Numerical Study of Superconducting-Tape Thermal Stability under the Effect of a Two-Dimensional Hyperbolic Heat Conduction Model*

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Thermal stability of a superconducting tape subjected to thermal disturbances under the effect of a two-dimensional hyperbolic heat conduction model is numerically investigated. The thermal inertia of the material, the finite speed of heat transfer, the temperature dependence of the Ohmic heat generation, and the finite duration and finite length of the thermal disturbances are taken into consideration in the mathematical model. Two types of superconductor tapes are considered, Types II and I. It is found that hyperbolic model predicts a wider stable region as compared to the predictions of its parabolic counterpart, the hyperbolic model shows a higher peak at early stages and the difference between the two models is significant at higher values of disturbance intensity and smaller values of time. The effects of different design, geometrical and operating conditions on superconducting tape thermal stability are also studied.

Key Words: Superconductors, Tape-Type Superconductors, Thermal Stability, Hyperbolic Model, Non-Fourier Conduction Model

1. Introduction

Superconducting tapes are one of the most promising candidates for future energy problems because they provide an efficient mean of energy transport to reduce unnecessary energy loss. Knowledge on the thermal stability and quench characteristics of superconducting tapes and wires are of fundamental importance to the design, operation and performance of superconducting devices such as motors, generators, fault-current limiters, power distribution networks and cellular phone base stations. These devices must be designed in such a way that they are stable against any thermal disturbances. For example, a tiny conductor motion can initiate a normal zone in a high current density magnet. Other examples of disturbances are absorption of particulate or infrared radiation, or short-term failure of cooling. Disturbances

immediately transform into heat pulses that increase the temperature of the superconductor. A fully thermal stability denotes a situation where a superconductor can carry the operating current without resistance at all times even if a localized thermal disturbance has been released.

Superconductor thermal stability, using different superconductor types, geometries, assumptions, applications, operating conditions, and different models has been considered by many researchers^{(1)–(18)}. Most of the previous works have investigated the superconductor stability based on the classical Fourier's heat diffusion model^{(1)–(14)}. However, very few studies have considered the stability based the hyperbolic heat conduction^{(15)–(18)}. It is well-known that the diffusion heat conduction equation based on the Fourier's law leads to the unreasonable result that heat propagates at an infinite speed a conducting medium. This concept can be impractical from a physical point of view. To resolve this dilemma, the Cattaneo equation was proposed⁽²⁰⁾. This lead to a wave type of heat conduction equation called the hyperbolic heat conduction equation. In contrast to

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the classical model, the hyperbolic model depicts a finite speed of heat propagation and remedies this unphysical point of view. The classical heat conduction model can give a good results for most of the situations encountered in the daily life. However, it fails to adequately describe problems involving a high rate of temperature change, such as heat flux in an extremely short period of time (i.e. when the foregoing time is comparable with the relaxation time), for very low temperatures near absolute zero (cryogenic temperatures), extremely high frequency sources, temperature and/or time dependent heat source and for describing heat conduction from a thin tape with a dimension of order of the heat carrier's mean free path⁽²⁰⁾. In operation, superconducting tapes are subjected to all or some of these situations. The solution of the thermal wave equation is of special important in the emerging fields of heat transfer microscale and nanoscale thickness tapes. Usually these partial differential equations describe physical processes when the diffusion equation is inadequate and quantum analyses are too bulky. A marked improvement of temperature data makes the thermal wave equation a feasible model for analyzing thermal characteristics of materials having dimensions of a few nanometers. Although it is difficult to measure temperatures within small devices of nanometer size, a microscale heat conduction equation remains the only possible method.

To the best of the authors' knowledge, the thermal stability of composite superconducting tape based on a two-dimensional hyperbolic heat conduction model (that considers the formation and behavior of a normal zone in both transverse and longitudinal directions) has not been investigated. This is the objective of the present work. In this paper, the stability of Type I and II superconducting tape under different operating, design, and geometrical parameters is analyzed. Also, the effect of the time rate of change of the disturbance initial temperature and the disturbance duration time on the stability will be investigated.

Nomenclature

- a : superconducting tape width (m)
- A : superconducting tape cross sectional area of (m²)
- As : superconducting tape aspect ratio (a/b)
- b : superconducting tape thickness (m)
- B : dimensionless disturbance intensity
- C : heat capacity (J m⁻³ K⁻¹)
- Di : external thermal disturbance per unit volume (Wm⁻³)
- f : volume fraction of the stabilizer in the super-

conducting tape

g : internal heat generation by Joule heating (Wm⁻³)

$g_{\max}$: maximum Joule heating with the whole current in the stabilizer (Wm⁻³)

$G_{\max}$: dimensionless Joule heating source

h : convective heat transfer coefficient (Wm⁻²K⁻¹)

J : current density (Am⁻²)

k : superconducting tape thermal conductivity (Wm⁻¹K⁻¹)

q : heat flux vector (Wm⁻²)

Q : dimensionless maximum Joule heating

t : time (s)

T : temperature (K)

T_c : critical temperature (K)

T_{c1} : current sharing temperature (K)

T_i : initial temperature (K)

T_o : ambient temperature (K)

x, y : coordinates defined in Fig. 1 (m)

Greek symbols

α : thermal diffusivity (m²s⁻¹)

β : dimensionless time

β_i : dimensionless duration time

η : dimensionless y -variable

η_o : superconducting tape dimensionless thickness

θ : dimensionless temperature

θ_{c1} : dimensionless current sharing temperature

θ_1 : dimensionless maximum temperature

Δx : width of superconducting tape subjected to heat disturbances (m)

Δy : thickness of superconducting tape subjected to heat disturbances (m)

ξ : dimensionless x -variable

ξ_o : superconducting tape dimensionless width

ρ_o : stabilizer electrical resistivity (Ω m)

τ_q : relaxation time of heat flux (s)

ω : speed of heat propagation (ms⁻¹)

Subscripts

i : initial

sc : current sharing

$\max$: maximum

o : ambient

2. Analysis

Consider a thin superconducting tape of a finite width ($2a$) and a thickness ($2b$) carrying an electric current normal to the xy plane as shown schematically in Fig. 1. The superconducting tape may be of Type I or II. In Type I the superconductor is a non-composite type while in type II the superconductor consists of a superconducting strands embedded in a high purity metal. The superconducting tape exchanges heat with the surrounding coolant at a rate

Fig. 1 Physical model of superconducting tape

given by a uniform and constant heat transfer coefficient h . A conductor section of width $2\Delta x$ and a thickness of $2\Delta y$ is instantly heated from the heat disturbance source up to a temperature T_i , exceeding the critical superconductor temperature T_{c1} at a given current. Also, this sectional area is subjected to an external disturbance of a finite duration. The temperature field in the normal zone and the quenching process are governed by the energy equation coupled with the hyperbolic constitution law. With the assumption of constant and isotropic thermal properties, the governing equations are given as

$$C \frac{\partial T(x, y, t)}{\partial t} = k \nabla \cdot \mathbf{q} + g(T) + D(x, y, t) \quad (1)$$

$$\mathbf{q}(x, y, t) + \tau_q \frac{\partial \mathbf{q}(x, y, t)}{\partial t} = -k \nabla T(x, y, t) \quad (2)$$

The salient feature of the hyperbolic model lies in the involvement of the thermal relaxation time τ_q which is basically a measure of thermal transmission time between points in the heated domain. A typical value of the relaxation time for metals has been reported to be on the order of 10^{-11} second. The constitution law of Eq.(2) assumes that the heat flux vector (the effect) and the temperature gradient (the cause) across a material volume occur at different instants of time and the delay time between the heat flux and the change in temperature gradient is the relaxation time. In Eq.(2) it is assumed that τ_q is small so that the first order Taylor series expansion of $\mathbf{q}(t + \tau, x, y)$ is a good representation for the heat flux.

Eliminating the heat flux vector $\mathbf{q}$ between Eqs. (1) and (2), leads to the following two-dimensional hyperbolic heat conduction equation

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha} \frac{\partial^2 T}{\partial t^2} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{1}{k} \left[g(T) + \tau_q \frac{\partial g(T)}{\partial t} \right] + \frac{1}{k} \left[D + \tau_q \frac{\partial D}{\partial t} \right] \quad (3)$$

Equation (3) describes the behavior of a damped temperature wave with a finite speed equal to $\omega = \sqrt{\frac{\alpha}{\tau_q}}$. For $\tau_q \rightarrow 0$, $\omega \rightarrow \infty$, Eq.(3) will become the classical heat diffusion equation, this means that the inertial $\frac{\tau_q}{\alpha} \frac{\partial^2 T}{\partial t^2}$ term is neglected. The heat propagation speed is usually approximated by the velocity of free electron gas in the cases of metal, and by the speed of sound in crystals in non-metals. At room temperature, these speeds are of order of 9×10^4 m/s and 5×10^3 m/s.

In the literature the thermal disturbance are simulated as a two parts. The first one simulates an instantaneously imposed heat pulse, and is taken into consideration as an initial condition, hence, this disturbance raises the conductor temperature, instantaneously, at $t=0$ over the critical temperature^{(2),(4),(9)-(16)}. The second part $D(x, y, t)$ is taken into consideration as an externally imposed heat source in the governing equation, and can be represented mathematically by Eq.(4) below^{(3),(5),(7),(9),(17)}. Each part of the disturbance was considered alone in the literature. However, the disturbance, considered in this study, consists of two parts each of them has rectangular heat pulse form thus the combined effect of both parts will investigated. The external disturbance is given as

$$D(x, y, t) = \begin{cases} D_i & \text{for } 0 < x < \Delta x, 0 < y < \Delta y \\ & \text{and } 0 < t < t_i \\ 0 & \text{outside this region} \end{cases} \quad (4)$$

The steady capacity of the Ohmic heat source is given as

For Type I superconductor :

$$g(T) = \begin{cases} 0 & \text{for } T \leq T_{c1} \\ g_{\max} & \text{for } T \geq T_{c1} \end{cases} \quad (5)$$

and for Type II superconductor :

$$g(T) = \begin{cases} 0 & \text{for } T \leq T_{c1} \\ \frac{g_{\max}}{f} \frac{T - T_{c1}}{T_c - T_{c1}} & \text{for } T_{c1} < T < T_c \\ \frac{g_{\max}}{f} & \text{for } T \geq T_c \end{cases} \quad (6)$$

where $g_{\max} = \rho_o J^2$ is the Ohmic heat generation with the whole current in the stabilizer. The current sharing function $g(T)$ given in Eq.(6) is based on the assumption that the critical current density of the superconductor is a linear function of temperature at a fixed field.

Due to the symmetry of the normal zone, the analysis is limited to the half zone, i.e., to the domain that lies within $0 \leq x \leq a$ and $0 \leq y \leq b$. Equation (3) assumes the following initial and boundary conditions :

$$\begin{aligned}
T(x, y, 0) &= T_i & \text{for } 0 < x \leq \Delta x, & \text{and } 0 \leq y \leq \Delta y \\
T(x, y, 0) &= T_o & \text{for } x > \Delta x & \text{and } \Delta y \leq y \leq b \\
\frac{\partial T(x, y, 0)}{\partial t} &= \dot{T}_i, & \text{for } 0 \leq x \leq a \text{ and } 0 \leq y \leq b \\
k \frac{\partial T(0, y, t)}{\partial x} &= 0, & \frac{\partial T(a, y, t)}{\partial x} &= -h(T - T_o) \\
k \frac{\partial T(x, 0, t)}{\partial y} &= 0, & \frac{\partial T(x, b, t)}{\partial y} &= -h(T - T_o)
\end{aligned} \tag{7}$$

The above boundary conditions feature convective cooling on the surfaces and symmetry at the center of the conductor. The above initial conditions are based on the assumption that the disturbance energy is initially deposited in a centrally located rectangular area of $4(\Delta x \times \Delta y)$. The width and the thickness of the rectangular heat source are variables, with a values closed to zero simulating a line heat source. It is more convenient to rewrite Eqs. (3) - (7) using the following dimensionless parameters:

$$\begin{aligned}
\xi &= \frac{\omega x}{2a}, & \eta &= \frac{\omega y}{2a}, & \beta &= \frac{t}{2\tau_q}, \\
\theta &= \frac{T - T_o}{T_c - T_o}, & \Delta\xi &= \frac{\omega \Delta x}{2a}, & \Delta\eta &= \frac{\omega \Delta y}{2a}, \\
B &= \frac{T_i - T_o}{T_c - T_o}, & B_i &= \frac{2h\sqrt{a\tau_q}}{k}, & \xi_o &= \frac{\omega a}{2a}, & \eta_o &= \frac{\omega b}{2a}, \\
G &= \frac{4\tau_q g(T)}{C(T_c - T_o)}, & \phi_i &= \frac{4\tau_q D_i(x, y, t)}{C(T_c - T_o)}, & Q &= \frac{4\tau_q \rho_o J^2}{C(T_c - T_o)}
\end{aligned}$$

As a result, Eq. (3) is reduced to

$$\frac{\partial^2 \theta}{\partial \beta^2} + 2 \frac{\partial \theta}{\partial \beta} = \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} + \left(Q + \frac{1}{2} \frac{\partial Q}{\partial \beta} \right) + \left(\phi + \frac{1}{2} \frac{\partial \phi}{\partial \beta} \right) \tag{8}$$

The heating source in its dimensionless form is rewritten as:

For Type I superconductor:

$$G = \begin{cases} 0 & \text{for } \theta \leq \theta_{c1} \\ Q & \text{for } \theta > \theta_{c1} \end{cases} \tag{9}$$

and for Type II superconductor:

$$G = \begin{cases} 0 & \text{for } \theta \leq \theta_{c1} \\ \frac{Q}{f} \frac{\theta - \theta_{c1}}{1 - \theta_{c1}} & \text{for } \theta_{c1} < \theta < 1 \\ \frac{Q}{f} & \text{for } \theta \geq 1 \end{cases} \tag{10}$$

The dimensionless external thermal disturbance is:

$$\phi(\xi, \eta, \beta) = \begin{cases} \phi_i & \text{for } 0 < \xi < \Delta\xi, 0 < \eta < \Delta\eta \text{ and } 0 < \beta < \beta_i \\ 0 & \text{outside this region} \end{cases} \tag{11}$$

and the initial and boundary conditions are reduced to

$$\begin{aligned}
\theta(\xi, \eta, 0) &= B & \text{for } 0 < \xi \leq \Delta\xi & \text{and } 0 \leq \eta \leq \Delta\eta, \\
\theta(\xi, \eta, 0) &= 0 & \text{for } \xi > \Delta\xi \leq \xi_o & \text{and } \Delta\eta \leq \eta \leq \eta_o, \\
\frac{\partial \theta(\xi, \eta, 0)}{\partial \beta} &= \dot{\theta}_i, & \text{for } 0 \leq \xi \leq \xi_o & \text{and } 0 \leq \eta \leq \eta_o, \\
\frac{\partial \theta(0, \eta, \beta)}{\partial \xi} &= 0, & \frac{\partial \theta(\xi_o, \eta, \beta)}{\partial \xi} &= -B_i \theta, \\
\frac{\partial \theta(\xi, 0, \beta)}{\partial \eta} &= 0, & \frac{\partial \theta(\xi, \eta_o, \beta)}{\partial \eta} &= -B_i \theta
\end{aligned} \tag{12}$$

2.1 Thermal stability criterion

The temperature distribution within the superconducting tape has the following general form $\theta = \theta(\xi, \eta, \beta, B, Q, B_i, As, \Delta x \times \Delta y, f, \theta_i)$. It is obvious that the superconducting-tape maximum temperature occurs at $\xi=0$ and $\eta=0$. The temperature at this location is the largest possible one, as heat needs to transmit across the maximum distance in order to be dissipated through convection on the surface. Thus, if the temperature distribution is stable at this location,

then it will be stable elsewhere. This maximum temperature is referred to by θ_1 where $\theta_1 = \theta(0, 0, \beta, B, Q, B_i, As, \Delta x \times \Delta y, f, \theta_i)$. For each combination of $B, Q, B_i, As, \Delta x \times \Delta y, \theta_i$ and f two behaviors can be featured; (a) θ drops below 1 which means that the normal zone shrinks to zero and the superconductor is stable and (b) θ does not drop below 1 indicates that the normal zone grows and the superconductor is unstable. We are, in particular, interested in the marginal case when the conditions

$$\theta_1=1, \quad \frac{\partial \theta_1}{\partial \beta}=0 \quad (13)$$

occur simultaneously. For each combination of B , Q , B_i , As , $\Delta x \times \Delta y$, θ_i and f , we may find the critical value of Q , which is referenced to Q_c , where for each, $Q \leq Q_c$ the two conditions given in Eq.(12) are satisfied. The stability criterion is then: $Q < Q_c$ for collapse (stable) and $Q > Q_c$ for growth (unstable).

2.2 Solution methodology

The governing equations have been solved numerically by means of a finite difference technique consisting of Alternating-Direction-Implicit iterative (ADI) method of Peaceman and Rachford⁽¹⁹⁾. Applying forward and central differences for time and spatial derivative in Eq.(8), a tridiagonal system of equations is set up on a uniform grid and is solved using Gauss-Seidel iteration scheme. Thus, temperature at an advance point in time $\beta=(n+1)\Delta\beta$, is computed using the solution at $\beta=(n)\Delta\beta$ ($n=0$ corresponds to the initial conditions). The basic time step used for the calculation is $\Delta\beta=5 \times 10^{-5}$. The iteration continue until the desired results are obtained with a convergence criteria

$$\frac{\theta_{n+1}(i,j) - \theta_n(i,j)}{\theta_{n+1}(i,j)} \leq \epsilon \quad (14)$$

where n refers to time and i and j refer to space coordinates. The value of ϵ is chosen as 10^{-5} . A grid independence study was carried out with 31×31 , 41×41 , 61×61 mesh size. Because of the computational cost and accuracy considerations 41×41 mesh size was used in this investigation.

3. Results and Discussion

To evidence the accuracy of the code developed in this study, a comparison between the present numerical results with those of Bejan and Tien⁽⁴⁾ was made. In reference⁽⁴⁾ the thermal stability of superconducting composite media was studied using the classical heat diffusion model, and the Joule heating was approximated by neglecting the existence of the current sharing region (i.e., $T_{c1}=T_c$ or $\theta_{c1}=1.0$). Figure 2 illustrates the comparison of stability map obtained in this study and that presented in Ref.(4). This figure shows an excellent agreement between the results obtained using the presented code and those from the literature. This have established confidence in our results and enabled us to move forward. Unfortunately, no comparisons with experimental data on this problem were conducted because data do not exist at this time.

The result presented in this study are based on NbTi superconducting tape with the following specifications:

$$b=2 \mu\text{m}, As=5, T_o=4.2 \text{ K}, T_c=9.6 \text{ K}, T_{c1}=0.5 \times$$

$$(T_c + T_o), k=0.11 \text{ Wm}^{-1}\text{K}^{-1}, \alpha=2.2 \times 10^{-5} \text{ m}^2\text{s}^{-1}, \rho_o=6 \times 10^{-7} \Omega\text{m}, \phi_i=1 \times 10^{12} \text{ Wm}^{-3}, \tau_i=4.5 \text{ ps}, h=1 \times 10^4 \text{ Wm}^2\text{K}, \Delta x=60 \text{ nm}, \Delta y=15 \text{ nm}, \tau_q=0.1 \text{ ps}, J=1 \times 10^{10} - 4 \times 10^{10} \text{ Am}^{-2}.$$

Figures 3 and 4 show the transient response of the superconducting-tape maximum temperature θ_1 at different heating sources Q for Type II and Type I superconducting-tape, respectively. It is clear that the stability collapses as the heating source increases. Also, it can be seen that Type II is more stable than

Fig. 2 Comparison between the results obtained in this study and the results reported by Bejan and Tien⁽⁴⁾ ($\theta_{c1}=1$, $\theta=0.0$, $\beta=0$, $\xi=0.0$, $\eta=0.0$ and $\Delta\xi \times \Delta\eta=0.0$)

Fig. 3 Effect of dimensionless Joule heating on Type II superconducting tape thermal stability based on the two-dimensional wave model ($B=2$, $f=0.5$, $\phi=0.5$, $B_i=0.18$, $\theta_{c1}=0.1$, $\dot{\theta}=0.0$, $\xi=0.0$, $\eta=0.0$, $\beta_i=0.45$, $As=5$ and $\Delta\xi \times \Delta\eta=1.0$)

Fig. 4 Effect of dimensionless Joule heating on Type I superconducting tape thermal stability based on the two-dimensional wave model ($B=2$, $B_i=0.18$, $\phi=0.5$, $\dot{\theta}=0.0$, $\xi=0.0$, $\eta=0.0$, $\beta_i=0.45$, $As=5$ and $\Delta\xi\times\Delta\eta=1.0$)

Type I superconducting-tape. Type II remains stable for heating source smaller than 0.95, while, Type I, remains stable for heating source smaller than 0.46 only. The justification for this advantage of Type II superconducting-tape is its ability to redistribute the current excess to the critical current from the superconductor into the metal matrix within the normal zone. Knowing that the normal state resistivity of the superconducting material is much greater than that of the matel. Thus, current redistribution will minimizes the Joule heating effect through the superconducting tape.

Figures 5 and 6 show the transient response of the superconducting-tape maximum temperature θ_1 at different values of Biot number (B_i) for a Type II and Type I superconducting-tape, respectively. It is obvious that the stability collapses as the value of Biot number decreases. Moreover, these two figures provide another evidence that Type II is more stable than Type I.

Figure 7 shows the transient response of the superconducting-tape maximum temperature θ_1 at different initial disturbance area ($\Delta\xi\times\Delta\eta$) and for Type II superconducting-tape. The stability collapses as the disturbance area increases. For a line heat disturbance ($\Delta\xi\times\Delta\eta=0$) the superconducting tape may be stable for any value of Q .

Figure 8 shows the transient response of the superconducting-tape maximum temperature θ_1 at different disturbance duration time β_i and for Type II superconducting-tape. The parameter β_i represents the dimensionless disturbance duration time which is

Fig. 5 Effect of Biot number on Type II superconducting tape thermal stability based on the two-dimensional wave model ($B=2$, $f=0.5$, $Q=1.25$, $\theta_{c1}=0$, $\dot{\theta}=0.0$, $\xi=0.0$, $\beta_i=0.45$, $As=5$ and $\Delta\xi\times\Delta\eta=1.0$)

Fig. 6 Effect of Biot number on Type I superconducting tape thermal stability based on the two-dimensional wave model ($B=2$, $Q=1.0$, $\phi=0.5$, $\dot{\theta}=0.0$, $\xi=0.0$, $\beta_i=0.45$, $As=5$ and $\Delta\xi\times\Delta\eta=1.0$)

the time within which the second part of the disturbance is maintained within the normal zone. The stability collapses as disturbance duration time increases.

Figure 9 displays the transient response of the superconducting-tape maximum temperature θ_1 at different current sharing temperatures θ_{c1} . It is clear that as the current sharing temperature decreases, the stability collapses. The reason for this behavior is that the quantity of heat generation increases as θ_{c1}

Fig. 7 Effect of dimensionless disturbance area on composite superconducting tape thermal stability based on the two-dimensional wave model ($B=2$, $f=0.5$, $\phi=0.5$, $Q=0.25$, $B_i=0.18$, $\theta_{c1}=0.5$, $\dot{\theta}=0.0$, $\xi=0.0$, $\eta=0.0$, and $As=5$ and $\beta_i=0.45$)

Fig. 8 Effect of dimensionless disturbance duration time on Type II superconducting tape thermal stability based on the two-dimensional wave model ($B=2$, $f=0.5$, $\phi=0.5$, $Q=0.25$, $\theta_{c1}=0.1$, $\dot{\theta}=0.0$, $B_i=0.18$, $\xi=0.0$, $\eta=0.0$, $As=5$ and $\Delta\xi \times \Delta\eta=1.0$)

increases. However, this effect is negligible. This is obvious since as θ_{c1} increases, the superconducting tape can sustain a localized thermal disturbance having a higher intensity before a normal zone can be initiated.

Figure 10 shows the transient response of the superconducting-tape maximum temperature θ_1 at different time-rates of change $\dot{\theta}_i$ of the disturbance

Fig. 9 Effect of dimensionless current sharing temperature on Type II superconducting tape thermal stability based on the two-dimensional wave model ($B=2$, $f=0.5$, $\phi=0.5$, $Q=0.95$, $B_i=0.18$, $\dot{\theta}=0.0$, $\xi=0.0$, $\eta=0.0$, $\beta_i=0.45$, $As=5$ and $\Delta\xi \times \Delta\eta=1.0$)

Fig. 10 The effect of initial time-rate of temperature ($\dot{\theta}_i$) on type II-superconducting tape thermal stability based on the two-dimensional wave model ($B=2$, $f=0.5$, $\phi=0.5$, $Q=0.35$, $B_i=0.18$, $\theta_{c1}=0.1$, $\beta_i=0.45$, $\xi=0.0$, $\eta=0.0$, $As=5$ and $\Delta\xi \times \Delta\eta=1.0$)

initial temperature. The stability collapses as $\dot{\theta}_i$ increases.

The axial temperature distributions at different times and for Type II superconducting-tape based on the two heat conduction models are plotted in Figs. 11 and 12. The temperature distributions predicted by the hyperbolic model are quite different from that predicted its parabolic counterpart. It is illustrated

Fig. 11 Comparison between temperature profiles obtained based on diffusion model and wave model for type II-superconductor ($B=2$, $f=0.5$, $\phi=0.5$, $Q=0$, $B_i=0.18$, $\theta_{c1}=0.1$, $\dot{\theta}=0.0$, $\beta_i=0.0$, $As=5$, $\eta=0.0$ and $\Delta\xi \times \Delta\eta=0.15$)

Fig. 12 Comparison between temperature profiles obtained based on diffusion model and wave models for type II-superconductor (For $B=2$, $f=0.5$, $\phi=0.5$, $Q=0.5$, $B_i=0.18$, $\theta_{c1}=0.1$, $\dot{\theta}=0.0$, $\beta_i=0.45$, $\eta=0.0$, $As=5$ and $\Delta\xi \times \Delta\eta=1.5$)

that the parabolic model predicts instantaneous energy propagation through the superconducting tape while the hyperbolic model shows a wave nature of the energy propagation. Since hyperbolic model takes into account the thermal inertia effect that accounts for a finite time necessary to start the heat flow, a small time must be pass prior to heat flow initiation. This fact explains why the most significant difference

Fig. 13 The stability criterion for Type II superconducting tape based on the two different heat conduction models ($f=0.5$, $\phi=0.5$, $\theta_{c1}=0.05$, $\dot{\theta}=0.0$, $\beta_i=0.45$, $\xi=0.0$, $\eta=0.0$, $As=5$ and $\Delta\xi \times \Delta\eta=1.0$)

Fig. 14 The stability criterion for Type I superconducting tape based on the two different heat conduction models ($\dot{\theta}=0.0$, $\beta_i=0.45$, $\xi=0.0$, $\eta=0.0$, $As=5$ and $\Delta\xi \times \Delta\eta=1.0$)

between the two models occurs close to the tape surface and the effect is more noticeable at smaller times. It is clear from these figures that the wave model predicts a sharp discontinuous jumps in the axial temperature distribution. It is clear from Fig. 12

that these jump discontinuities decays with the dimensionless time. These phenomena continue till the heat wave has damped out. Furthermore, Figs. 11 and 12 shows that diffusion model underestimate the temperature at the early stages of time and overestimates it at large times.

Figures 13 and 14 show a stability map in terms of the critical Joule heating Q_c and the disturbance intensity B for both superconducting-tape types based on the two heat conduction models. It is clear that, the hyperbolic model predicts a wider stable region as compared to the predictions of the diffusion heat conduction. The deviations between the predictions of the two models vanish as the value of B decreases and the value of Biot number increases. It is obvious that as B increases, the conductor ability to sustain Q_c , while remaining stable decreases. Furthermore, these two figures show that the stability region for Type II superconductors is wider than that of Type I. The effect of Biot number is insignificant at small values of B .

4. Concluding Remarks

Thermal stability of superconducting tape is investigated under the effect of the of a two-dimensional hyperbolic heat conduction model. The parameters which are found to affect the superconducting tape thermal stability are the disturbance intensity B , volume fraction of the stabilizer f , initial disturbance area ($\Delta\xi \times \Delta\eta$), Biot number B_i , Joule heating source Q , current sharing temperature θ_{c1} , the initial temperature rate of change $\dot{\theta}_i$ and the disturbance duration time β_i . It is found that the two-dimensional hyperbolic model predicts a wider stable region as compared to the its parabolic counterpart, the hyperbolic model shows a higher peak at early stages and the difference between the prediction of the two models is significant at higher values of disturbance intensity and smaller values of time. Furthermore, this study shows that a Type II superconductor is more stable than for Type I. The superconducting tape stability improves as $\Delta\xi \times \Delta\eta$, B , β_i , $\dot{\theta}_i$ and Q decrease and as B_i , and θ_{c1} increase. However, the influence of the current sharing on the superconducting tape stability is negligible.

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