Numerical Analysis of Heat and Fluid Flow in Pulse Tube Refrigerator*

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A numerical study has been performed to analyze the working principle of refrigeration in basic pulse tube refrigerator. There is a mechanism called "surface heat pumping" as one of the working principles of refrigeration for basic pulse tube refrigerator of the first generation. This is the mechanism that heat is pumped from the cold end to the hot end by the successive heat exchange between the pulse tube wall and the working gas. Transient axisymmetric two-dimensional equations of continuity, momentum and energy were solved by means of TVD scheme. The transient behaviors of pressure and gas temperature, and heat transfer from the working gas to the tube wall are analyzed. Also the behavior of surface heat pumping is explained by analyzing the axial movements and temperature changes of gas particles.

Key Words: Refrigeration, Compressible Flow, Viscous Flow, Oscillatory Flow, Pulse Tube, Basic Type, Heat Exchange, Numerical Analysis

1. Introduction

Recently, studies on pulse tube refrigerators are being made all over the world because they are expected to be used in the field of medicine and space, and for cooling of superconducting devices. The biggest advantage is that they have no moving parts at low temperatures, which leads to high reliability, simplicity, long life and low vibration.

Pulse tube refrigerator was first developed in 1963 by Gifford and Longsworth⁽¹⁾ which is now called basic type and classified as the first generation. Orifice type classified as the second generation was developed in 1984 by Mikulin et al.⁽²⁾, and double inlet type classified as the third generation was developed in

1990 by Zhu et al.⁽³⁾. Other types developed after that are classified as the third generation. The lowest temperature and COP of pulse tube refrigerators have been developed by past many studies and now become equal to those of other types of refrigerator. Although theoretical studies have been made by many researchers, detailed heat and fluid flow in the pulse tube is not clear.

There is a mechanism called surface heat pumping⁽⁴⁾ for indicating the working principle of refrigeration in basic pulse tube refrigerator. This is the mechanism that heat is pumped from the cold end to the hot end by the successive heat exchange between the pulse tube wall and the working gas. However, it is difficult for this mechanism to explain the actual working principle of refrigeration because it includes the assumptions that the working gas is compressed and expanded adiabatically, and heat is exchanged successively until the temperatures of the pulse tube wall and the working gas become locally equal each other in the pulse tube.

Hozumi et al.⁽⁵⁾ reported a numerical study in which the axisymmetric two-dimensional viscous compressible equations were solved directly to make clear the working principle of refrigeration in basic pulse tube refrigerator. They assumed the pulse tube

* Received 25th April, 2003 (No. 03-4043)

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wall to be adiabatic and neglected heat exchanges between the pulse tube wall and the working gas. However it is considered that heat exchanges between the pulse tube wall and the working gas are very important in basic pulse tube refrigerator, and those can not be neglected in order to analyze heat and fluid flow in actual pulse tube.

In the present study, a numerical simulation in which heat exchanges between the pulse tube wall and the working gas as convective heat transfer are considered has been performed to analyze the behavior of heat and fluid flow in the pulse tube and make clear the physical working principle of refrigeration. Equations used for numerical simulation are continuity, momentum and energy equations, and are solved faithfully. In this paper the transient behaviors of pressure and gas temperature, and heat transfer from the working gas to the tube wall are analyzed. Also the behavior of surface heat pumping is explained by analyzing the axial movements and temperature changes of gas particles.

2. Nomenclature

- a : thermal diffusivity of stainless steel [m^2/s]
 A : heat transfer area [m^2]
 A' : heat transfer area per unit volume [m^2/m^3]
 e : total of internal energy and kinetic energy [J/kg]
 f : friction force [N/kg]
 $\mathbf{f}$: friction force vector [N/kg]
 m : mass of working gas [kg]
 M : mass of regenerator material [kg]
 p : pressure [Pa]
 Q : heat transfer rate per unit volume [W/m^3]
 r : radius [m , mm]
 t : time [s]
 T : temperature [K]
 V : velocity [m/s]
 $\mathbf{V}$: velocity vector [m/s]
 x : length from the open end of regenerator

[m , mm]

x_p : length from the boundary between regenerator and pulse tube in pulse tube [m , mm]

x_{rg} : length from the open end in regenerator [m , mm]

α : heat transfer coefficient [$\text{W}/(\text{m}^2 \cdot \text{K})$]

Φ : energy dissipation function [W/m^3]

λ : thermal conductivity [$\text{W}/(\text{m} \cdot \text{K})$]

μ : coefficient of viscosity [$\text{Pa} \cdot \text{s}$]

ρ : density of working gas [kg/m^3]

τ : shearing stress [Pa]

Subscripts

gas: working gas

in: open end of regenerator

r: radial direction

rm: regenerator material

w: wall (stainless steel layer)

x: axial direction

xr: surface of *x-r*

3. Physical Model

Physical model for numerical simulation is shown in Fig. 1. The pulse tube is a stainless steel pipe of 150 mm in length, 5 mm in inner diameter and 1 mm in thickness, containing the heat exchanger of 30 mm in length. The dimensions of regenerator are 60 mm in length and 12.5 mm in inner diameter. It is assumed that the woven 200 mesh wire screens of copper are filled as regenerator material. Heat exchanger is a part of pulse tube and the effect as heat exchanger is represented by keeping the thick line shown in Fig. 1 at constant temperature of 290 K. The temperature at the open end of regenerator is also kept at constant temperature of 290 K. The outer wall of the pulse tube is assumed to be adiabatic as shown by the diagonal in Fig. 1, however heat exchange between the pulse tube wall and the working gas is assumed as a convective heat transfer. The air is selected as a working gas. The oscillating flow is generated by changing the pressure at the open end of regenerator

Fig. 1 Physical model

as a sinusoidal wave.

It is selected as the initial condition that the pressure is 0.1 MPa and temperatures of the working gas, stainless steel pipe and regenerator material are 290 K. It is chosen as the operating condition that the pressure ratio is 2 and operating frequency is 1 Hz.

4. Governing Equations

The basic assumptions are as follows:

1. The flow in the regenerator is one-dimensional and that in the pulse tube is axisymmetric two-dimensional.
2. It is considered to be compressible flow in the regenerator and viscous compressible flow in the pulse tube.
3. The working gas is an ideal gas.
4. The term of gravity in energy equation is neglected.

Under these assumptions, the basic governing equations are as follows.

Continuity equation for gas

$$\frac{D\rho}{Dt} = -\rho \operatorname{div} \mathbf{V} \quad (1)$$

Momentum equation for gas

$$\frac{D\mathbf{V}}{Dt} = -\frac{1}{\rho} \operatorname{grad} p + \mathbf{f} \quad (2)$$

where in case of gas calculation in regenerator, $\mathbf{f}$ is neglected. In case of gas calculation in pulse tube, $\mathbf{f}$ is as follows:

$$\mathbf{f} = (f_x, f_r) \quad (3)$$

$$f_x = \frac{1}{\rho} \left[\frac{\partial \tau_{xx}}{\partial x} + \frac{1}{r} \frac{\partial (r \tau_{xr})}{\partial r} \right] \quad (4)$$

$$f_r = \frac{1}{\rho} \left[\frac{\partial \tau_{xr}}{\partial x} + \frac{1}{r} \frac{\partial (r \tau_{rr})}{\partial r} + \frac{\tau_{\theta\theta}}{r} \right] \quad (5)$$

where

$$\tau_{xx} = 2\mu \frac{\partial V_x}{\partial x} - \frac{2}{3} \mu \operatorname{div} \mathbf{V} \quad (6)$$

$$\tau_{\theta\theta} = 2\mu \frac{V_r}{r} - \frac{2}{3} \mu \operatorname{div} \mathbf{V} \quad (7)$$

$$\tau_{rr} = 2\mu \frac{\partial V_r}{\partial r} - \frac{2}{3} \mu \operatorname{div} \mathbf{V} \quad (8)$$

Energy equation for gas

$$\frac{De}{Dt} = -\mathbf{V} \cdot \frac{1}{\rho} \operatorname{grad} p + \mathbf{V} \cdot \mathbf{f} - \frac{p}{\rho} \operatorname{div} \mathbf{V} + \frac{\Phi}{\rho} + \frac{Q}{\rho} \quad (9)$$

where in case of gas calculation in regenerator, $\mathbf{f}$ and Φ are neglected. Also Q is expressed as

$$Q = \lambda_{gas} \nabla^2 T_{gas} + \alpha A' (T_{rm} - T_{gas}) \quad (10)$$

where

$$A' = \frac{A}{m/\rho} \quad (11)$$

In case of gas calculation in pulse tube, Q is expressed as

$$Q = \lambda_{gas} \nabla^2 T_{gas} \quad (12)$$

Energy equation for regenerator material

$$c_{rm} M \frac{\partial T_{rm}}{\partial t} + \alpha A (T_{gas} - T_{rm}) = 0 \quad (13)$$

Thermal conductive equation for pulse tube wall

$$\frac{\partial T_w}{\partial t} = \alpha \nabla^2 T_w \quad (14)$$

These equations for gas were solved by means of TVD scheme⁽⁶⁾.

For the boundary conditions at the open end of regenerator, the pressure oscillation is taken as follows:

$$p_{in} = p(t) \quad (15)$$

The gas temperature is kept at constant temperature as follows:

$$T_{in} = \text{const} \quad (16)$$

5. Numerical Results

5.1 The behaviors in initial several cycles

It was firstly confirmed that the sinusoidal pressure wave was generated at the center point in the pulse tube by a boundary condition given at the open end of regenerator. It was also confirmed that the pressure wave was almost the same at any point in the pulse tube.

Figure 2 shows the behavior of the gas temperatures at the cold end in the pulse tube ($x_p = 5$ mm), the hot end in the pulse tube ($x_p = 145$ mm) and the cold end in the regenerator ($x_{rg} = 55$ mm). Also all points shown in Fig. 2 are at the center points in radial direction. It can be seen that the temperature at the cold end in the pulse tube does not increase compared with the temperature at the hot end during compression. It is considered that this is due to the gas flowing into the pulse tube after cooled by passing through the regenerator. Therefore, the temporal average gas temperature during a cycle at the cold end in the pulse tube decreases rapidly in the first cycle as shown in

Fig. 2 The behavior of gas temperatures in the pulse tube and regenerator in initial 10 cycles

Fig. 3. It can be seen that the temperature decreases gradually at the cold end after its rapid decrease. It is considered that this is due to gradual temperature decreases of pulse tube wall and regenerator material having large heat capacity. On the other hand, though the temperature at the hot end in the pulse tube increases rapidly in the first cycle, after that does not change as much as the temperature at the cold end because of the heat exchanger kept at constant temperature.

It can be seen that the change in gas temperature at the cold end in the regenerator is not as large as that in the pulse tube. Therefore, only the behaviors of the temperatures at the cold end ($x_{rg}=55$ mm), hot end ($x_{rg}=5$ mm) and the center point ($x_{rg}=30$ mm) in the regenerator are shown in Fig. 4. It can be seen that the temperature decrease at the cold end in the regenerator is the largest among three points, though the change in gas temperature is very small as shown

Fig. 3 The behavior of temporal average gas temperature in the pulse tube and regenerator in initial 10 cycles

Fig. 4 The behavior of temporal average gas temperature in the regenerator in initial 10 cycles

in Figs. 3 and 4. Therefore, it is expected that enormous time is needed to achieve a cyclic steady-state. Also it was also confirmed that the changes in temperature of regenerator material and stainless steel layer are very small like that in gas temperature in regenerator.

Figure 5 shows the total amount of heat transfer from the working gas to the tube wall during a cycle in the first cycle at each point in axial direction. The total amount is calculated by integrating transient heat transfer at each point. The heat transfer from the working gas to the wall is defined as positive and reversely the heat transfer from the wall to the working gas as negative. It can be seen that the heat is transferred from the wall to the working gas at the part from $x_p=0$ to about 20 mm and is getting small as x_p is large, however at the side of closed end from about 20 mm heat transfer is reversed and heat is removed from the working gas.

The result shown in Fig. 5 is corresponding to the result of Fig. 3. Actually heat is transferred from the wall to the working gas at the cold end because of decrease in gas temperature and heat is transferred from the working gas to the wall at the hot end because of increase in gas temperature.

5.2 The behaviors in a cyclic steady-state

It was found that an enormous time is needed to achieve a cyclic steady-state because the changes in gas temperature in regenerator and in temperature of regenerator material and tube wall are very small compared with that in gas temperature in pulse tube. The calculations of gas in pulse tube and in regenerator, regenerator material and tube wall are included in general calculation. Because most parts of general calculation are occupied by the calculation of gas in pulse tube, if this calculation is removed, the computational speed for other three calculations

Fig. 5 Heat transfer from the working gas to the wall in the pulse tube during a cycle in the first cycle

becomes faster. Therefore, after general calculation was made in several cycles, the change in gas temperature in pulse tube is assumed to be cyclic steady-state, and only the calculations of gas temperature in regenerator and regenerator material and tube wall were made. These two calculations, general calculation and the calculation except that of gas in pulse tube, were made alternately in order to achieve a cyclic steady-state.

Figure 6 shows the temperature waves during a cycle in a cyclic steady-state. The horizontal axis shown in Fig. 6 indicates the time measured from the start of compression. It can be seen that these waves are qualitatively very similar to those shown in Fig. 2 and the temperature at the cold end during a cycle is lower than the initial temperature of 290 K. It is considered that the gas temperature at the cold end decreases gradually until a cyclic steady-state is achieved because of gradual decrease in temperature at the cold end in the regenerator material and tube wall.

Figure 7 shows the total amount of heat transfer from the working gas to the tube wall at each point in axial direction during a cycle in a cyclic steady-state. It can be seen that the heat transfer is qualitatively similar with that in Fig. 5, however heat transfer from the working gas to the tube wall at the cold end is larger by two times because the gas temperature at the cold end in a cyclic steady-state is lower than that in the first cycle. Also it can be seen that the neutral point moves to the side of closed end compared with that in Fig. 5.

Figures 8 and 9 show the temperature distributions in axial direction in a cyclic steady-state during compression and expansion respectively. The time shown in Figs. 8 and 9 indicates the elapsed time from

Fig. 6 The behavior of gas temperature in the pulse tube during a cycle in a cyclic steady-state

Fig. 7 Heat transfer from the working gas to the wall in the pulse tube during a cycle in a cyclic steady-state

Fig. 8 Temperature distribution during compression in a cyclic steady-state

Fig. 9 Temperature distribution during expansion in a cyclic steady-state

the start of compression. For compression process, the temperature at the time of the start of compression is lower than initial temperature of 290 K at any point in the pulse tube as shown in Fig. 8. It can be seen that the temperatures in the pulse tube during compression increase with time until the time reaches 0.3 s, while the temperatures at 0.4 s and 0.5 s are lower than the temperature at 0.3 s because the temperature decrease by heat transfer to the pulse tube wall is larger than the temperature increase by compression. Reversely it can be seen that the temperatures in the pulse tube during expansion decrease with time until the time reaches 0.8 s, while temperatures at 0.9 s and 1 s are higher than the temperature at 0.8 s because the temperature increase by heat transfer from the pulse tube wall is larger than the temperature decrease by expansion.

5.3 Analysis on surface heat pumping

The model shown in Fig. 1 is compared with an adiabatic model that has no stainless steel layer and no heat exchanger at the hot end. Although the refrigeration effects do not occur in the adiabatic model because heat is not transferred to the tube wall in any part in the pulse tube by the complete adiabatic inner wall. This comparison will make clear the behavior of heat exchange between the tube wall and working gas in the pulse tube in the model shown in Fig. 1. Hereafter the model shown in Fig. 1 is referred as the analyzed model and the above-mentioned model is referred as the adiabatic model.

Figure 10 shows the behavior of gas temperatures in the first cycle. The maximum temperatures at the hot end for analyzed model and adiabatic model are about 310 K and 360 K, respectively. It is considered that the difference between these two models is generated because heat is transferred from the working gas

Fig. 10 The behavior of gas temperatures in the first cycle in analyzed model and in adiabatic model

to the tube wall and the change in gas temperature is limited during compression in analyzed model but not the case in adiabatic model. The difference between these two models in temperature at the cold end during compression is small, while the temperature of analyzed model decreases compared with that of adiabatic model during expansion and lowest temperature achieved in case of analyzed model is nearly 270 K. This reason is described later.

Figure 11 shows the T - x_p diagram of two gas particles which are located at $x_p=5$ mm and 120 mm respectively in axial direction and the center in radial direction in the first cycle in analyzed model and adiabatic model. The closed circles indicate the points at the time of the start of compression. The closed squares mean the points at the time of the end of compression or start of expansion. The closed triangles show the points at the time of the end of expansion. Because the cycle shown in Fig. 11 is not a cyclic steady-state, the curves illustrated are not closed. It can be seen that in both models the particles move toward the hot end during compression and reversely they move toward the cold end during expansion, however these temperature changes are different entirely. During compression, in adiabatic model the temperatures increase with time, while in analyzed model the temperatures do not increase remarkably compared with those in adiabatic model because temperature increase is limited by existence of the tube wall. During expansion, in adiabatic model the temperature decreases with time passing near the reverse routes to that in compression, while in analyzed model the temperature which is lower than that in adiabatic model at the start of expansion decrease much more by expansion. In Fig. 10 the difference between these two models at the cold end is consid-

Fig. 11 The comparison of the change in temperatures and the movements of gas particles in analyzed model with those in adiabatic model

Fig. 12 The comparison of the change in temperatures and the movements of gas particles in analyzed model at $r=0$ mm with those at $r=2$ mm

ered as follows. During compression this difference is very small because the gas cooled by regenerator flows in the pulse tube and temperature difference between the tube wall and working gas is very small in analyzed model. During expansion this difference is very large because the particles at the right side of the cold end which are already cooled at the start of expansion pass through the cold end as cooled further by pressure decrease in analyzed model.

Figure 12 shows the $T-x_p$ diagram for particles of initial location at $x_p=5$ mm, 70 mm and 120 mm in axial direction and at $r=0$ mm (as shown by solid line) and 2 mm (as shown by broken line) in radial direction at the time of the start of compression with wall temperature in a cyclic steady-state in analyzed model. In Fig. 12 the closed and open circles mean the points at the time of the start of compression at $r=0$ mm and 2 mm respectively. The closed and open squares indicate the points at the time of the end of compression (or start of expansion) at $r=0$ mm and 2 mm respectively. It can be seen that the movements of the particles in axial direction at the side of the cold end are larger than those at the side of the hot end, and the amplitudes of temperature change for particles at $r=2$ mm are smaller than those at $r=0$ mm. Also it can be seen that though the temperatures of each particle increase in compression, they decrease at the end of compression because heat is transferred from the working gas to the tube wall, and reversely though the temperatures of each particles decrease in expansion they increase at the end of expansion because heat is transferred from the tube

wall to the working gas. It is confirmed that the particles of initial location at $x_p=5$ mm move to the location at nearly $x_p=80$ mm and give the heat to the tube wall during compression. It is also confirmed that the particles at nearly $x_p=80$ mm at the time of the start of compression receive the heat from the tube wall and carry the heat to the hot end. In other word, it is found that heat is pumped from the cold end to the hot end.

6. Conclusion

A numerical simulation of viscous compressible flow in basic pulse tube refrigerator has been performed to analyze the behavior of heat and fluid flow in the pulse tube and make clear the physical working principle of refrigeration. The results are as follows:

1. The rapid decrease in temporal average gas temperature at the cold end during the first cycle is due to the gas flowing into the pulse tube after cooled by regenerator, and subsequent gradual decrease is due to gradual decrease in temperature of the tube wall and the regenerator material at the cold end.
2. The total amount of heat transfer from the working gas to the tube wall during a cycle at each point in axial direction in the pulse tube was confirmed in the first cycle and in a cyclic steady-state.
3. The behavior that heat is pumped from the cold end to the hot end is made clear by observing the temperature changes of particles in the pulse tube.

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