Numerical Simulation on Annular Hall Energy Conversion Device under Wide Range of Operating Condition

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The characteristics of the annular Hall MHD device under the wide range of operating conditions are investigated in detail by carrying out time-dependent three-dimensional numerical simulations with the purpose of providing fundamental data for the future industrial application of the device. Because of the configuration of the device, it can be operated both as generator and accelerator. Under power extraction, the highest enthalpy extraction of 5\% for the load resistance of 3.0\,\Omega and the highest isentropic efficiency of 15\% for 2.0\,\Omega are obtained. The nonuniform plasma induced by weak ionization of seed forms a clockwise spiral structure in $\theta-z$ plane. Due to this unsteady behavior of the discharge, output electrical power fluctuates periodically at the frequency of approximately 20–40\,kHz. Finally, the Lorentz efficiency is evaluated under power input. The highest Lorentz efficiency of 58\% can be obtained for $-0.5\,kV$.

Key Words: Magnetohydrodynamics, Energy Conversion, Power Generator, Accelerator, Supersonic Flow, Computational Fluid Dynamics

1. Introduction

The magnetohydrodynamic energy conversion can be categorized as (1) the extraction of electrical energy from working gas (energy), (2) the conversion of the electrical energy into the kinetic energy of gas by means of an external input of the electric power. The engineering application in the former case is expected as the generator (MHD electrical power generation), and in the latter case as the accelerator or electro-magnetic propulsion.

The researches on the non-equilibrium MHD generator have been conducted at Tokyo Institute of Technology and its performance and the physical phenomena in the generator have been made clear both experimentally(1),(2) and numerically(3),(4). The shape of the generator considered here is the ‘disk-shape’ and the MHD channel is sandwiched between two disk walls. This ‘disk-shape’ is of great advantage to the generator because of the easiness of realizing the strong enough magnetic field perpendicular to the flow, however, is not applicable as the accelerator because of its shape. On the other hand, as shown in Fig. 1, the device with a cylindrical shape (hereinafter this device is referred to as ‘annular Hall MHD device’) has been proposed(5). Because of this cylindrical geometry of the channel, this device can work both as the generator and the accelerator. Furthermore, since a pair of ring-shaped electrode is installed in this device, induced arc does not stay at one position but rotates azimuthally by Lorentz force, thus severe damage on the electrode is avoided and the long lifetime of electrode can be expected. Although this device has such an attractive feature mentioned above, the detail investigations on this device have not been made due to the difficulty of realizing the strong radial magnetic field needed for the efficient energy conversion.

The progress in hypersonic air-breathing engine is outstanding and it is reported recently that the performance of scramjet engines can be improved by MHD energy bypass(6),(7). In this MHD energy bypass scramjet engine, an MHD generator installed in the engine inlet controls the flow and extracts the flow energy as an electrical power. This electrical power is bypassed into an MHD accelerator installed downstream of the combustor. Then, the flow after passing through a combustor is accelerated by the MHD accelerator. Since an MHD power generator and an MHD accelerator in this system are the key tech-
technology for improving engine performance, the importance of the researches on MHD technology are now growing.

This annular Hall MHD device presented in this paper can be considered as one of the potential applications for the MHD energy bypass system. Therefore, in this study, with the purpose of providing fundamental data for the future application of this device, the characteristics of the annular Hall MHD device under the wide range of operating conditions are investigated in detail by carrying out time-dependent three dimensional numerical simulations.

**Nomenclature**

- $B$: Magnetic flux density [T]
- $c_p$: Specific heat at constant volume [J·kg⁻¹·K⁻¹]
- $E_i$: Electric field vector [V·m⁻¹]
- $E_s$: Total energy per unit volume [J·m⁻³]
- $e$: Charge of an electron [C]
- $I$: Current [A]
- $j_i$: Current density vector [A·m⁻²]
- $k$: Boltzmann constant [J·K⁻¹]
- $k_i$: Ionization rate coefficient of $i$ th particle [m³·s⁻¹]
- $k_{ij}$: Recombination rate coefficient of $i$ th particle and $j$ th particle [m³·s⁻¹]
- $m_e$: Mass of an electron [kg]
- $m_h$: Mass of heavy particle [kg]
- $n_i$: Number density of electron [m⁻³]
- $n_i$: Number density of $i$ th particle [m⁻³]
- $n_i^+ = n_i^+ = k_j n_i n_j - k_{ij} n_i^+ n_j^+$
- $n_e$: Number density of electron [m⁻³]
- $p_g$: Static gas pressure [Pa]
- $q_i$: Heat flux vector of working gas [J·m⁻²·s⁻¹]
- $r$: Radius [m]
- $r_{inner}$: Inner radius of annular Hall MHD device [m]
- $r_{outer}$: Outer radius of annular Hall MHD device [m]
- $R_L$: External load resistance [Ω]
- $t$: time [s]
- $T_e$: Electron temperature [K]
- $T_g$: Static gas temperature [K]
- $\vec{u}_i$: Gas velocity vector [m·s⁻¹]
- $V_h$: Cathode voltage to the anode [V]

**Greek**

- $\beta$: Hall parameter
- $\delta_{ij}$: Kronecker's delta
- $\epsilon_i$: Ionization energy of $i$ th particle [J]
- $\mu_m$: Molecular viscosity coefficient [kg·m⁻¹·s⁻¹]
- $\mu_t$: Turbulent viscosity coefficient [kg·m⁻¹·s⁻¹]
- $\tilde{\nu}_e$: Total collision frequency for electron with heavy-particle [s⁻¹]
- $\nu_{ei}$: Averaged collision frequency for electron with $i$ th particles [s⁻¹]
- $\phi$: Electrical potential [V]
- $\rho$: Density of the working gas [kg·m⁻³]
- $\sigma$: Electrical conductivity [S·m⁻¹]
- $\Pi_{ij}$: Stress tensor [kg·m⁻¹·s⁻¹]
- $\tau_{ij}$: Viscosity stress tensor [kg·m⁻¹·s⁻¹]

2. Basic Equations and Numerical Procedure

2.1 Basic equations

For non-equilibrium plasma in annular Hall MHD device, electron and heavy particles are under equilibrium state for different temperature, resulting from much smaller collision cross sectional area between electron and heavy particles. To describe the non-equilibrium plasma with such characteristic, two-temperature model is adopted and then governing equation for charged particles and heavy particles are expressed separately.

2.1.1 Governing equations for charged particles

A non-equilibrium MHD plasma consists of noble gas atoms, noble gas ions, seed atoms, seed ions and electrons. The governing equations for charged particles are as follows.

**Ion Continuity Equation**

$$\frac{\partial n_i^+}{\partial t} + \nabla \cdot (\vec{n}_i^+ \vec{u}) = n_i^+ = k_i n_i n_j - k_{ij} n_i^+ n_j^+ \quad (1)$$

**Generalized Ohm's Law**

$$j_i = \sigma E_i, \quad j_0 = \frac{\sigma}{1 + \beta^2} (E_{th} - \beta E_e - \beta \mu u_0 B + u_0 B) \quad (2)$$

$$j_e = \frac{\sigma}{1 + \beta^2} (E_{th} + E_e - u_0 B + \beta u_0 B)$$

$$\sigma = \frac{e^2 n_e}{m_e \tilde{\nu}_e}, \quad \beta = \frac{eB}{m_e \tilde{\nu}_e} \quad (h = \text{seed, noble gas, ion})$$

**Electron Energy Equation**

$$\frac{\dot{\tilde{E}}_e}{\sigma} = 3n_e k (T_e - T_0) \sum \frac{m_e}{m_i} \tilde{\nu}_e + \sum \frac{n_i^+}{2} \left( \frac{3}{2} k T_e + \epsilon \right) \quad (3)$$

As for the ionization rate term ($\dot{n}_i^+$) in Eq. (1), ionization by the electron collisions and three body recom-
bination are considered. The quasi-steady electron energy equation (3) is adopted, since the relaxation time in electron temperature is shorter than that in electron density. In Eq. (3), the electron radiation loss is neglected in this simulation because the electron radiation loss is much smaller than the collision loss by the order of $10^{-2} - 10^{-3}$.

### 2.1.2 Governing equations for heavy particles

The governing equations for heavy particles can be described as follows.

#### Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$  \hspace{1cm} (4)

#### Momentum Equation

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \cdot \tau_{ij} + \vec{j} \times \vec{B}$$  \hspace{1cm} (5)

where, $\tau_{ij} = (\mu_n + \mu_l) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right)$.

#### Energy Equation

$$\frac{\partial E_i}{\partial t} + \nabla \cdot (E_i \vec{u}) = \nabla \cdot (\Pi_{ij} \vec{u}^* - \vec{j} B)$$  \hspace{1cm} (6)

$$E_i = \rho \left( c_i T_i + \frac{1}{2} |\vec{u}|^2 \right)$$

The coefficients of turbulent viscosity used in $\tau_{ij}$ and turbulent thermal conductivity are calculated on the basis of Baldwin-Lomax turbulent model. (11)

#### 2.1.3 Maxwell equation

By adapting conventional MHD approximations, Maxwell equations are reduced to the following elliptic equation:

$$\nabla \times \vec{E} = \vec{0}$$  \hspace{1cm} (7)

and

$$\nabla \cdot \vec{j} = 0.$$  \hspace{1cm} (8)

#### 2.2 Numerical procedures

The ion continuity equation (1), the momentum equation (4), the momentum equation (5), and the energy equation (6) of the hyperbolic equation are solved by the CIP method applied to curvilinear coordinate extended to the three-dimensional case. The substitution of Eq. (2) into Eq. (8) reduces to the following elliptic equation:

$$\frac{\partial}{\partial r} \left[ \frac{\sigma r}{1 + \beta^2} \frac{\partial \phi}{\partial \theta} + \frac{\partial}{\partial \theta} \left[ \frac{\sigma \phi}{1 + \beta^2} - \beta \frac{\partial \phi}{\partial z} - \beta \frac{u_B - u_B}{B} \right] \right]$$

$$+ \frac{\partial}{\partial z} \left[ \frac{\sigma r}{1 + \beta^2} \left( \beta \frac{\partial \phi}{\partial \theta} + \frac{\partial \phi}{\partial z} + u_B - \beta \frac{u_B}{B} \right) \right] = 0$$  \hspace{1cm} (9)

by introducing a potential function $\phi$ defined from Eq. (7) as

$$E_r = -\frac{\partial \phi}{\partial r}, \quad E_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta}, \quad E_z = -\frac{\partial \phi}{\partial z}.$$  \hspace{1cm} (10)

The elliptic equation (9) is discretized with a finite difference method, and is solved by Bi-CGSTAB method. (12) In this numerical simulation, the effect of sheath is neglected for simplicity. However, it is important to consider the voltage drop near the electrode due to the electron sheath and this is a work to be done in the near future. The electron temperature is obtained by solving the non-linear algebraic equation of (3) with bi-section method.

#### 2.3 Calculation conditions and boundary conditions

The cross sectional view of the annular Hall MHD device and the coordinate system is shown in Fig. 1. An external magnetic field is applied radially and the ring-shaped anode and cathode are installed coaxially. The applied magnetic flux density has a distribution in $r$-direction as shown in Fig. 2. The distribution is simply given as $B(r) = 3.0 \tau_{inner}(z = 0, 2)$, and the optimization of the magnetic field has not yet made in this simulation. The highest magnetic flux density is 3.0 T at the radius of 20.0 mm and the lowest is 0.75 T at the radius of 80.0 mm.

The calculation region covers from the inlet of the throat to the exit of the channel along the $z$ direction, between the walls in the normal ($r$-) direction and from 0 to $\pi$ along the azimuthal ($\theta$-) direction to capture the discharge structure induced by ionization instability and also to lighten computational load. The inner radius $r_{inner}(z)$ and the outer radius of the device $r_{outer}(z)$ are given as $r_{inner}(z) = -0.1z + 0.04$ and $r_{outer}(z) = 0.1z + 0.06$, respectively. In the nozzle region ($z = 0.0$ mm – 34.0 mm), Lorentz force is assumed not to be effective and only fluid properties are solved. The time step in simulation is determined as $\Delta t = 0.04$ μsec from the assumption that $\tau_{Tr} < \Delta t < \tau_{Te}$, where $\tau_{Tr}$ and $\tau_{Te}$ are relaxation times of electron temperature and electron number density, respectively. The number of grid points in $r$, $\theta$ and $z$ directions are given as 31, 30 and 101 in the present simulation.

The boundaries of $\theta = 0$ and $\theta = \pi$ are joined with the periodical condition. As for the inlet boundary condition for Eq. (1), the ion number densities are given by assuming

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**Fig. 2** Distribution of magnetic flux density
Table 1 Operating conditions of annular Hall MHD device

<table>
<thead>
<tr>
<th>Working gas</th>
<th>He-Cs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total enthalpy flux</td>
<td>4.8 MW</td>
</tr>
<tr>
<td>Seed fraction</td>
<td>6.0×10⁻⁵</td>
</tr>
<tr>
<td>Load resistance</td>
<td>0.1~4.0 Ω</td>
</tr>
<tr>
<td>V_h</td>
<td>-1.5kV~0.1kV</td>
</tr>
<tr>
<td>Inlet swirl ratio</td>
<td>0.0</td>
</tr>
<tr>
<td>Magnetic flux density</td>
<td>0.75~3.0 T</td>
</tr>
<tr>
<td>Inlet stagnation temperature</td>
<td>2000 K</td>
</tr>
<tr>
<td>Inlet stagnation pressure</td>
<td>202.6 kPa</td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>0.46 kg/s</td>
</tr>
</tbody>
</table>

Saha equilibrium for the fixed inlet electron temperature of 3000 K. All the physical values at the inlet boundary are fixed to the operating parameters listed in Table 1. Inlet axial velocity is assumed to be uniform along radius. For the exit boundary conditions, all the physical properties are given by the free boundary condition, i.e., differential values along the z-direction are zero.

3. Results and Discussion

The characteristics of annular Hall MHD device were investigated numerically by changing either external load resistance or external applied voltage under the operating condition listed in Table 1.

The voltage-current characteristics obtained from the numerical simulation are shown in Fig. 3. In this figure, the positive voltage implies that the electrical power is extracted from the plasma, thus, under this condition, this device works as the MHD generator. On the contrary, by applying the inverse voltage (the negative voltage), electrical power can be input into the plasma. This electrical power is partly converted into kinetic energy of the working gas, therefore, under this condition, this device works as the MHD accelerator. In the following sections, the operating characteristics of the annular Hall MHD device under power extraction and power input are discussed.

3.1 Operating characteristics of annular Hall MHD device under power extraction

For an MHD electrical power generation, enthalpy extraction ratio and isentropic efficiency are often used as the performance indicator. Enthalpy extraction ratio is defined as the ratio of electrical output power to the total thermal input to the generator. Isentropic efficiency is the ratio between the actual work and the isentropic work of the working gas through expansion process. Enthalpy extraction ratio and isentropic efficiency are plotted as a function of load resistance in Fig. 4. As will be described later, output power fluctuates due to the ionization instability for lower load resistances. The amplitude of the fluctuation is shown in this figure with error bar. The highest enthalpy extraction ratio of 5% for the load resistance of 3.0Ω and the highest isentropic efficiency of 15% for 2.0Ω are obtained. These efficiencies are not so high for the MHD electrical power generation (30% of enthalpy extraction ratio and 80% of isentropic efficiency are expected for the commercial application of MHD electrical power generation[14]). This is because the magnetic field is very weak for the power generation and the optimization of the channel shape and the operating condition are not perfectly made. It can be feasible to improve the performance of the device as an MHD power generator by utilizing a higher magnetic flux density and by modifying operating conditions and the channel geometry.

Figure 5 shows the $r-z$ two-dimensional distribution of electron temperature along $\theta=\pi/2$ under the load resistance of 2.0Ω. Under this condition, discharge becomes steady and forms uniform structure in $\theta$-direction. Since the upstream electron temperature is 5 000~6 000 K, cesium as seed material is fully ionized. It is found from this figure that the electron temperature has a nonuniform dis-
Fig. 5  $r-z$ two-dimensional distribution of electron temperature along $\theta = \pi/2$ under the condition of the external load resistance of 2.0Ω

Fig. 6  Axial velocity profile at $r = 50$ mm and along $\theta = \pi/2$ under the load resistance of 2.0Ω

Fig. 7 Three-dimensional discharge structure (electron-temperature) under the load resistance of 0.5 Ω

It is found from Figs. 3 and 4 that the output power decreases for the load resistances lower than 2.0Ω. For the lower load resistances ($< 1.0$Ω), large enough Joule heating which leads to the fully ionized seed plasma cannot be obtained at the inlet of the device. This lack of Joule heating induces the spatially nonuniform distribution of electron number density in $\theta$-direction at the inlet, which is caused by the ionization instability. The electron temperature disturbances resulting from the instability propagate downstream at flow velocity. According to the linear and nonlinear theory of ionization instability in nonequilibrium plasma, the angle between this wave vector and the current is found to be almost $\pi/2^{(15),(16)}$. As the result of that, the discharge becomes unsteady and forms the nonuniform structure called ‘streamer’ in the plain perpendicular to the magnetic field$^{(15),(16)}$. Under this configuration of the device, the discharge is found to form a clockwise spiral structure in $\theta-z$ plane as shown in Fig. 7. Although it’s not shown in the figure, this discharge structure does not affect the flow field. Due to this unsteady behavior of the discharge, output electrical power fluctuates periodically at the frequency of approximately 20–40 kHz. This frequency is the same order of magnitude as the one estimated from the residence time of working gas in the device. Figure 8 (a) shows the input/output power fluctuation frequency as a function of averaged output/input power, and Fig. 8 (b) shows the power spectrum of output power fluctuation for $R_L = 0.5$Ω. As seen in Fig. 8 (a), under the power generation, the frequency increases by lowering the load resistance. Lowering the load resistance decreases the loading factor, and then the Faraday current density decreases. This decrease in Faraday current weakens the interaction between plasma and working gas, as a result of that, gas velocity increases. Since disturbances propagate at flow velocity, fluctuation frequency increases with lowering the load resistance. On the other hand, through electrical power input into plasma, electron temperature increases by Joule heating. Then, the fluctuation frequency decreases and the discharge is finally stabilized for $V_h < -150$ V.
3.2 Operating characteristics of annular Hall MHD device under power input

Figures 9 and 10 show the profiles of the electrical potential and \( \theta \)-component of current density \( (j_\theta) \) at \( r = 50 \text{ mm} \) along \( \theta = \pi/2 \) for \( V_h = -0.15 \text{ kV} \) and \(-1.5 \text{ kV} \), respectively. It is found from Fig. 9 that in the case of \( V_h = -0.15 \text{ kV} \), the gradient of the electrical potential becomes positive (negative electrical field) in the upstream region and it becomes negative (positive electrical field) in the downstream region. The former corresponds to the ‘power extraction’ and the latter to the ‘power input’. It is understandable from Fig. 10 that in the case of \( V_h = -0.15 \text{ kV} \), \( j_\theta \) has positive value under power extraction, therefore, Lorenz force acts on the working gas as retarding force. While, under power input, \( j_\theta \) becomes negative and Lorenz force acts as accelerating force. Under the condition of \( V_h = -1.5 \text{ kV} \), \( j_\theta \) has negative value and Lorenz force acts as accelerating force throughout the channel, except near the downstream edge of cathode. From the discussion above, under the condition of \( V_h = -0.15 \text{ kV} \) as shown in Fig. 11, working gas is decelerated by the Lorenz force attributed to the negative electric field in the upstream region, so that, an effective acceleration cannot be obtained and exit velocity is lower than that without MHD interaction. Therefore, to obtain an effective acceleration, the applied voltage needs to be controlled so that the electrical potential becomes negative in the whole region of the channel.

Figure 12 shows the \( r - z \) two-dimensional distributions of electron temperature and static gas temperature along \( \theta = \pi/2 \) under the condition of \( V_h = -1.5 \text{ kV} \). Under this condition, plasma is stable and uniform plasma along \( \theta \)-direction can be maintained. As seen in this figure, the electron temperature becomes locally higher in downstream near the outer wall. Since the working gas is accelerated and has a relatively high velocity in the downstream, the Joule heating attributed to the electromotive force becomes large. This is the reason why the locally higher electron temperature is generated in the downstream. The phenomenon that the electron temperature becomes higher near the outer wall can be explained in the same way as found in the previous section. The temperature of heavy particle becomes \(~1900 \text{ K}\) in the downstream near the outer wall because the translational energy...
of the electron is transferred to the heavy particle through collisions. As seen in this figure, plasma is still thermally non-equilibrium even under this high voltage.

Figure 13 shows the Lorentz efficiency as a function of $|V_h|$. The Lorentz efficiency is defined as:

$$\eta_L = \frac{\text{Work done by Lorentz force}}{\text{Total input power}}$$

$$= \frac{\iint |\vec{d} \cdot (\vec{j} \times \vec{B})| dV}{V_h I}$$

(11)

$$= 1 - \frac{\iint |\vec{j}^2| dV}{\sigma V_h I}$$

(12)

It is understandable from this figure that the highest Lorentz efficiency of 58% can be obtained for $V_h = -0.5$ kV. For $V_h$ lower than 0.5 kV ($V_h > -0.5$ kV), sufficient electrical conductivity cannot be obtained owing to the lack of Joule heating. Thus, $j_\theta$ providing acceleration force is not so high, which results in the low Lorentz efficiency. On the other hand, For $V_h < -0.5$ kV, most of the input power is transferred to the internal energy of heavy particles through an excessive Joule heating. This fact also leads to the reduction of Lorentz efficiency.

4. Conclusion

The characteristics of the annular Hall MHD device under the wide range of the operating conditions are investigated in detail by carrying out time-dependent three-dimensional numerical simulations. The following conclusions can be drawn.

(1) Under power extraction, the highest enthalpy extraction ratio of 5% for the load resistance of 3.0 $\Omega$ and the highest isentropic efficiency of 15% for 2.0 $\Omega$ are obtained.

(2) Attributed to a weakly ionized seed with low electron temperature, the discharge becomes unsteady and forms the nonuniform structure of 'streamer' in $\theta-z$ plane. Under this configuration of the device, the discharge forms a clockwise spiral structure. Due to this unsteady behavior of the discharge, output electrical power fluctuates periodically at the frequency of approximately 20–40 kHz.

(3) The highest Lorentz efficiency of 58% can be obtained for $V_h = -0.5$ kV. For $V_h > -0.5$ kV, sufficient electrical conductivity cannot be obtained owing to the lack of Joule heating. Thus, $j_\theta$ providing acceleration force is not so high, which results in the low Lorentz efficiency. On the other hand, For $V_h < -0.5$ kV, most of the input power is transferred to the internal energy of heavy particles through an excessive Joule heating. This fact also leads to the reduction of Lorentz efficiency.

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