The Use of Inverse Methods for the Design and Control of Radiant Sources*

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Design methods for thermal systems with predominant radiation are given, followed by applications of these methods to idealized but practical engineering systems. It is argued that such designs in general present inverse mathematical problems, in that an outcome (the desired output of the systems) is prescribed, and the necessary inputs (geometry, heater placement, heater power distribution) are to be found that will achieve the desired output. Such inverse problems require some methods for handling their ill-conditioned nature. Two general techniques are discussed: Regularization methods, which remove or ameliorate the ill-conditioned portion of the problem at the expense of some degree of accuracy; and optimization methods, which replace the ill-conditioned problem with a well-posed problem that must be solved repetitively through a systematic approach to a useful solution. Applications of both methods to a variety of radiative transfer problems are discussed and demonstrated, including problems in which heater power is determined, problems in which the geometry of the enclosure must be determined, and problems with a prescribed transient power distribution on the processed material that must be provided by the heaters. Problems with conduction and/or convection in addition to radiation are also discussed, as are problems with specularly reflecting surfaces, participating media between the heaters and the processed materials, and enclosures with complex geometries.

Key Words: Inverse, Radiation, Design, Control, Participating Media

1. Introduction

Inverse analysis implies the need for the prediction of the necessary conditions in some region of a system (boundary conditions, energy source strengths, etc.) that will result in prescribed or observed conditions over another region of the system.

The interest in inverse solutions historically arose because experimental observations of temperature or heat flux were sometimes unavailable at the physical location where they were needed. These quantities then had to be inferred from measurements at a more accessible location. These problems tend to be mathematically ill-conditioned, and the prediction of the conditions on the remote boundary can result in

multiple solutions, physically unrealistic solutions, or solutions with unexpected oscillations in space and time. Treatment of this type of problem has resulted in the development of a number of methods for overcoming the ill conditioned nature of the equations describing inverse solutions. For problems dominated by conduction, there are texts and monographs available that demonstrate many of these solution methods⁽¹⁾⁻⁽⁴⁾. Tikhonov⁽⁵⁾ is often credited with developing the first systematic treatment of inverse problems of this type.

Another application of inverse methods is to the *design and control* of thermal systems. In these problems, the designer specifies the expected output of the thermal system being designed (e.g., the required temperatures and heat fluxes on an object to be heat treated, dried, cooked, cured, annealed, subjected to CVD, or otherwise thermally processed), and wishes to find the necessary inputs to the thermal system that will provide this desired output. The unknown conditions may be the required steady or transient temperature and energy input distributions to heaters or

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burners; heater or burner locations; or oven or furnace geometry. Such problems result in equation sets that are mathematically ill-conditioned. This occurs because multiple conditions are specified on some parts of the system being designed (the design portion), and the unknown conditions on other parts of the system are to be determined. In addition, the number of unknowns in the design may be less than, equal to or greater than the number of equations that describe the system. The traditional solution strategy has been to use trial and error solutions with guidance by experience to attain a viable solution.

For thermal systems dominated by radiative transfer (Fig. 1) the problem is complicated because the thermal input at any location on the design portion is affected by all radiant sources in the system. Thus, the mathematical form of the inverse solution is a set of integral equations that must be solved simultaneously. Some of these equations will be Fredholm equations of the first kind, which are notoriously ill-conditioned. This implies that control of distributed radiative sources may be tricky, especially where a transient recipe is prescribed for the temperature and heat flux distributions on the design surface.

Recent review papers discuss inverse methods for radiative transfer devoted to analysis of experimental data⁽⁶⁾ and to earlier work in design and control^{(6),(7)}. In this review, we concentrate on ongoing research on thermal design and control in systems with predominant radiation using both inverse analysis and optimization methods.

Nomenclature

- A : matrix of coefficients
 A : area
 $\mathbf{b}$: vector of known coefficients
 B : approximate Hessian matrix
 $C_i(\Phi)$: design constraints

Heater surface
 Design surface
 Re-radiating surface

Fig. 1 Radiant furnace with design surface 2 [prescribed $T(x_2, t)$ and $q(x_2, t)$], heater surfaces 5–7 with unknown distributed inputs $q(x_5, t)$ and unknown temperature $T(x_5, t)$, adiabatic surfaces 1, 3, 4, 8

- E : emissive power
 $F(\Phi)$: objective function to be minimized
 $\bar{F}$: exchange factor
 H : Hessian matrix
 N : number of points or increments
 $\mathbf{p}$: search direction
 q : net radiative heat flux
 $\mathbf{r}$: residual vector
 T : absolute temperature
 t : time
 u : location along enclosure boundary
 $\mathbf{U}, \mathbf{V}$: orthogonal vectors
 V : volume
 $\mathbf{x}$: solution vector
 x : location along design surface
 α : optimization step size; Tikhonov regularization parameter
 Φ : set of optimized design parameters
- Subscripts
 B : blackbody
 DS : design surface
 HS : heater surface
 i, j, k : surface or volume element index
 k : iteration number
 n : time increment index
 s : at radiative source (heaters)

2. Methods

2.1 Regularization

A direct or explicit solution to the design problem requires use of an inverse formulation. Such inverse problems are ill-posed, and the discretized set of equations for a numerical solution is ill conditioned. Ordinary techniques (e.g., Gauss-Seidel, Gauss elimination or LU decomposition) may result in non-physical solutions with high amplitude fluctuations and/or complex absolute temperatures.

To achieve an accurate and reasonable solution, the explicit system must be *regularized* by modifying the ill conditioned system. The solution is then subject to some error and the level of regularization must be selected so that the accuracy of the solution satisfies the application's needs.

For a system of equations such as $A\mathbf{x}=\mathbf{b}$, regularization is usually performed following one of two approaches:

1. Modifying the ill-conditioned coefficient matrix that defines the system to a well conditioned one.
2. Modifying the system to include a shape constraint for the solution.

Before considering regularization techniques, the ill conditioned behavior of the system can be examined using singular value decomposition (SVD)⁽⁸⁾. The SVD of an arbitrary $M \times N$ matrix A

is $A = USV^T$, where U and V are orthogonal matrices and S is a diagonal matrix of singular values, so that $S_{1,1} > S_{2,2} > \dots > S_{N,N} \geq 0$. The inverse matrix is $A^{-1} = VS'U^T$, where $S'_{i,i} = 1/S_{i,i}$. If the condition number of this matrix ($S_{1,1}/S_{N,N}$) is large, small singular values dominate the inverse matrix and the solution becomes unstable.

The most popular regularization techniques are *truncated singular value decomposition* (TSVD), *conjugate gradient method* (CGM) and *Tikhonov regularization* (TR).

TSVD is based on SVD. The solution uses the pseudo-inverse matrix that is formed by filtering or truncating small singular values. The solution using the p largest singular values becomes

$$x_n = \sum_{k=1}^p V_{n,k} \frac{b_m U_{m,k}}{S_{k,k}} \quad (n=1, 2, \dots, N) \quad (1)$$

where p has a value smaller than the rank of $A^{(9)}$. Retaining different numbers of singular values yields alternative solutions. Those with acceptable accuracy present alternatives to the designer.

The L-curve displays the variation of the residual norm against the solution norm, and is used to select an appropriate solution. The change in residual norm represents the change in accuracy while change in solution norm represents the change in the smoothness of the solution.

The typical trend of the L-curve for a discrete ill-posed system is displayed in Fig. 2. The L-curve has a corner where further regularization significantly increases the norm of the residual. The optimal or desired solution usually lies close to the corner.

The CGM aims at minimizing the functional $F(x) = [A(x_{\text{exact}} - x)] \cdot (x_{\text{exact}} - x)$, and has a minimum where the residual vector equals to zero. For inverse problems, the solution achieved after each step provides a unique solution with different accuracy and smoothness characteristics. The N different solutions can then be used to construct an L-curve to select the

Fig. 2 Typical L-curve for a discrete ill-posed system

optimal solution. Small memory needs for storing intermediate results, computation economy, robust convergence characteristics, and the ability to store and use the original matrix makes CGM the method of choice for large systems^{(4),(10)}.

Tikhonov's regularization scheme modifies the equation system by including a shape constraint. The objective function to be minimized is

$$F(x) = [\|Ax - b\|]^2 + \sum_{i=1}^p \alpha_i^2 [\|L_i(x - x_0)\|]^2 \quad (2)$$

For a p -th order scheme, L_i approximates the discretized i -th derivative operator and α_i is the i -th order regularization parameter. Using a small regularization parameter will result in an accurate solution by putting the emphasis on minimizing the norm of the residual. Using a large regularization parameter will result in a solution with improved smoothness characteristics. For the standard or zeroth order TR scheme, $p=0$ and L_i becomes the identity matrix I , leading to the set of linear equations $(A^T A + \alpha_0^2 I)x = A^T b + \alpha_0^2 Ix_0$. Using the correct regularization parameter achieves an optimal solution that is smooth and accurate enough for the designer's needs. Solutions with different regularization levels can be used to construct an L-curve, and the required regularization parameter can be selected accordingly.

In a forward problem one boundary condition, either the temperature or the heat flux/source term, is specified for every surface or medium element. In many inverse problems, there are elements in the system where both of the boundary conditions are specified (design elements), there are elements where no condition is specified (heater or burner elements), and there are others where only one condition is specified.

For a transient system, the energy equation for a design surface element is:

$$\begin{aligned} & \left[\sum_{j=1}^{N_V} E_j^n V_j \tilde{F}_{j-i} + \sum_{j=1}^{N_S} E_j^n A_j \tilde{F}_{j-i} \right]_{T^n \text{ unknown}} \\ &= \rho_i c_{p,i} \delta A_i \frac{T_i^{n+1} - T_i^n}{\Delta t} + E_i^n A_i - q_{\text{cond}_i}^n - q_{\text{conv}_i}^n \\ & - \left[\sum_{j=1}^{N_V} E_j^n V_j \tilde{F}_{j-i} + \sum_{k=1}^{N_S} E_k^n A_k \tilde{F}_{k-i} \right]_{T^n \text{ known}} \quad (3) \end{aligned}$$

For a system in steady state, the $(T_i^{n+1} - T_i^n)$ term becomes zero. Equation (3) is in terms of exchange factors from element i to j , $\tilde{F}_{i-j}$. These represent the rate of energy emitted by element i that is absorbed by element j including intermediate scattering or reflection paths. Exchange factors can be used for problems with participating media and/or specularly reflecting surfaces. The exchange factors for the system must usually be calculated only once unless there are moving objects, or the geometry of the system changes.

All the terms on the right hand side of Eq. (3) are

known and the terms on the left hand side are not. It is thus a discretized ill-conditioned Fredholm equation of the first kind and a system that includes these must be regularized to achieve a reasonable solution.

2.2 Optimization

Like conventional design, optimization techniques use an iterative process to arrive at the final design configuration. The performance of a particular design configuration is evaluated at each iteration. If it does not satisfy the design requirements, the configuration is modified and checked again. This process is repeated until a satisfactory design configuration is identified.

The efficiency of this process and the quality of the final design depend on how much the design performance improves at each iteration. Optimization techniques modify the design configuration based on sensitivity information and sophisticated numerical algorithms that maximize the improvement between successive iterations. Consequently, optimization techniques require far fewer iterations than the forward design methodology, and the final solution is usually near optimal.

The first step of the optimization technique is to specify a set of n design parameters, Φ , which control the design configuration. Next, an objective function, $F(\Phi)$, that quantifies the merit of a particular design is defined such that the minimum of $F(\Phi)$ corresponds to the ideal design. Design constraints, $c_i(\Phi)$, which define the domain of the design parameters may be imposed. Once the problem is posed in this form, the goal of the analysis is to identify the set of design parameters Φ^* , where

$$F(\Phi^*) = \text{Min}(F(\Phi)), \quad \Phi \in \mathbb{R}^n \quad (4)$$

subject to

$$c_i(\Phi^*) = 0, \quad i = 1 \dots m' \quad (5)$$

and

$$c_i(\Phi^*) \leq 0, \quad i = m' + 1 \dots m. \quad (6)$$

Consider the enclosure shown in Fig. 3. The goal is to find the enclosure configuration that results in a uniform heat flux of q_s^{target} over the design surface. The design parameters in Φ specify the enclosure geometry and heat flux distribution over the heater surface. The objective function is defined as the variance of the realized heat flux distribution from the desired heat flux distribution, evaluated at N_{DS} discrete points over the design surface,

$$F(\Phi) = \frac{1}{N_{DS}} \sum_{j=1}^{N_{DS}} [q_{sj}(\Phi) - q_{sj}^{\text{target}}]^2. \quad (7)$$

Constraints are used to restrict the dimensions of the enclosure geometry and to ensure that the heat flux distribution over the heater surface remains strictly non-negative.

Gradient-based tools are most often employed to

Fig. 3 Formulation of a radiant enclosure optimization problem

minimize smooth objective functions, and work according to the following scheme: at the k th iteration, the set of design parameters Φ_k is checked to see if it minimizes $F(\Phi)$, usually by seeing if $|\nabla F(\Phi_k)| \approx 0$. If not, a search direction, $\mathbf{p}_k$, is selected based on the curvature of $F(\Phi)$ at Φ_k . Once a search direction has been chosen, a scalar step size, α_k , is found. Finally, a new set of design parameters is formed by taking a “step” in the $\mathbf{p}_k$ direction,

$$\Phi_{k+1} = \Phi_k + \alpha_k \mathbf{p}_k \quad (8)$$

This process continues until $\Phi_k \approx \Phi_{k-1}^{(11),(12)}$.

The performance of gradient-based minimization methods depends on how the search direction is chosen. One of the simplest methods is steepest descent, where $\mathbf{p}_k$ is set equal to the negative of the gradient,

$$\mathbf{p}_k = -\nabla F(\Phi_k), \quad (9)$$

the direction of steepest descent. Many iterations are required to reach Φ^* , and accordingly this technique is rarely used.

Newton's method relies both on the first- and second-order curvature information to choose a search direction. Newton's direction is found by solving

$$H(\Phi_k) \mathbf{p}_k = -\nabla F(\Phi_k), \quad (10)$$

where the Hessian matrix $H(\Phi_k)$ contains the second-order objective function sensitivities, i.e. $H_{pq}(\Phi_k) = \partial^2 F(\Phi_k) / \partial \Phi_p \partial \Phi_q$.

The quasi-Newton method uses first-order curvature information “built-up” over successive iterations to form a matrix $\mathbf{B}_k$ which approximates the Hessian matrix. The search direction is then found by solving

$$\mathbf{B}_k \mathbf{p}_k = -\nabla F(\Phi_k) \quad (11)$$

Quasi-Newton methods require more iterations to reach Φ than Newton's method, but often require less computational effort.

3. Applications: (Using Regularization)

3.1 Wafer RTP design

At this point the question arises: Are inverse approaches able to accurately design a real thermal

process? An experimental validation is necessary, where a real thermal system is modeled and the boundary conditions are predicted using these methods to achieve prescribed conditions on the design surface.

Such a study was done for a steady state problem with dominant radiative heat exchange. The experimental apparatus is a radiatively heated chamber designed to reproduce the main characteristics of rapid silicon wafer processing. The chamber is a test bed developed by the University of Texas, NIST, the SensArray Corporation and International SEMATECH^{(13),(14)} for calibration of commercial temperature sensors. The apparatus is an axisymmetric radiatively heated chamber composed of a cylindrical stainless steel, water cooled vacuum vessel that contains the design surface (in this case, the silicon wafer to be processed) and the heating elements, composed of three independently powered concentric zones, plus radiation shields placed on the bottom, top and sides. The details of the chamber are in Refs. (13)–(15).

Optical properties (reflectivity and specular to diffuse ratio) were measured for the molybdenum radiation shields and silicon wafer. For the other components, standard optical properties were assumed.

The system was instrumented to map temperature fields on the wafer and shields.

The goal is to predict the power input to the innermost and outermost heating zones (the middle zone is rarely used in the particular experimental apparatus) needed to keep the silicon wafer at a uniform temperature. The radiative heat transfer was modeled using the Monte Carlo method to handle the complicated geometries, shading effects and spectral and directional dependencies of the optical properties. The Conjugate Gradient method is used for solution.

First, a temperature of 873 K was imposed on the wafer and the necessary heater power inputs were calculated. The optimal solution was 168 W for the inner zone and 508 W for the outer zone. When this power was applied to the physical system, a steady state wafer temperature of 882.4 K was achieved, with a temperature uniformity of ± 6.3 K. The experimental error is ± 1.0 K. The measured temperatures of the shields compared well with the predicted temperatures, showing a maximum difference of about +35 K for the upper and side shields and about -160 K for the bottom shields (where conduction was important and was not modeled). For a second case, a temperature of 1073 K was imposed on the wafer and the power input requirements were calculated. Measurements gave a wafer temperature of about 1039.9 K

with a wafer uniformity of about ± 2.9 K. For this case the agreement on the wafer was poorer, but the agreement on the shields was about the same as for the first case.

Considering the complexity of the system and the uncertainty involved with the thermal properties, we can conclude that the numerical simulation was able to adequately predict the required boundary conditions. The wafer temperature was predicted within 3%, and fair uniformity was achieved.

3.2 Furnace design

Inverse boundary condition design can be applied to steady and transient applications. Howell et al.⁽¹⁶⁾ use inverse design for solving a two-dimensional enclosure that contains participating media and a three-dimensional evacuated enclosure. Erturk et al.⁽¹⁷⁾ compare three regularized solution techniques (TSVD, CGM and Bi-CGM) for a three-dimensional enclosure problem with an absorbing emitting and anisotropically scattering medium. Both of these studies consider steady radiating systems where other modes of heat transfer are ignored.

França et al.⁽¹⁸⁾ present the design of a drying system, where a participating gas flows through the system. The solution for the problem considers multimode heat transfer and the resulting system is non-linear. TSVD is used in an iterative scheme with relaxation to handle the non-linearity. França et al.⁽⁷⁾ is a comprehensive review of applications of the inverse design of steady systems, including solution of purely radiating enclosure problems and enclosure problems with multi-mode heat transfer.

The inverse formulation is also applicable for transient systems. Erturk et al.⁽¹⁹⁾ consider the design of the evacuated enclosure shown in Fig. 1. The goal is to heat the design object following a specified temperature history while keeping it spatially isothermal. The inputs required for the heaters are determined using the CGM at every time step using a constant regularization level to prevent instabilities along time (Fig. 4).

A similar approach is also applicable for systems where the object moves as it is heated. The main difference is replacing the local time derivative in Eq. (3) and updating the exchange factors for changing geometry⁽²⁰⁾.

3.3 Control

If the heater inputs presented in Fig. 4 are applied in a physical system, there will be some difference between the measured and desired conditions on the design surface because the models used for the solution are based on various assumptions and approximations and the accuracy of the radiative property data is often questionable. To use the model-based inverse

Fig. 4 Power input required to heat the design object to follow the temperature history

control approach, some means of correction of the predicted responses must be employed. This can be accomplished through the use of a control algorithm. An artificial neural net (ANN) based control algorithm in Ref.(20) was able to significantly reduce the difference.

4. Applications: (Using Optimization)

4.1 Design optimization of radiant enclosures

The simplest types of design problems involve diffuse-walled enclosures with transparent media. Daun et al.⁽¹¹⁾ describe an optimization methodology for finding the optimal heater settings that produce a desired temperature and heat flux distribution over a design surface. In this methodology, the radiosity distribution is solved using the infinitesimal-area analysis technique of Daun and Hollands⁽²¹⁾, and the objective function is minimized using Newton's method. Because of the form of the governing integral equations, the radiosity sensitivities can be found efficiently by post-processing the radiosity solution; these sensitivities are then used to calculate the elements in the gradient vector and Hessian matrix, at every iteration. This technique was applied to solve the design problem shown in Fig. 5.

The optimal heater settings are shown in Fig. 6 and the resulting heat flux distribution on the design surface was in excellent agreement with the required distribution.

A similar technique for optimizing the geometry of diffuse-walled radiant enclosures is in Ref.(12). The enclosure surfaces are represented by Bézier curves, and the design parameters are their control points. The first- and second-order objective function sensitivities are calculated by post-processing the radiosity solution, and Newton's method is used to

Fig. 5 Heater setting optimization of a diffuse-walled enclosure (Daun et al.⁽¹¹⁾)

Fig. 6 Optimal heater settings for the design problem shown in Fig. 5 (Daun et al.⁽¹¹⁾)

minimize the objective function.

Optimization techniques have recently been extended to solve problems involving enclosures that contain participating media and multimode problems. Hosseini Sarvari et al.⁽²²⁾ present a methodology for designing diffuse-walled radiant enclosures containing absorbing and scattering media to find the heat flux distribution over the heater surface that produces the desired heat flux distribution over the temperature-specified design surface. This approach has been extended to solve problems where conduction through the participating media is considered⁽²³⁾.

Fedorov et al.⁽²⁴⁾ solved a multimode enclosure problem to find the steady-state heater inputs that would result in a prescribed temperature history over the design surface in a continuous roll-through furnace with an isothermal non-participating medium and a constant heat transfer coefficient over the design surface. The Levenberg-Marquardt method⁽²⁵⁾ was used to minimize the objective function.

Geometric optimization of enclosures containing specularly-reflecting surfaces is encountered in design

Fig. 7 Geometric optimization of an enclosure containing both specular and diffuse walls (Daun et al.⁽²⁷⁾)

of solar collectors, light fixtures, and infrared heaters. Traditionally, these types of enclosures are designed either by trial-and-error or through non-imaging optics techniques based on ray-tracing⁽²⁶⁾. Recently, Daun et al.⁽²⁷⁾ presented a stochastic optimization technique that uses a Monte Carlo simulation coupled with a Kiefer-Wolfowitz minimization algorithm^{(28),(29)} to identify the enclosure geometry that produces the desired temperature and heat flux distributions over the design surface. This technique was demonstrated by solving the design problem shown in Fig. 7; the heat flux distributions over the design surface based on the initial and optimal heater settings are shown in Fig. 8, and it is seen that the optimal solution is very close to the desired solution of $q_r(x) = -1.0$.

Ashdown⁽³⁰⁾ has also used genetic algorithms to design reflectors and lenses for illumination applications.

4.2 Multimode problems with participating media

Research on inverse radiation problems in participating media and combined with other modes of heat transfer can be divided into two classes:

- (1) Inverse heat source problems: Estimation of a heat source distribution in the medium in such a way that a specified condition is satisfied.
- (2) Inverse boundary design problems: Estimation of unknown conditions over some part of the boundaries in such a way that both temperature and heat flux distributions approach the prescribed desired profiles over the design surface.

For inverse heat source problems Kudo et al.⁽³¹⁾ have used the SVD regularization technique to estimate the temperature and heat load distributions within the gas region from the profiles of the heat flux and the temperature of wall elements. França et al.^{(32)–(35)} used the inverse technique based on TSVD to solve the radiative heat source problem in a 2-D

Fig. 8 Initial and optimal heat flux distributions over the design surface for the enclosure shown in Fig. 7 (Daun et al.⁽²⁷⁾)

rectangular enclosure in which both the temperature and the heat flux distributions are satisfied on the four walls of the enclosure; to determine the distribution of gas temperature in a rectangular enclosure so that the two conditions were satisfied on the design surface; and for the case where the heat transfer involves both radiation and conduction.

Siewert⁽³⁶⁾ used an analytical technique to solve a class of inverse radiative transfer problems relevant to finite and semi-infinite plane parallel media. Fedorov et al.⁽²⁴⁾ used Levenberg-Marquardt optimization to analyze the combined convective, conductive and radiative heat transfer during heating of a continuously moving load in an industrial radiant oven.

Oguma and Howell⁽³⁷⁾ introduced an inverse Monte Carlo method for a class of inverse radiation problems. A modified TSVD technique was applied to the inverse problem of radiative exchange within enclosures composed of diffuse-gray surfaces by Harutunian et al.⁽³⁸⁾. Matsumura et al.⁽³⁹⁾ applied TSVD to inverse analysis of the radiative heat transfer within a 2-D enclosure with diffuse-gray surfaces enclosing an isothermal absorbing-emitting medium. Morales et al.⁽⁴⁰⁾ presented an inverse boundary design procedure for radiating enclosures containing an isothermal participating medium using MSTVD. França et al.⁽⁴¹⁾ investigated inverse boundary design for combined radiation and convection heat transfer using TSVD. Ruperti et al.⁽⁴²⁾ developed an inverse procedure to estimate the surface temperature and heat flux distributions from simulated transient temperatures measured within a semitransparent slab. Recently Hosseini Sarvari et al.⁽⁴³⁾ developed an optimization technique based on the CGM to solve the inverse boundary design problem of radiative heat transfer in irregular 2-D participating media.

5. Unresolved Problems and Future Work

Although a considerable amount of progress has been made in developing methods for design of radiating systems, there remain some serious questions. In specifying the conditions on the design surface, the design engineer may choose conditions for which there is no physical solution. For example, some heat flux distributions on an isothermal surface may not be achievable without unacceptable heater conditions (coolers in place of heaters, or negative or imaginary absolute temperatures on the heaters). Such solutions may satisfy the mathematical specification of the design constraints, but they are not acceptable physical solutions. An *a priori* determination of the existence of acceptable physical solutions would save a lot of fruitless calculation, but is not yet at hand.

Automation of the optimization and inverse solutions would allow the designer to set an acceptable accuracy for the design, and the algorithms outlined above could then proceed to achieve an acceptable solution without user intervention to determine the best number of singular values, the best Tikhonov parameter, the correct number of conjugate gradient steps, the best level of optimization, etc. However, it remains to determine the best approach to such designer-friendly methods.

6. Concluding Remarks

The methods and applications of thermal design and control that can be applied by viewing design and control as inverse problems have been outlined and discussed. Some of the remaining research issues have also been addressed.

It is hoped that the thermal design community will see the advantages of these methods, and begin to adopt them in the design of actual thermal systems where radiation plays an important or dominant role.

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