Combustion Oscillation Analysis
of Premixed Flames at Elevated Pressures

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A new analytical time lag flame model based on Bloxidge’s flame model was introduced, which calculates the combustion oscillation of premixed flame to take into account the distribution of heat release rate and flame speed that was calculated by analytical formulas dependent on pressure, temperature, fuel-to-air ratio and velocity. The transfer matrix technique using the new flame model was applied to the calculation of acoustic resonance characteristics. To verify the model, combustion oscillation experiments were performed for methane-air premixed flames stabilized by a swirl burner at elevated pressures in a range of 0.6 – 0.9 MPa. The fluctuating pressure had a maximum peak at a specific value of $f\tau_f$, where $f$ is the resonance frequency and $\tau_f$ is the passing time of premixed gas through the flame zone. The analytical model could simulate the dependency of the fluctuating pressure local peak on the fuel-to-air ratio and the static pressure.

Key Words: Oscillating Combustion, Premixed Combustion, Flame, Turbulent Flow, Swirling Flow, Gas Turbine, Elevated Pressure, Acoustic Resonance, Transfer Matrix Analysis

1. Introduction

Damping combustion oscillation is one of the most important tasks for the development of low NOx combustors, because this oscillation tends to occur with lowered NOx emissions and may cause irreparable damage to combustors. Premixed flame has been widely used to reduce NOx, but combustion oscillation is induced due to the flame instability. The combustion oscillation can be a self-excited oscillation generated by the feedback loop of acoustic systems including flames. The combustion oscillation occurs when Rayleigh’s criterion is satisfied, that is, the fluctuating pressure and the fluctuating heat release rate are in phase for the oscillation period. The condition is achieved by the phase matching of the acoustic time, which is set by the combustor length, and the combustion time, which is characterized by the burning time of premixed gas. Analytical models for combustion oscillation have been reviewed(1), (2). But the onset of combustion oscillation has not been well predicted, because the phenomenon is affected by many factors such as pressure, temperature, fuel-to-air ratio, velocity, geometrical configuration and so on. It is very important to clarify the dependency of the combustion oscillation on these factors when investigating the phenomenon and designing combustors. The objectives of the present study were to elucidate the relationships between these factors and to develop a new analytical model for combustion oscillation.

The transfer matrix approach is useful to calculate the acoustic mode in a one-dimensional network system. Bohn and Deuker(3) calculated the stability of combustion oscillation by evaluating an acoustic amplifying factor using the transfer matrix. The analytical results were crucially dependent on the transfer matrix of the flame, but so far the flame model has not been well established. Rayleigh’s criterion shows that the combustion oscillation is strongly coupled with the unsteady heat release rate.
Bloxidge et al.\(^{(4)}\) studied a premixed flame in a duct and introduced an experimental formulation for the fluctuating heat release rate in relation to the inlet velocity fluctuation. The development of flame model was continued for premixed flames\(^{(5)}\).

In the present study, a new flame model for the transfer matrix analysis was introduced by using Bloxidge’s model, assuming a mean heat release rate distribution and a turbulent flame speed model for evaluating the flame length. The authors’ HTA (Hyperbolic Tangent Approximation) model\(^{(6)}\) was applied to the calculation of turbulent flame speed which is dependent on pressure, temperature, fuel-to-air ratio and velocity. To verify the model, combustion oscillation experiments were performed for methane-air premixed flames stabilized by a swirl burner with elevated pressure.

2. Nomenclature

\(A_{O_2}\): Mass Fraction of \(O_2\) in Air (–)
\(a\): Speed of Sound (m/s)
\(C_p\): Specific Heat Capacity (J/kgK)
\(E_0\): Activation Energy (K)
\(F/A\): Fuel-to-Air Ratio (–)
\(f\): Frequency (Hz)
\(G\): Acoustic Amplification Factor (–)
\(G_f\): Flame Response Function (–)
\(k\): Wave Number (1/m)
\(L_f\): Total Flame Length (m)
\(L_m\): Flame Length until Heat Maximum (m)
\(l\): Flow Path Length (m)
\(l_t\): Taylor Microscale (m)
\(M_{O_2}\): Molecular Weight of \(O_2\) (kg/kmol)
\(P\): Static Pressure (Pa)
\(p\): Fluctuating Pressure (Pa)
\(Q\): Mean Heat Release Rate per Unit Length (W/m²)
\(q\): Fluctuating Heat Release Rate per Unit Length (W/m²)
\(R_f\): Turbulent Reynolds Number (–)
\(r_B\): Center Rod Radius of Burner (m)
\(r_o\): Air Path Outer Radius of Burner (m)
\(S\): Flow Path Cross Section Area (m²)
\(S_f\): Turbulent Flame Speed (m/s)
\(S_u\): Laminar Flame Speed (m/s)
\(T\): Absolute Temperature (K)
\(U_f\): Axial Velocity at Flame (m/s)
\(U_u\): Axial Velocity at the Outlet of the Burner (m/s)
\(u\): Fluctuating Velocity (m/s)
\(v_t\): Turbulent Velocity (m/s)
\(W\): Acoustic Energy (J/m²)
\(x\): Axial Length (m)
\(Y\): Fuel Mass Fraction (–)
\(\alpha\): Frequency Factor (mol/m³s)
\(\Gamma\): Attenuation Factor
\(\Delta W\): Oscillation Energy (J/m²)
\(\eta\): Fluctuating Mass Flow Rate (kg/s)
\(\theta\): Flame Angle (rad)
\(\lambda\): Heat Conduction Rate (W/mK)
\(\nu\): Laminar Kinematic Viscosity (m²/s)
\(\tau_f\): Lag Time through Flame (s)
\(\tau_m\): Lag Time until Heat Maximum (s)
\(\chi\): Heat Release Parameter = \((T_b - T_u)/T_u\)
\(\omega\): Angular Velocity (rad/s)
\(i\): Flow Path Number
\(b\): Burned Side of Flame
\(u\): Unburned Side of Flame
\(I\): Imaginary Part
\(R\): Real Part

3. Test Facility

Figures 1 and 2 show the test facility and test burner, respectively. The test section consists of a straight pipe of 200 mm inner diameter and the premixed burner is located at the center of the pipe. A choke nozzle is set at the outlet of the test section to clarify the outlet acoustic boundary condition. An inlet pipe (inner diameter of 66 mm and a length of 151 mm) is attached to the burner and connected to the premixer (inner diameter of 200 mm). The sectional area difference between the premixer and the inlet pipe makes the acoustic boundary condition nearly open. Methane is injected and mixed with air within a mean standard deviation of the fuel concentration distribution at the inlet of the burner of about 2%. A swirl burner which has
an ignition fuel nozzle at the center and a swirl vane angle of 30° is used. A baffle plate of 4% area opening is installed in the premixer to prevent back-fire.

The experimental conditions are as follows:

- **Pressure**: 0.51 – 1.48 MPa
- **Inlet Temperature**: 235 – 395°C
- **Fuel-to-Air Ratio**: 0.03 – 0.045
- **Flow Rate**: 0.27 – 0.52 kg/s
- **Burner Outlet Velocity**: 30 – 45 m/s
- **Pressure**: 0.51 – 1.48 MPa, and inlet temperature is 370 – 395°C mainly. Burner outlet velocity can be changed by inserting a ring at the outlet of the burner but it is fixed at ~40 m/s by using a ring with a 53 mm inner diameter. Strain gauge-type pressure transducers 0.35 m from the combustion chamber wall with 50 m long damping tubes are used for measuring pressure fluctuations. The amplitudes and frequencies of fluctuating pressures agree with those of the piezo type reference pressure gauge directly attached to the combustion chamber wall within 10% accuracy.

### 4. Analytical Method

In this study, physical properties are assumed to be constant in the lateral direction and to change only in the axial direction. Stability of the acoustic wave is evaluated by linear analysis. The stability of combustion oscillation is calculated by Bohn and Deuker’s method(3) using an acoustic amplifying factor. The relationship between the fluctuating mass flow rate and the fluctuating mass flow rate

\[
\eta_F = \eta_u + \eta_b
\]

where subscripts \( u \) and \( b \) stand for unburned and burned, respectively. \( G_u \) and \( G_b \) can be calculated by setting the inlet and outlet boundary conditions. \( G_j \) and \( G_f \) are functions of \( G_j \), which is the ratio of the fluctuating mass flow rate at both sides of the flame, where, \( G_j \) is designated the flame response function. Bohn and Deuker(3) used the following simplified time lag model.

\[
G_f = \frac{\eta_b}{\eta_u} = \frac{1}{j \omega \tau_f } (1 - e^{-j \omega \tau_f})
\]

\[
\tau_f = \frac{L_f}{U_f}
\]

where the time dependency of the solution is expressed by \( e^{j \omega t} \) and \( \tau_f \) is the lag time and the passing time through the flame zone. To take account of the effect of fluctuating heat release rate, Bloxidge’s flame model(4) is used.

\[
q(x) = \frac{u_u}{2 \pi j \omega B} Q(x) e^{-j \omega x}
\]

\[
\tau(x) = x / U_f
\]

In this model, the fluctuating heat release rate changes sinusoidally in the axial direction and it can be calculated from the fluctuating inlet velocity, mean heat release rate and passing time at the position \( x \). To eliminate \( M^2 \) order terms, where \( M \) is Mach number, the following relationship can be deduced between fluctuating inlet/outlet mass flow rate and fluctuating heat release rate.

\[
\eta_L C_p T_b = \eta_u C_p T_u + \int_0^{L_f} q(x) dx
\]

Mean heat release rate is assumed to have the following triangular distribution.

\[
Q(x) = \frac{Q_m}{L_m} (0 < x < L_m)
\]

\[
Q(x) = \frac{Q_m}{L_f - L_m}(x - L_m) (L_m < x < L_f)
\]

The flame response function \( G_f \) is introduced from Eqs. (8) – (10) by integration.

\[
G_f = \frac{T_f}{T_b} \left[ 1 + \frac{2(T_b - T_f)}{\omega^2 \tau_m} \frac{1}{j \omega T_u} \frac{L_f}{L_f - L_m} e^{-j \omega x} - 1 \right]
\]

\[
\tau_m = \frac{L_m}{U_f}, \quad \tau' = \frac{2 \pi \tau B}{U_u} \frac{L_f}{U_f}
\]

Here, \( U_u \) is the velocity at the outlet of the burner. The passing time is calculated by the following formula using turbulent flame speed \( S_t \) where \( \theta \) is flame angle.

\[
\tau_f = \frac{L_f}{U_f} = \frac{1}{U_f} \frac{\sqrt{(r_f^2 - r_0^2) U_u \cos \theta}}{S_t \tan \theta}
\]

Here, \( U_f \) is not equal to \( U_u \) and it is assumed to be proportional to the velocity \( U_u \) because the flame is divergent radially.

\[
U_f = 0.43 \times U_u
\]

The proportional factor 0.43 is calculated at half of the flame length, assuming that the premixed gas is diverged along the flame front and the velocity is reduced proportional to the inverse of the squared radius.

The dependency of pressure, fuel-to-air ratio, and velocity can be considered through \( S_t \). Turbulent flame speed can be calculated by the authors’ HTA model(6) which assumes the temperature profile across the flame is a hyperbolic tangent profile and the reaction rate is an Arrhenius type.

\[
S_t = S_u (1 + (0.056 R_e)^2)^{0.5}
\]

\[
S_u = \rho^{-0.5} \sqrt{\frac{\lambda}{C_p M_O_2} \left( Y_2 \frac{1}{\chi^2} \left( \frac{T_b}{E_0} \right)^2 + Y_3 \frac{1}{\chi^2} \left( \frac{T_b}{E_0} \right) \right)}
\]
Here, \( R_t \) is the turbulent Reynolds number and it is calculated from the turbulent velocity \( v_t \), Taylor microscale \( l_t \) and laminar kinematic viscosity \( \nu \).

\[
R_t = \frac{v_t l_t}{\nu} \tag{20}
\]

\( S_t/S_u \) is proportional to \( R_t \) at high \( R_t \), which is the same as the formula by Andrews et al.\(^{(7)} \). The pressure dependency of \( R_t \) is mainly introduced by laminar kinematic viscosity at the condition of constant velocity. Figure 3 shows experimental results of \( v_t, l_t, \) and \( R_t \), for cold flows in the same combustion chamber\(^{(8)} \). Flow is not the same as that of the present study but the grid generated turbulence. Average axial velocity is fixed as 6 m/s and a hot wire anemometer is used for turbulence measurement. Turbulent velocity increases, the Taylor microscale decreases and \( R_t \) increases with increasing pressure.

The power has not been fixed and varies from 0.5 to 0.25 for various investigators\(^{(7),(9)} \). The power of 0.62 introduced from the authors’ experimental results\(^{(8)} \), which covered the pressure range of this study, is used. The correlation is set as follows, assuming that the kinematic viscosity is proportional to the inverse of pressure.

\[
R_t = \left( \frac{0.14 U_{\omega R} B}{\nu} \right)^{0.62} \tag{21}
\]

Here, the constant 0.14 is fitted so that the calculated flame length agrees with the experimental data in this study. The pressure dependencies of some quantities are shown in Fig. 4. Laminar flame speed of methane decreases with static pressure, but turbulent Reynolds number increases with it. Turbulent flame speed increases with static pressure, because turbulent Reynolds number is more sensitive and flame length decreases with static pressure. Temperature dependency is also considered for viscosity, heat conduction rate and heat capacity.

Examples of the flame response function are shown in Fig. 5. The imaginary part of the response function is proportional to the oscillation energy generated at the flame. The oscillation energy generated at the flame can be calculated because of Rayleigh’s criterion and the approximation of \( O(M^2) \) by the difference of acoustic energy between the leading and the following edges of the flame.

\[
\Delta W = W_b - W_u
\]

\[
= \int (p_{R_b} u_R p_{R_b} - p_{R_u} u_R) dt
\]

\[
= (p_{R_b} u_R + p_{R_u} u_R) - (p_{R_b} u_R + p_{R_u} u_R) \tag{22}
\]

Here, dots indicate the quantity including \( e^{j \omega t} \). In general, the absolute value of acoustic energy at the leading edge is less than that at the following edge, because acoustic energy is zero in a uniform medium and enthalpy fluctuation at the flame is transported downstream. Then \( W_b \gg W_u \) and oscillation energy would be approximately proportional to the imaginary part of the response function as follows.

\[
\Delta W \approx (p_{R_b} u_R - p_{R_u} u_R) G_j^f \tag{23}
\]

The sign of the oscillation energy is dependent on \( p_{R_u}, u_u \) and \( G_j^f \) and the value in parentheses depends on the flame region in the acoustic standing wave. The parenthesized term is negative where the standing wave changes from a node to an antinode axially and positive where the standing wave changes from an antinode to a node axially. The response function \( G_j^f \) varies with \( f \tau_f \) and \( L_m/L_f \).
as shown in Fig. 5. GI is changed from positive/negative to negative/positive alternately with increasing $f \tau_f$ and it diminishes gradually at large $f \tau_f$ because of the triangular distribution of the heat generation rate. Combustion oscillation occurs when $f \tau_f$ satisfies the oscillation condition $\Delta W > 0$. Pressure, $F/A$, inlet velocity and temperature affect $\tau_f$ through inlet velocity $U_u$ and flame length $L_f$ which is dependent on flame speed and turbulent Reynolds number.

Fluctuating pressure mode in the acoustic system is calculated by the transfer matrix calculation including the flame. The transfer matrix of the flame is written as follows:

$$
\begin{bmatrix}
  p_b \\
  \eta_b
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 \\
  0 & G_f
\end{bmatrix}
\begin{bmatrix}
  p_u \\
  \eta_u
\end{bmatrix}.
$$

(24)

where the amplitude and the phase of the fluctuating pressure is assumed to be unchanged on both sides of the flame.

In the analysis, the calculational domain covers the area from the baffle plate inside the premixer to the choke orifice at the outlet of the combustion chamber. The inlet boundary is assumed to be open ($p = 0$) and the outlet boundary is assumed to be closed ($\eta = 0$). The flow path is divided into 8 flow path elements: the baffle plate, the premixer, the inlet pipe, the swirl vane, the ring, the flame, the combustion chamber and the choke nozzle, which are considered to be composed of straight pipes with the same area. For each pipe, a simple transfer matrix is assumed as follows.

$$
\begin{bmatrix}
  p_{i+1} \\
  \eta_{i+1}
\end{bmatrix} =
\begin{bmatrix}
  \cos(kl) & -j\varsigma \sin(kl) \\
  -j\varsigma \sin(kl) & \cos(kl)
\end{bmatrix}
\begin{bmatrix}
  p_i \\
  \eta_i
\end{bmatrix}.
$$

(25)

$$
\varsigma = \frac{a}{S}
$$

(26)

The attenuation is considered by using an imaginary wave number. The ratio $\Gamma$ of the imaginary part to the real one of the wave number is assumed by the following expression for a smooth pipe.

$$
\Gamma = -1.24 \times 10^{-5} a \sqrt{\frac{\pi}{4S\omega}}
$$

(27)

The matrix given by Eq. (25) is also used to calculate the acoustic impedance upstream and downstream from the flame and to calculate the acoustic amplifying factor $G$. The stability of combustion oscillation can be evaluated by $G$ and Eq. (22). The oscillation energy $\Delta W$ can be calculated by the acoustic resonance mode and Eq. (22). Both methods are applied to the analysis. Here, the modes for various conditions are normalized using the properties at the outlet of the combustion chamber.

5. Results

Figure 6 shows the time averaged light emission distribution of the flame, which is measured by a CCD camera from a window located at the side wall of the test section (Fig. 1). Figure 6 also shows the mean light emission distribution, which is calculated using the light emission at the centerline. The flame diverges from the outlet of the burner at 30 deg, which is almost the same as that of the swirl angle. The axial mean light emission distribution is nearly triangular as assumed to be in Eqs. (9) and (10). $L_m/L_f$ is 0.65 and the value is used in the analysis. This kind of image processing clarifies that flame length is shortened with increasing pressure and that the high intensity region is transported from upstream to downstream. Such observations are important to evaluate the characteristics of combustion oscillation because the combustion time alters the ratio of combustion to acoustic times.

Figure 7 shows experimental and analytical axial modes of fluctuating pressure. Here, inlet temperature is 385°C, fuel-to-air ratio is 0.04 and pressure is 1.21 MPa. The pressure mode of 350 Hz has a nodal point in the comb-
bustion chamber and antinodes at both ends of the chamber. The pressure mode of 110 Hz has no nodal point in the combustion chamber and has antinodes at the outlet of the chamber. The 110 Hz oscillation is the Helmholtz type oscillation with a quarter wave in an acoustic system including a combustion chamber and an inlet pipe. The analysis using the transport matrix method can simulate the shapes of these modes although the lower frequency does not agree with the experimental data. Here, the analysis results are normalized to fit with the experimental results because the transport matrix method cannot calculate the absolute amplitude of fluctuating pressure.

The combustion oscillation analysis is applied to the experiment where inlet temperature is 388°C, F/A is 0.03 – 0.04 and pressures are 0.66 and 0.88 MPa. Only the 350 Hz oscillation is observed in the fluctuating pressure spectrum at these experimental conditions. Figure 8 shows Bode’s diagrams of the acoustic amplifying factor G for the simple time lag model and the present model. Here, the quantity along the ordinate is set to $e^{\phi_0}$ for the phase $\phi$ of G as a matter of convenience so that the resonance occurs where the ordinate for the phase is one. The phase change is 360° at the discontinuity of the phase diagram in Fig. 8 (a). The 80 Hz and 350 Hz resonances are generated by the simple time lag model but the 80 Hz resonance is dominant. The analysis results are inconsistent with the experimental results. By using the present model, on the other hand, the phase $\phi$ is not zero at 80 Hz and the resonance is suppressed. The peak which appears around 350 Hz is dominant.

Figure 9 shows the imaginary part of the response functions of flame $G'_j$. The $G'_j$ of the simple time lag model is always negative and, for the resonant frequency, the amplitude of $G'_j$ of 80 Hz is larger than that of 350 Hz. The 80 Hz oscillation is dominant. For the present model, there is a single resonant frequency as shown in Fig. 8 and the amplitude of $G'_j$ has its local maximum around the 350 Hz resonant frequency.

Fluctuating pressure at resonant frequency which is obtained by experiment is shown in Fig. 10. The ordinate is the ratio of fluctuating pressure and static pressure. Fluctuating pressure has its peak at a specific F/A and the peak moves from large F/A to small F/A with higher pressure. Analytical results of the acoustic amplifying factor using the present model are shown in Fig. 11. Here, the ordinate is the excess of the absolute value of G from one, and instability of small F/A with higher pressure is indicated. That is consistent with experimental data. The absolute value along the ordinate cannot be compared between analysis and experiment because a linear analysis is employed in this calculation. However, the pressure and the fuel-to-air ratio dependency of the analysis agree with experimental data. Following the discussion on Figs. 5

![Fig. 8 Bode’s diagram ($T_u = 388^\circ$C, $F/A = 0.038$, 0.66 MPa)](image)

![Fig. 9 Flame response functions ($T_u = 388^\circ$C, $F/A = 0.038$, 0.66 MPa)](image)

![Fig. 10 F/A dependency of fluctuating pressure (Experiment)](image)
and 9, combustion oscillation occurs at specified $f \tau_f$. As shown in Fig. 4, turbulent flame speed increases with pressure and flame length decreases with pressure. Therefore, the resonance occurs at low $F/A$ with increasing high pressure because flame length is kept constant with the same velocity.

Figure 12 shows the oscillation energy calculated by Eq. (22) using the transfer matrix of flame. Figure 12 is almost the same as Fig. 11. The 80 Hz oscillation also does not appear in this calculation. The acoustic stability can be evaluated by the simple transport matrix method using the flame matrix and without using the acoustic amplifying factor. The method using the flame transport matrix is flexible and applicable to a complex geometry including junctions of flow paths which often appear in real configurations of commercial combustors.

Figure 13 shows the 350 Hz oscillation mode calculated by the transfer matrix method using the flame transfer matrix of the present model. The fluctuating pressure mode has antinodes both at the inlet and outlet of the combustion chamber and the fluctuating velocity mode has a node at the outlet of the burner. Acoustic energy $W$ changes from negative to positive through the flame axially and oscillation energy $\Delta W$ is transferred from thermal hydraulic energy to acoustic energy. Oscillation energy generated at the flame is transported from upstream to downstream. The dissipation of acoustic energy is balanced with the energy loss along the combustion chamber wall. The positive going traveling wave is larger than the negative going traveling wave in the combustion chamber because acoustic energy vanishes for the standing wave in the uniform medium.

Figure 14 shows $f \tau_f$ calculated by the analysis. Combustion oscillation occurs when $f \tau_f$ is equal to 1.5 where the GI $f$ has its local maximum in Figs. 5 and 9. As mentioned above, combustion oscillation for an almost completely premixed flame can be predicted by calculating the acoustic resonance mode, the response function and $f \tau_f$.

6. Conclusion

A new analytical time lag flame model based on Bloxidge’s flame model was introduced for calculating combustion oscillation of premixed flame to take into account the distribution of heat release rate and flame speed that was calculated by analytical formulas dependent on pressure, temperature, fuel-to-air ratio and velocity. Here, time lag $r_f$ could be calculated by analytical formulas with the authors’ HTA (Hyperbolic Tangent Approximation) model. To verify the model, combustion oscillation experiments were performed for methane-air premixed flames stabilized by a swirl burner at elevated pressures ranging 0.6–0.9 MPa.

The simple time lag model could not calculate the dominant frequency of oscillation. The new model, how-
ever, could simulate the pressure and $F/A$ dependency of the maximum fluctuating pressure peak. The fluctuating pressure had its maximum peak at the specific value of $f \tau_f$ where $f$ is the frequency of resonance. The combustion oscillation for almost completely premixed flame, therefore, could be predicted by calculating the acoustic resonance mode, the response function and $f \tau_f$.

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References


