Study on Interfacial Phenomena of Magnetic Fluids*

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The interfacial phenomena of magnetic fluids subjected to a normal magnetic field are studied experimentally. To begin with, the effect of the shape and the dimension of the transparent containers on the interfacial phenomena is examined for two kinds of magnetic fluids, ferricolloid W-40 and ferricolloid HC-50. The cross-sectional shape of the container is made to be circular, square and hexagon. The dimension of the container is set to 55 mm, 70 mm and 85 mm. The critical magnetic induction values $B_c$ calculated on the basis of the present experiment are compared with those obtained from the theoretical analysis by Cowley & Rosensweig, where at these values of $B_c$ the interfacial phenomena begin to be built up. Finally, on the hysteresis in generating the transition between three kinds of deformation modes on the interface, the present observation results are compared with those obtained from the analysis by A. Gailitis. It was concluded that the shape and the dimension of the containers had no effect on the interfacial phenomena, and the critical value $B_c$ calculated by measured data agreed with that of the theoretical analysis. And also, the mechanism of the hysteresis was clarified mathematically in the manner of the analysis by Gailits.

Key Words: Fluid Dynamics, Magnetic Fluid, Stability, Flow Visualization

1. Introduction

Magnetic fluid is a suspension in which ferromagnetic fine particles having a diameter of approximately 10 nm are dispersed in a mother liquid, such as water or kerosene, to form a kind of solid-liquid mixed phase fluid(1)–(3). Recently, with the advancement of particle size reduction technology and the development of new materials, the use of magnetic fluids in instruments in various fields is expanding(4)–(6). Under these circumstances, numerous investigations have been conducted to clarify the basic characteristics of magnetic fluids under magnetic fields(7)–(9).

If a stationary magnetic field is applied perpendicular to the flat surface of a magnetic fluid, projecting type surface deformation occurs at a specific magnetic field strength (critical magnetic field). However, there are numerous unknowns concerning the effects of the size and shape of the container(10) on the surface deformation when a magnetic field is applied perpendicular to a magnetic fluid in a container having a relatively large surface area.

In the present report, using containers of various sizes and shapes, we investigated the generation and growth of surface deformation caused by surface instability when a magnetic field is applied perpendicular to the surface of a magnetic fluid. As a result, we clarified the effects of container size and shape on the surface deformation for two kinds of magnetic fluids, based on either water or kerosene.

In 1967, Cowley and Rosensweig(11)–(13) carried out a theoretical analysis on the surface deformation of a kerosene-based magnetic fluid and calculated that the critical magnetic field strength for the occurrence of deformation is $84 \times 10^{-4}$ T. In addition, A. Gailitis carried out a theoretical analysis on the surface deformation of a kerosene-based magnetic fluid. Gailitis introduced the variable $\varepsilon = (B_0^2 - B_c^2) / B_0^2 - 1)$, which is dependant on the external magnetic field ($B_0$) and critical magnetic field ($B_c$), and clarified the range of $\varepsilon$ values(12),(14) that generate three types of deformation modes (flat surface, hexagonal patterns, and square patterns) on the surface of a magnetic fluid. In addition, he investigated hysteresis in which $\varepsilon$ for the transition of deformation patterns is different between increasing and decreasing external fields. He showed that three types of deformation patterns are present during both the increase and decrease of the external field and determined the range of these patterns.

In the present report, the above-mentioned theoretical analysis for the kerosene-based magnetic fluid is extended...
to a water-based magnetic fluid. A comparison was made between the observed results and the theoretical analysis results, and the effectiveness of the theoretical analysis of hysteresis by A. Gailitis is discussed based on the experimental results and observations of the present study.

2. Experimental Apparatus and Method

A schematic view of the experimental apparatus is shown in Fig. 1. Helmholtz coils having a height of 110 mm were used to apply a magnetic field. The containers for the experiments were made of transparent acrylic resin having a thickness of 5 mm so that the deformation of the surface could be easily observed. Circular, square, and hexagonal cross-sectional areas, the dimensions of which are represented by the inner diameter, the inner side length, and the incircle diameter, respectively, were adopted. The container size was set at 55 mm, 70 mm, or 85 mm.

In the present experiments, ferricollloid W-40 (Taiho Kogyo), a water-based magnetic fluid, and ferricollloid HC-50 (Taiho Kogyo), a kerosene-based magnetic fluid, were used. The liquid height of the magnetic fluid in the container was set at 30 mm.

The height of the projecting type deformation (spike height) and the distance between adjacent spikes (spike pitch) were measured for the surface deformation that took place on the fluid surface when a stationary magnetic field was applied to the magnetic fluid.

The spike height, i.e. the difference in height between the top of the spike and the bottom of the spike, was measured using a traveling microscope as follows. A stationary magnetic field was applied perpendicular to the magnetic fluid in a container by passing a DC current through the coils. Then, spikes (or projecting type surface deformation) due to the deformation of the fluid surface appeared over the entire surface. The measurement was conducted at four positions near the center of the surface. Using the traveling microscope, the spike pitch was also determined at four positions by measuring the horizontal distance between the top of the spike used for the above-described height measurement and the top of the neighboring spike. In both cases, an L-shaped arm with an aluminum needle was attached to the supporting section of the microscope barrel, and the needle tip was moved to the top of the spike and then to the bottom section of the spike to measure the respective positions.

3. Experimental Results and Discussion

3.1 Outline of surface deformation

When the magnetic field strength was increased to the critical value, cone-shaped projections (spikes) appeared near the center of the magnetic fluid surface, which had initially been flat. The critical value was $90 \times 10^{-4}$ T for the water-based magnetic fluid and $70 \times 10^{-4}$ T for the kerosene-based magnetic fluid. When the magnetic field strength was increased approximately $10^{-3}$ T beyond the critical value, the spikes spread over the entire surface.

The spike patterns of the surface, viewed from the top, were initially circular, and gradually changed to orderly hexagonal patterns similar to a honeycomb (Fig. 2 (a)). When the magnetic field passed the critical value, the spike patterns changed. Thus, neigh-
boring spikes of some sections merged, and the number of spikes on the surface began to decrease. During this time, the spike patterns, viewed from the top, changed from hexagons to pentagons and eventually changed to squares (lattice structure) (Fig. 2 (b)).

When the magnetic field was further increased, the spikes increased in height, as if competing. Since neighboring spikes interfered with each other, orderly patterns were no longer observed, and the distances among spikes became irregular.

On the other hand, when the magnetic induction was decreased from approximately $400 \times 10^{-4}$ T, the height of the spikes decreased, and lattice-like orderly square patterns reappeared. The patterns changed in reverse order (from pentagons to hexagons, and from hexagons to circles) were compared to that observed as the magnetic field was increased. However, the magnetic induction at the pattern change was different from that observed when the magnetic field was increased; thus, hysteresis was exhibited.

### 3.2 Effects of container size on surface deformation

The change in spike height with container size is shown in Fig. 3. When the container size was varied, the spike height under the same magnetic induction changed little for containers of the same shape. Thus, container size was not observed to have an effect on surface deformation.

The change in spike pitch with container size is shown in Fig. 4. For circular containers, no difference in spike pitch was observed among containers of different sizes. However, the kerosene-based magnetic fluid in the square containers and the water-based magnetic fluid in the hexagonal containers both exhibited a variation of approximately 2 mm. Thus, the spike pitch was relatively constant, except for localized variations.

### 3.3 Effects of container shape on surface deformation

The change in spike height with container shape is shown in Fig. 5. The figure shows an upward-sloping curve with an increase in the magnetic field strength for all container shapes. Container shape was observed to have no particular recognizable effect on spike height.

The change in spike pitch with container shape shows a variation of approximately 2 mm in spike pitch for the containers of each shape, except for the 55 mm hexagonal container, form the Fig. 4. However, the spike pitch was nearly constant, and container shape was observed to have no particular recognizable effect on spike pitch.

### 3.4 Comparison of the analysis results by Cowley and Rosensweig and the results of the present measurement

Cowley and Rosensweig reported that the minimum value $M_{\text{crit}}$ (critical magnetization) at which the deformation of the magnetic fluid surface can be maintained is expressed by the following equation:

$$\mu_0 M_{\text{crit}}^2 (1 + 1/r) = 2 \sqrt{\Delta \rho g}$$

where $B$ is the magnetic induction, $H$ is the magnetic field strength, $\Delta \rho$ is the difference in density ($\rho_1 - \rho_2$), $g$ is the
gravitational acceleration, \( r \) is the composite relative magnetic permeability, \( \mu_0 \) is the vacuum magnetic permeability, and \( \alpha \) is the surface tension of the magnetic fluid.

In this case, the wavenumber \( k_{\text{crit}} \) (critical wavenumber) is expressed by the following equation:

\[
k_{\text{crit}} = \sqrt{g\Delta\rho/\alpha}
\]  

Equation (1) can be rearranged to obtain the following equation:

\[
M_{\text{crit}} = \left\{ \frac{2(\mu_0)(1+\mu_0/\mu)(\rho_1-\rho_2)g\alpha}{2}\right\}^{1/2}
\]  

(a) The representative dimension is 55 mm

(b) The representative dimension is 70 mm

(c) The representative dimension is 85 mm

The theoretical value for the critical magnetization is obtained from Eq. (3) by assigning the following values: density of the water-based magnetic fluid (W-40), \( \rho_1 = 1.40 \times 10^3 \text{ kg/m}^3 \); surface tension, \( \alpha = 0.028 \text{ N/m} \) (\( \rho_1 \) and \( \alpha \) are the manufacturer’s test values); magnetic permeability, \( \mu = 1.68 \) (the measured value); and density of air, \( \rho_2 = 1.23 \text{ kg/m}^3 \), to obtain
In this case, the theoretical value for the critical magnetic induction $B_{\text{crit}}$ is

$$B_{\text{crit}} = \mu_0 \times M_{\text{crit}} = 89.2 \times 10^{-4} \text{T}.$$ 

This value is in good agreement with the measured value (approximately $90 \times 10^{-4} \text{T}$) of the magnetic induction, where the deformation appeared on the surface of the magnetic fluid (Fig. 6).

At the critical magnetization, the theoretical distance between spikes (pitch) is expressed by the following equation:

$$l_{\text{crit}} = \frac{4\pi}{\sqrt{3k_{\text{crit}}}} \tag{4}$$

Therefore, we can obtain

$$l_{\text{crit}} = 0.0098 \text{m} = 9.8 \text{mm}$$

from Eq. (4) by assigning material property values similar to those of W-40. The measured spike pitch at the start of surface deformation was approximately 10 mm. Thus, a good agreement with the theoretical value could be confirmed.

Similar calculations were performed for the kerosine-based magnetic fluid ($\rho_1 = 1.39 \times 10^3 \text{kg/m}^3$; surface tension, $\alpha = 0.026 \text{N/m}$ (both are the manufacturer’s test values); magnetic permeability, $\mu = 2.55$ (the measured value)), and the following value for the critical magnetization was obtained:

$$M_{\text{crit}} = 3.65 \times 10^3 \text{A/m},$$

and we then have

$$B_{\text{crit}} = \mu_0 \times M_{\text{crit}} = 45.8 \times 10^{-4} \text{T},$$

which does not agree well with either Cowley’s calculated value ($89.4 \times 10^{-4} \text{T}$) or the measured value (approximately $70 \times 10^{-4} \text{T}$) (Fig. 7). On the other hand, from Eq. (4), the following value was obtained for the spike pitch:

$$l_{\text{crit}} = 0.0100 \text{m} = 10.0 \text{mm}.$$ 

The measured spike pitch was approximately 10 mm, indicating good agreement with the theoretical value for the water-based magnetic fluid.

A comparison of the measured and theoretical values for all conditions applied in the present study is shown in Fig. 8.

### 3.5 Comparison between the analysis results of A. Gailitis and the results of the present measurement

The analysis of A. Gailitis showed that the surface of the magnetic fluid maintains three kinds of stable equilibrium states (flat surface, hexagonal patterns, and square patterns) depending on the strength of the applied magnetic field. This analysis is outlined in Fig. 9.
A transition from the flat surface to the hexagonal patterns takes place when the magnetic field strength becomes equal to $B_c$. However, once the hexagonal patterns are generated, the surface remains in a stable state, even after returning to a subcritical state ($B_0 < B_c$). When $e = (B_0^2/B_c^2 - 1)$ takes a certain negative value, the surface will change back to a flat surface. Likewise, compared with the magnetic field strength at which the transition from the hexagonal patterns to the square patterns occurs, the magnetic field strength at which the reverse transition occurs is smaller.

Thus, the hysteresis in the surface deformation was examined by filling containers of circular, square, or hexagonal cross-sectional shapes with the water-based magnetic fluid (W-40) or kerosene-based magnetic fluid (HC-50) up to a level of 30 mm.

Clear hysteresis, such as that analyzed by A. Gailitis, was recognized only in square containers for both magnetic fluids. For the other types of containers, neither a transition from a flat surface to a hexagonal pattern nor a transition from a hexagonal pattern to a square pattern could be clearly identified. In certain regions of the magnetic field, two or three hexagonal, pentagonal, or square deformation patterns coexisted.

The hysteresis for the water-based magnetic fluid is shown in Fig. 10. In the same manner as A. Gailitis, the displacement from the flat surface (height of spike) was plotted as the ordinate and $e = (B_0^2/B_c^2 - 1)$ was plotted as the abscissa. In this case, the critical magnetic induction $B_c$ was $88 \times 10^{-4}$ T and the magnetic permeability of the magnetic fluid $\mu$ was 1.68.

When a magnetic induction of $102 \times 10^{-4}$ T (where $B_0^2/B_c^2 - 1 = 0$) was reached during the gradual increase in the external magnetic field, the deformation spread across the entire surface forming orderly hexagonal patterns of spikes (Fig. 2 (a)). This stable state lasted for a time, and when $200 \times 10^{-4}$ T ($B_0^2/B_c^2 - 1 = 2.84$) was reached, the spikes rearranged themselves and the hexagonal patterns changed to square lattice patterns (Fig. 2 (b)).

On the other hand, when a magnetic induction of $170 \times 10^{-4}$ T ($B_0^2/B_c^2 - 1 = 1.78$) was reached during the gradual decrease in the magnetic field from the state of stable square patterns (approximately $220 \times 10^{-4}$ T), the surface returned to hexagonal patterns and became sta-
The results of the present experiment are shown in Fig. 13 (b). Gailitis’s analysis results are shown in Fig. 13 (a), and the \( \frac{\varepsilon}{\gamma^2} \) values.

When the magnetic induction was further decreased to \( 97 \times 10^{-4} \text{T} \) (\( B_1^2/b_1^2 - 1 = 0.10 \)), the spikes disappeared and the surface became flat and stabilized.

The hysteretic for the kerosene-based magnetic fluid is shown in Fig. 11. In this case, the magnetic induction \( B_e \) was \( 61 \times 10^{-4} \text{T} \) at the critical magnetic field, and the magnetic permeability of the magnetic fluid \( \mu \) was 2.55.

When a magnetic induction of \( 61 \times 10^{-4} \text{T} \) (where \( B_e^2/b_1^2 - 1 = 0 \)) was reached during the gradual increase of the external magnetic field, deformation appeared across the entire surface forming orderly hexagonal patterns of spikes (Fig. 12 (a)). This stable state lasted for a time, and when \( 140 \times 10^{-4} \text{T} \) (\( B_e^2/b_1^2 - 1 = 4.27 \)) was reached, the spikes rearranged themselves and the hexagonal patterns changed to square lattice patterns (Fig. 12 (b)).

On the other hand, when a magnetic induction of \( 128 \times 10^{-4} \text{T} \) (\( B_e^2/b_1^2 - 1 = 3.20 \)) was reached during the gradual decrease of the magnetic field from the state of stable square patterns, the surface returned to hexagonal patterns and became stable. When the magnetic field was further decreased to \( 57 \times 10^{-4} \text{T} \) (\( B_e^2/b_1^2 - 1 = -0.13 \)), the spikes disappeared and the surface became flat and stabilized.

As shown in Fig. 13, the results in Fig. 11 were arranged according to Gailiti’s analysis method by plotting \( \frac{\varepsilon}{\gamma^2} \) (\( \varepsilon = B_1^2/b_1^2 - 1 \), \( \gamma = 3(\mu - 1)/4(\mu + 1) \)) as the abscissa. Gailiti’s analysis results are shown in Fig. 13 (a), and the results of the present experiment are shown in Fig. 13 (b).

By comparing these results, values of \( \frac{\varepsilon}{\gamma^2} \) were obtained for the transition point: hexagonal patterns \( \rightarrow \) square patterns, which takes place during an increase in the magnetic field, and for the two transition points: square patterns \( \rightarrow \) hexagonal patterns, and hexagonal patterns \( \rightarrow \) flat surface, which take place during a decrease in the magnetic field. These are listed in Table 1.

### 4. Conclusions

In the present study, the surface deformation caused by a magnetic field applied perpendicular to the surface of two types of magnetic fluids in containers having different sizes and shapes were experimentally investigated. The effects on the deformation of container size, container shape, and type of magnetic fluid were investigated.

In addition, the theoretical analyses of the surface deformation of a kerosene-based magnetic fluid by Cowley and Rosensweig and by A. Gailitis were discussed. These analyses were extended to a water-based magnetic fluid (W-40), and measurement using a kerosene-based magnetic fluid (HC-50) was performed in order to compare the experimental results with the results of the theoretical analyses.

The following information was obtained in the present study:

1. The heights of projecting type deformation (spike heights), which appeared on the surface of the magnetic fluid, increased monotonically with increasing magnetic induction. No difference in height was observed in relation to container size or shape. Compared with the water-based magnetic fluid, the kerosene-based magnetic fluid had a higher absolute value for the spike height and a higher rate of increase with respect to the change of the magnetic field, revealing a distinct difference between these two fluids.

2. The distances between the tops of spikes (spike pitches) were observed to be approximately constant, regardless of the strength of the applied magnetic field. However, in some cases, a minor variation was observed, revealing the need for further investigation.

3. When the analysis method of Cowley and Rosensweig was applied to the water-based magnetic fluid, the calculated magnetic induction at the critical magnetic field, where the surface deformation starts, was \( 89.2 \times 10^{-4} \text{T} \), which is approximately equal to the measured value \( 90 \times 10^{-4} \text{T} \). Thus, the theoretical equation by Cowley et al. for the critical magnetic field was made clear to apply to the water-based magnetic fluid. For the kerosene-based magnetic fluid, the magnetic induction at

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**Table 1** Values of \( \frac{\varepsilon}{\gamma^2} \) which Gailitis calculated, and the corresponding values calculated from present experimental results

<table>
<thead>
<tr>
<th>Transition from hexagonal pattern to square one</th>
<th>Transition from square pattern to hexagonal one</th>
<th>Transition from hexagonal pattern to flat surface</th>
<th>Decreasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>104</td>
<td>7.44</td>
<td>-0.24</td>
<td></td>
</tr>
<tr>
<td>211</td>
<td>168</td>
<td>-8.27</td>
<td></td>
</tr>
</tbody>
</table>
the critical magnetic field as calculated by Cowley was $84 \times 10^{-4}$ T. The value calculated in the present study was $45.8 \times 10^{-4}$ T and the measured value was $70 \times 10^{-4}$ T, indicating that good agreement was not obtained.

(4) For both the water-based magnetic fluid and the kerosene-based magnetic fluid, hysteresis involving three kinds of surface deformation patterns was observed. By applying Gailitis’s theoretical method, the surface deformation could be expressed as a function of $(B_c^2 / B_c^2 - 1)$, where $B_c$ is the magnetic induction at the critical magnetic field. The magnetic inductions obtained by Gailitis’s theoretical analysis and the corresponding values obtained in the present experiment are listed in Table 1. These discrepancies in the results may be due to the difference between Gailitis’s assumption $|\mu - 1| \ll 1$ concerning the magnetic permeability $\mu$ of the magnetic fluids and the present consideration based on the measured values. More investigations by using the numerical analysis are necessary to make clear the effects of magnetic force, surface tension and gravity force on the surface deformation of the magnetic fluids.

References