelet component appears falling off rather rapidly, up to about 15% of the measured value for $\overline{v_i^2}/(\overline{v^2})_{\max}$ when the central frequency increases from $2f_0$ to $4f_0$.

For the screen wake, $\overline{f_i g_i}/(\overline{fg})_{\max}$ of f_0 remains everywhere largest value and accounts for almost 40%, 29% and 40% of $(\overline{u^2})_{\max}$, $(\overline{v^2})_{\max}$ and $(\overline{uv})_{\max}$, respectively. The decrease is rather rapid, up to 17–22% of the measured $(\overline{fg})_{\max}$, when the central frequency increases from f_0 to $2f_0$. This indicates that the greatest contribution to the total Reynolds stresses comes from the large-scale structure in the screen wake.

The above observation indicates that the most significant contribution to the Reynolds stresses comes from the turbulence structures of f_0 and $2f_0$ in the triangular cylinder wake. However, in the screen wake, the contribution to Reynolds stresses comes primarily from the turbulence structures of f_0 ; the role played by those of $2f_0$ appears to be less significant than that in the triangular wake. This is a significant difference between the two wakes.

It is pertinent to comment that $\overline{v_i^2}/(\overline{v^2})_{\max}$ (58%) corresponding to f_0 and $2f_0$ overwhelms $\overline{u_i^2}/(\overline{u^2})_{\max}$ (38%) in the triangular wake, which is similar to the behaviour of the coherent contribution from the large-scale structures in solid bluff-body wakes⁽²²⁾. In contrast, $\overline{u_i^2}/(\overline{u^2})_{\max}$ (40%) at f_0 exceeds $\overline{v_i^2}/(\overline{v^2})_{\max}$ (29%) in the screen wake, resembling the behavior of the coherent contribution from the large-scale structures in a turbulent near-wake⁽⁵⁾. The different behavior between the large-scale structures in the two wakes corroborates the proposition that their vortex generation mechanisms are not the same.

4.2.2 Vorticity variance Figure 11 presents the normalized spanwise measured vorticity variance, $\overline{\omega_z^2}/(\overline{\omega_z^2})_{\max}$, and the wavelet components, $\overline{\omega_{zi}^2}/(\overline{\omega_z^2})_{\max}$, where $(\overline{\omega_z^2})_{\max}$, as shown in Table 3, is the maximum value of the measured vorticity variance. The varia-

Fig. 11 Spanwise vorticity variance of the measured and the wavelet components at various central frequencies. The symbols are as in Fig. 10.

tion of $\overline{\omega_{zi}^2}/(\overline{\omega_z^2})_{\max}$ with y/h is qualitatively similar to $\overline{\omega_z^2}/(\overline{\omega_z^2})_{\max}$. Both $\overline{\omega_z^2}/(\overline{\omega_z^2})_{\max}$ and $\overline{\omega_{zi}^2}/(\overline{\omega_z^2})_{\max}$ drop towards the free-stream. The drop is particularly fast for the screen wake. Interestingly, $\overline{\omega_{zi}^2}/(\overline{\omega_z^2})_{\max}$ of the triangular cylinder corresponding to f_0 is now smallest, accounting for about 10% of the total vorticity variance. This indicates a relatively small contribution to the total vorticity variance from the large-scale structures. This contribution increases considerably at $2f_0$, up to about 16%, and continues to rise as the central frequency increases, reaching the maximum, about 24%, at $8f_0$. Evidently, the intermediate- and relatively small-scale structures make a major contribution to the total vorticity variance. In contrast to the triangular wake, the difference in $\overline{\omega_{zi}^2}/(\overline{\omega_z^2})_{\max}$ between different central frequencies is smaller for the screen wake. The value of $\overline{\omega_{zi}^2}/(\overline{\omega_z^2})_{\max}$ at f_0 even reaches largest near the centerline, accounting for about 25%, though giving way to the component at $4f_0$ for $y/h \geq 0.5$. This could be because vorticity is generated under the effect of the shear layer instability in the wake, irrespective of its scales. On the other hand, in the triangular wake, the vorticity of larger scales, typically at f_0 , originates from the boundary layer separation from the triangular cylinder, while the vorticity of other scales could be due to the interactions between the large-scale structures or the effect of the shear layer instability in the wake, as in the screen wake. The observation again reflects the basic difference in the vortex formation mechanisms between the triangular cylinder and screen wakes.

5. Conclusions

(1) The vortical structures at the central frequency of f_0 are consistent with the conditionally averaged large-scale structures for both wake genera-

tors⁽⁶⁾. These structures, along with those of intermediate-scales exhibit a strong dependence upon the initial conditions. However, the small-scale structures at a central frequency more than $8f_0$ are less independent of the initial conditions.

(2) The contributions to the Reynolds stresses are comparable from the turbulent structures of f_0 and $2f_0$ in the triangular cylinder wake, their combined contribution accounting for 38%, 58% and 44% to $(\overline{uv})_{\max}$, $(\overline{u^2})_{\max}$ and $(\overline{v^2})_{\max}$, respectively. In the screen wake, however, the structures of f_0 make a contribution considerably higher than other wavelet components, responsible for almost 40%, 29% and 40% to $(\overline{uv})_{\max}$, $(\overline{u^2})_{\max}$ and $(\overline{v^2})_{\max}$, respectively.

(3) For the triangular cylinder wake, the relatively small-scale structures of $8f_0$ contribute most, about 24%, to the spanwise vorticity variance. On the other hand, the large-scale structures contribute only 10% to $(\overline{\omega_z^2})_{\max}$. This is different in the screen wake. The contribution to the vorticity variance from the structures of f_0 is quite comparable to that of other components.

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