Analysis of Rotor Contact for Screw Compressor

Hirotaka KAMEYA**, Masakazu AOKI*** and Shigekazu NOZAWA****

A reliability evaluation of meshing rotor surfaces needs contact theory on a micro size area. We propose a method to evaluate whether a screw compressor rotor has enough contact fatigue strength. The method is based on analyses of three dimensional curvatures and the Hertz contact pressure. At a contact point one of the principal curvatures directs the rotor’s sealing line and this is calculated as the gap distribution. Another perpendicular curvature is calculated with an approximate arc to the rotor profile. As the contact point is an inflection point, the contact condition is first calculated on each side and then composed. Using our method we found the rotors of a typical screw compressor have enough strength. This is because the rotor contact pressure is calculated to be 0.57 times of a contact fatigue strength of their material.

Key Words: Screw Compressor, Hertz Contact Pressure, Curvature, Helix, Fatigue Strength

1. Introduction

A screw compressor is a kind of positive displacement rotary fluid machine. It is used in many industries as an air compressor, and as a refrigerant compressor for air conditioners. The structure of a screw compressor is shown in Fig. 1. It has a pair of meshing rotors, called male and female, with several helical lobes around them. The rotors and their casing bores create enclosed chambers in the helical grooves. When the rotors rotate, the chamber volume first gets larger, and then smaller. Gas is sucked into the chamber when the volume is enlarging, and compressed and discharged when the volume is reducing.

If the screw compressor is an oil injected type, one of the rotors rotates by a motor, and the other is driven by the first from contact with the lobe surface similar to a gear set. Even when the rotors are lubricated by oil, the contact points should be analyzed to see if their materials are strong enough. In fact a rotor surface may occasionally be damaged, especially if the compressor gets overloaded.

For example, Fig. 2 is pitting on a rotor surface.

To analyze the contact pressure between rotors using the Hertz contact theory, we have to know the normal load, Young’s modulus, Poisson’s ratio, and the principal curva-

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It is possible to calculate the contact pressure as an approximate value using the curvature on a transverse section instead of the principal curvatures. We, though, have not found papers about three-dimensional curvature analysis on screw compressor rotors. Therefore we propose a method of curvature analysis on a micro contacting area, and estimate contact pressure to be lower than material strength.

2. Nomenclature

\begin{align*}
a, b, c, d, e & : \text{points on female rotor} \\
f, g, h, i, j, k & : \text{points on male rotor} \\
K_{rt} & : \text{gap varying ratio with rotor rotation} \\
L & : \text{position on sealing line} \\
N & : \text{normal load at contact point} \\
\eta & : \text{overlap ratio of rotor lobe} \\
R_p & : \text{pitch circle radius} \\
R & : \text{rotation radius of contact point} \\
T & : \text{transmission torque} \\
X, Y, Z & : \text{coordinate of point on profile} \\
\sigma_H & : \text{Hertz contact pressure (stress)} \\
\theta & : \text{angle between rotational tangent and surface perpendicular}
\end{align*}

3. Profile and Sealing Line

Our rotor profile is defined as the transverse section in Fig. 3 and its specification is in Table 1. The profile constitutes four curves whose boundaries are marked a–e on the female and f–j on the male rotors. Each curve is defined as an arc, parabola, or generated curve by its partner curve. At every boundary, the curves connect smoothly.

A rotor surface consists of the envelope of the profile rotating and moving along its axis proportionally. The male and female rotors touch each other at one or three points on any transverse section or any rotational angle. From a three-dimensional viewpoint, these touching points link and become a sealing line, which divides high and low gas pressured chambers, as shown in Fig. 4.

We define distance \( L \) from the origin c or h as the positioning indicator on a sealing line. On the e or j side from the origin, \( L \) is expressed as positive, and on the a or f side, \( L \) is negative.

A sealing line is a theoretical contact line between the rotors, but in practice there is a little gap on the line. This is because a little gap is needed to avoid rotor interference caused by profile error, center distance error, thermal expansion, and vibration.

Therefore when one rotor drives another from contact, contact points between rotors are settled to be in a specific position on the profile.

Two principal curvatures on each rotor are needed around the contact point to calculate the contact pressure. One of the principal curvatures is along the sealing line and the other is perpendicular to it. Although the principal curvature doesn’t direct exactly same as the sealing line on the single rotor surface, it becomes to direct same when the male and female rotors mesh each other. The theoretical curvatures along the sealing line on both rotors are the same and their real relative curvature is almost zero at every point on the sealing line. Another principal curva-

<table>
<thead>
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<th>Table 1 Specifications of sample rotors</th>
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<tr>
<td>Tooth number of male rotor</td>
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<tr>
<td>Tooth number of female rotor</td>
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<tr>
<td>Diameter of male rotor</td>
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<tr>
<td>Diameter of female rotor</td>
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<tr>
<td>Center distance</td>
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<tr>
<td>Lead of male rotor</td>
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<td>Lead of female rotor</td>
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Fig. 3 Screw rotor profile

Fig. 4 Sealing lines on screw rotors
4. Curvature along Sealing Line

The smallest relative curvature along the sealing line is calculated as the gap distribution. The distribution supposes the gap is first given uniformly, and the male rotor drives the female rotor via surface contact.

Using a coordinate system on the transverse section shown in Fig. 5, surface point S has two vectors. Vector $\vec{W}$ is perpendicular to the surface, and vector $\vec{D}$ is a tangent of the rotation with radius $R$. A cosine of the angle $\theta$ between these vectors means a gap varying ratio with rotor rotation. The ratio $K_{rt}$ is expressed as follows, and its value along the sealing line is shown in Fig. 6.

$$K_{rt} = \cos \theta = \frac{Y}{|\vec{W}|}$$  \hspace{1cm} (1)

When we set the gap uniformly to 50 $\mu$m, the male rotor rotates 98 $\mu$m along its pitch circle to touch the female rotor’s trailing flank, and gap $G$ changes as:

$$G = 98K_{rt} + 50 \quad [\mu m]$$  \hspace{1cm} (2)

Then a point i on the male rotor meets a point d on the female, and they become the contact point d/i.

Around the contact point d/i, non-dimensional gap is shown in Fig. 7 as a solid line. It is described as a non-dimensional value comparing with a base of the rotor diameter. We approximate the gap line to an arc, as shown by the broken line. The approximate arc is used for the principal curvature along the sealing line.

A maximum difference between the calculated gap and the approximate arc around the contact point is smaller than roughness of the rotors. Then an effect of the difference on a contact pressure is too small.

The gap size of the neighborhood of the contact point d/i is less than the usual surface roughness or profile error. This means that the contact point may move along the sealing line through a range defined from a comparison between gap size and the roughness or the profile error.

5. Curvature Perpendicular to Sealing Line

Every point on the profile can be approximated to an arc as a second order tangent on a transverse section. If this arc moves along the rotor’s helix, it makes the envelope shown in Fig. 8. This envelope has same three-dimensional curvatures through the approximated point. A perpendicular curvature to the sealing line is selected from among these curvatures.

The perpendicular curvature is a function of the position on a sealing line. The male and female rotors both have it as shown in Fig. 9. They are described as non-dimensional values comparing with the rotor diameter as a base. They are uncontinuous lines with a step at the contact point d/i because the approximated arcs are different.
Fig. 9 Normal curvatures perpendicular to sealing line

on each side of the contact point. Both curvatures at the contact point marked in Fig. 9 are used to calculate the contact pressure.

6. Transmission Torque and Normal Load on Surface

According to the method from Fujiwara, the gas torque for each rotor is calculated with the profile and gas pressures that apply to the grooves[3]. A gas torque means the required torque to continue compressing against gas pressure. A male rotor gas torque is generally much larger than a female. Thus, the male rotor is set to the drive rotor and the female is driven by the male usually because the transmission torque is smaller. In this structure, transmission torque $T$ is made up of the female rotor gas torque and a little friction.

The transmission torque $T$ simultaneously acts at several contact points as the sum of normal loads $N_1, N_2, N_3$ and $N$, as shown in Fig. 10. The normal load component $N$ was averaged across time because the number of contact points varies according to the rotor rotation. The relational expression of the transmission torque and normal load is:

$$N = \frac{T}{nR\cos\theta} \quad (3)$$

Under a typical pressure condition for a refrigerant compressor, the transmission torque $T$ is 8.5 N·m. The normal load $N$ is calculated to be 85 N with expression (3).

7. Hertz Contact Pressure

The contact pressure at contact point $d/i$ is calculated on the $c/h$ and $e/j$ sides separately, because the contact point is an inflection point and both side curvatures are different from each other, as shown in Table 2. A normal load gives the approaching deformation and contact pressure using a method from Tanaka to calculate the elliptical Hertz contact [4]. Then variations of the approaching deformation and contact pressure are acquired on each side as functions of normal load, including the material properties (Young's modulus and Poisson's ratio). The modulation of these functions gives the normal load and contact pressure graphs, which have a common horizontal axis as in the approaching deformations in Figs. 11 and 12.

In Fig. 11, the normal loads are indicated as half the calculated values because each line means a half field contact. By summing these lines, the composed line becomes...
an integrated relationship between the normal load and the approaching deformation.

Using this composed line, the given normal load 85 N indicates a deformation of 0.8 \( \mu \)m. Then this deformation means a contact pressure of 0.305 \text{ [MPa] } at the c/h side and 0.57 at the e/j side, as in Fig. 12. The larger pressure 0.57 is important as an evaluation of the strength. These contact pressures are indicated as non-dimensional values comparing with the weakest material’s strength \( S \) which is explained in the next section.

8. Comparison Contact Pressure and Strength of Rotor Material

The contact fatigue strength data of several materials that are used as power transmission gears\(^{(5)}\) is available. The contact condition of gear surfaces is similar to screw rotors for contact pressure, rotational speed, slipping ratio, friction, lubrication, and materials. We think this data can be used to evaluate the screw rotor strength. We picked up some data on the materials from which the screw rotors can be made as shown in Fig. 13. They are different from each other according to their ingredients and processing. The weakest strength \( S \) of material C is used as a base of this comparison.

The screw rotor’s contact pressure 0.57\( S \) [MPa], as detailed in section 7, is less than the material strength of 1.05 \( S \) [MPa]. Even if the load increases, the contact pressure would not increase proportionally because the contact area would expand. Therefore, this comparison suggests the reliability of the rotor surface is sufficient.

9. Conclusions

We proposed a method to evaluate whether a screw compressor rotor has enough contact fatigue strength. The method and example used is summarized as follows:

1) A principal curvature along the sealing line is calculated as the gap distribution which is given by the gap various ratio of rotation. Another principal curvature is calculated as the envelope of an approximate arc of the rotor profile. The contact pressure can be calculated using these curvatures, material properties, and the normal load taken from the rotor profile and its gas pressure condition.

2) This method applied for an example rotor of a typical screw compressor and it was evaluated to be strong enough. This is because the rotor contact pressure is calculated to be 0.57 times of a contact fatigue strength of its material.

References


\( \text{(2) Kauder, K. and Dreischhoff, U., Verschleißschutz für Schraubenrotoren, VDI Berichte, (in German), Nr.859, (1990), p.324.} \)

