Study on Differences in Turbulence Statistics between Compressible and Incompressible Low-Reynolds Number Turbulent Channel Flows Using Semi-Local Scaling*

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The differences in turbulence statistics between compressible and incompressible turbulent channel flows are investigated at low-Reynolds numbers using semi-local scaling. DNS of the compressible turbulent channel flow between isothermal walls at low-Mach numbers and DNS of the incompressible flow are performed. It is revealed that the reduction in the pressure-strain correlation term is due to the reductions in RMS velocity-derivative fluctuations and in the absolute correlations between pressure and velocity-derivative fluctuations in the compressible turbulent channel flow at low-Mach and Reynolds numbers.

Key Words: Compressible Flow, Turbulent Flow, Numerical Analysis, Supersonic Flow, Incompressible Flow, Channel Flow, DNS, Compressibility Effect, Semi-Local Scaling

1. Introduction

In the last decade, direct numerical simulation (DNS) has been performed to investigate the detailed mechanism of wall-bounded compressible turbulent flows under different thermal boundary conditions. Coleman et al.⁽¹⁾, Lechner et al.⁽²⁾, Foysi et al.^{(3),(4)} and Sarkar et al.⁽⁵⁾ performed DNSs of the compressible channel flow between isothermal walls, while Guarini et al.⁽⁶⁾ conducted DNS of the boundary layer flow on adiabatic wall, and Maeder et al.⁽⁷⁾ simulated the compressible boundary layer developing on laminar adiabatic wall. Morinishi et al.^{(8),(9)} performed DNS of the channel flow between adiabatic and isothermal walls and clarified the differences in turbulence statistics between compressible turbulent flows near adiabatic and isothermal walls. In addition to the effect of different thermal boundary conditions on the compressible turbulent flow, for a better understanding of the mechanism it is necessary to compare turbulence statistics between compressible and incompressible turbulent flows. Fortunately, there is a wealth of valuable knowledge available on the turbulence statistics of incompressible turbulent flow. Recently, Foysi et al.⁽⁴⁾ compared DNS data

on the compressible flow with those for the incompressible flow presented by Moser et al.⁽¹⁰⁾, and they explained the reduction in the near-wall pressure-strain correlation term using a Green's-function analysis for the pressure field. Sarkar et al.⁽⁵⁾ also showed that the attenuation of the pressure-strain correlation was due to decreasing velocity derivative fluctuations. However, the investigation of turbulence statistics related to the pressure-strain term such as the RMS pressure fluctuation, RMS velocity fluctuation and correlation between pressure and velocity-derivative fluctuations is not enough to demonstrate why the pressure-strain correlation is reduced. For the compressible mixing layer and homogeneous shear flow, it is known that the RMS pressure fluctuation increases with the increase in the turbulent Mach number, as a result of which turbulent energy redistribution due to the pressure-strain correlation terms is suppressed^{(11),(12)}. However, it is not clear whether the mechanism behind the reduction in redistribution for the mixing layer and homogeneous shear flow is valid for the wall-bounded compressible turbulent flow.

It has been well established that turbulence statistics depend on the Reynolds number Re_τ in the incompressible wall-bounded turbulent flow at low Reynolds number^{(10),(13)–(15)}. Foysi et al.⁽⁴⁾ performed the DNS with Re_τ up to 1 030 in order to overcome the low-Reynolds number problem in the study of the compressible turbulent channel flow. On the other hand, to consider turbu-

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lence statistics at low-Reynolds number, we compare turbulence statistics between compressible and incompressible flows using the same semi-local wall unit y^* , which is based on the semi-local length scale, instead of the standard wall coordinate y^+ , while taking account of the effect of the semi-local friction Reynolds number, Re_τ^* , which is based on the semi-local friction velocity, local density and viscosity.

In this paper, we focus on Reynolds stresses and pressure-strain correlation terms at the location $y^* \simeq 15$ at which the streamwise turbulence intensity is maximum, and at $y^* \simeq 45$ at which the wall-normal turbulence intensity is maximum. The mechanism of the difference in turbulence statistics between wall-bounded compressible and incompressible turbulent flows is investigated using the DNS data on compressible turbulent channel flows at different combinations of Mach and Reynolds numbers, where the Mach number M is up to 1.5 and the Reynolds number Re is up to 3000, to improve the reliability of our results. We also conducted a DNS of incompressible turbulent channel flows at various Re_τ , since such DNS data have not been presented in any previous DNS database^{(10),(14),(15)}, are required for the present analysis. Although the present Mach numbers seem to be relatively low, comparing with the DNS of Foyi et al.⁽⁴⁾ with M up to 3.5, it is interesting to identify the qualitative difference in turbulence statistics between compressible and incompressible turbulent flows with semi-local scaling.

2. Outline of DNS Data

2.1 Compressible turbulent channel flow

The continuity, momentum and internal energy equations for a compressible flow are solved in the DNS. We use the DNS algorithm based on an eighth-order B-spline collocation method in the wall-normal (x_2) direction and the Fourier Galerkin method in the streamwise and spanwise (x_1 – x_3) directions. The time-advancement scheme is a third-order low-storage Runge–Kutta method. The detailed DNS algorithm was presented in Morinishi et al.⁽⁸⁾, and the validity of the present code was proven by comparing our results with those of an existing DNS⁽¹⁾ of compressible turbulent channel flows.

Five computational cases of different combinations of Reynolds number Re and Mach number M are considered at the Prandtl number $Pr = \mu c_p / \kappa = 0.72$ and the ratio of specific heats $\gamma = c_p / c_v = 1.4$ (where κ is thermal conductivity, μ is molecular viscosity, c_p is specific heat at constant pressure, and c_v is specific heat at constant volume). In this paper, ρ and T are the density and temperature, respectively. The Reynolds number, $Re = \rho_m U_m H / \mu_w$, is based on the bulk density $\rho_m = \int_{-H}^H \langle \rho \rangle dx_2 / (2H)$, bulk velocity $U_m = \int_{-H}^H \langle \rho u_1 \rangle dx_2 / (2H \rho_m)$, channel half-width H , and viscosity at the wall, and the Mach number, $M =$

Table 1 Computational cases of compressible flow

	Case A	Case B	Case C	Case D	Case E	Coleman et al. [1]	
Re	2000	2000	2000	2500	3000	3000	4880
M	1.0	1.25	1.5	1.5	1.5	1.5	3.0
Pr	0.72	0.72	0.72	0.72	0.72	0.7	0.7
N_1	80	80	80	120	120	144	144
N_2	150	150	150	180	180	119	119
N_3	80	80	80	120	120	80	80
L_1	$4\pi H$	$4\pi H$	$4\pi H$	$4\pi H$	$4\pi H$	$4\pi H$	$4\pi H$
L_2	$2H$	$2H$	$2H$	$2H$	$2H$	$2H$	$2H$
L_3	$4\pi H/3$	$4\pi H/3$	$4\pi H/3$	$4\pi H/3$	$4\pi H/3$	$4\pi H/3$	$4\pi H/3$

Table 2 Computational cases of incompressible flow

	Case 1	Case 2	Case 3	Moser et al. [10]		
Re_τ	100	150	300	180	395	590
N_1	80	128	128	128	256	384
N_2	80	128	160	129	257	257
N_3	80	128	128	128	256	384
L_1	$4\pi H$	$4\pi H$	$2\pi H$	$4\pi H$	$2\pi H$	$2\pi H$
L_2	$2H$	$2H$	$2H$	$2H$	$2H$	$2H$
L_3	$4\pi H/3$	$4\pi H/3$	$2\pi H/3$	$4\pi H/3$	πH	πH

U_m/a_w , is based on the bulk velocity and sound speed at the lower wall $a_w = (\gamma RT_w)^{1/2}$ ($R = (\gamma - 1)c_p/\gamma$ is the gas constant), where $\langle \rangle$ represents the Reynolds average over time and the x_1 and x_3 directions, and the subscript w denotes the Reynolds-averaged value at the wall. The Mach and Reynolds numbers (M , Re) of Cases A, B, C, D and E are (1.0, 2000), (1.25, 2000), (1.5, 2000), (1.5, 2500) and (1.5, 3000), respectively. The physical and numerical parameters are given in Table 1, in which the DNS parameters of Coleman et al.⁽¹⁾ at $(M, Re) = (1.5, 3000)$ and (3.0, 4880) are also indicated. (N_1, N_2, N_3) and (L_1, L_2, L_3) are the number of grid points and computational region in the x_1 , x_2 and x_3 directions, respectively. The no-slip wall boundary condition is used for velocity, and the upper and lower walls are isothermal without any temperature difference. We confirmed that the present DNS data had sufficient resolution and domain size by examining one-dimensional energy spectra and two-point correlations.

2.2 Incompressible turbulent channel flow

We performed the DNS of the incompressible turbulent channel flows at $Re_\tau = 100, 150$ and 300, using a DNS algorithm based on the Chebyshev-tau method in the wall-normal direction, and the Fourier Galerkin method in the streamwise and spanwise directions. The Reynolds number $Re_\tau = H/\delta_v$ is defined by the channel half-width H and viscous length scale $\delta_v = \nu/u_\tau$, where $u_\tau = (\tau_w/\rho)^{1/2}$ is the friction velocity, τ_w is the wall shear stress. The continuity and Navier–Stokes equations are solved by the modified Kleiser–Schumann method^{(16),(17)}. A mixed time marching method is used, in which the diffusion term is treated implicitly with the Crank–Nicolson method, and the third-order Runge–Kutta method is used for all other terms.

The physical and numerical parameters and grid resolution based on wall variables are shown in Table 2. The Re_τ values in Cases 1, 2 and 3 are 100, 150 and 300, re-

Fig. 1 Profiles of RMS velocity fluctuations: (a) Compressible flow and (b) Incompressible flow

spectively. In the table, DNS parameters of Moser et al.⁽¹⁰⁾ at $Re_\tau = 180, 395$ and 590 are also indicated. We confirmed that the resolution was comparable to that of Moser et al.⁽¹⁰⁾, which was carried out using the same spatial discretization method. The validity of the present simulation was also confirmed by comparing our results with those of previous DNS data^{(10), (14), (15)}.

3. Results with Semi-Local Scaling

3.1 Turbulence intensities

The profiles of turbulence intensities scaled by the semi-local friction velocity, $(u''_\alpha)^*_{rms} = (u''_\alpha)_{rms} / u_{\tau*} = \{u''_\alpha u''_\alpha\}^{1/2} / u_{\tau*}$ ($\alpha = 1, 2, 3$, no summation for α), for compressible flow are shown in Fig. 1 (a), and those of turbulence intensities scaled by the standard friction velocity, $(u'_\alpha)^+_{rms} = (u'_\alpha)_{rms} / u_\tau = \{u'_\alpha u'_\alpha\}^{1/2} / u_\tau$ ($\alpha = 1, 2, 3$), for incompressible flow are shown in Fig. 1 (b). In this paper, the Favre average of a quantity ϕ is given by $\{\phi\} = \langle \rho \phi \rangle / \langle \rho \rangle$, and $''$ represents the turbulent fluctuation with respect to the Favre average, and $'$ represents the turbulent fluctuation with respect to the Reynolds average. Note that instead of the standard wall unit $y^+ = y / \delta_v$ (y is distance from the wall), the semi-local wall unit $y^* = y / \delta_{v*}$ in the abscissa is used in the Fig. 1 (a), where $\delta_{v*} = \langle \mu \rangle / (\langle \rho \rangle u_{\tau*})$ and $u_{\tau*} = (\tau_w / \langle \rho \rangle)^{1/2}$ are semi-local viscous length scale and semi-local friction velocity, respectively. Even though the difference among five computational cases seems to be relatively small in compressible turbulent flow (Fig. 1 (a)), it becomes much larger when turbulence intensities are scaled by the standard friction velocity (see Foysi et al.⁽⁴⁾). Figure 1 (b) shows that turbulence intensities increase with the increase in Re_τ , as shown in a previous study of incompressible flow⁽¹⁴⁾.

In the compressible flow, the Reynolds number based on the semi-local viscous length scale, $Re_\tau^* = H / \delta_{v*}$, depends on the distance from the wall as shown in Fig. 2. Here we choose $y^* = 15$ and 45 as the optimal locations

Fig. 2 Profiles of semi-local friction Reynolds number in semi-local wall unit

Table 3 Values of Re_τ^* at $y^* = 15$ and 45

	Case A	Case B	Case C	Case D	Case E
$Re_\tau^* _{y^*=15}$	118	112	106	133	156
$Re_\tau^* _{y^*=45}$	114	107	99.8	125	147

to investigate the dependence of turbulence intensities on Re_τ^* , where $y^* = 15$ and 45 correspond to the locations where streamwise and wall-normal intensities are maximum, respectively (see Fig. 1). The values of Re_τ^* at $y^* = 15$ and 45 are also presented in Table 3. In Fig. 3, the values of $(u'_i)^*_{rms}$ ($i = 1, 2, 3$) and Re_τ^* are plotted on the ordinate and abscissa, respectively, at $y^* = 15$. Figure 4 shows the relation between $(u'_i)^*_{rms}$ ($i = 1, 2, 3$) and Re_τ^* at $y^* = 45$. In the present simulation of the incompressible turbulent channel flow, fluid properties such as $\langle \rho \rangle = \rho_w$ and $\langle \mu \rangle = \mu_w$ are constant. Therefore the values of $Re_\tau (= Re_\tau^*)$ and $(u'_i)^+_{rms} [= (u'_i)^*_{rms}]$ ($i = 1, 2, 3$) at $y^+ (= y^*) = 15$ and 45 are plotted for incompressible flow. The dependence of turbulence intensities on Re_τ^* for the compressible flow virtually agrees with that on Re_τ for incompressible. At $y^* = 15$, $(u'_1)^*_{rms}$ is larger, while $(u'_2)^*_{rms}$ and $(u'_3)^*_{rms}$ are smaller in compressible flow, compared with the incompressible flow. On the other hand, at $y^* = 45$, the difference in turbulence intensities between compressible and incompressible flows is barely discernible. The effect of M on

Fig. 3 Dependence of turbulence intensities on Re_τ^* at $y^* = 15$: (a) $(u''_1)_{rms}^*$, (b) $(u''_2)_{rms}^*$ and (c) $(u''_3)_{rms}^*$

Fig. 4 Dependence of turbulence intensities on Re_τ^* at $y^* = 45$: (a) $(u''_1)_{rms}^*$, (b) $(u''_2)_{rms}^*$ and (c) $(u''_3)_{rms}^*$

$(u''_1)_{rms}^*$ appears at $y^* = 15$ in addition to the effect of Re_τ^* . The DNS data of Coleman *et al.*⁽¹⁾ show that the effect is considerably large at $M = 3.0$, and the effect of M seems to be somewhat scattered. This corresponds to the result of Foyi *et al.*⁽⁴⁾ that the peak amplitudes of the streamwise turbulence with $M = 3.5$ do not collapse with inner scaling. The data of Case E ($M = 3.0$ and $Re = 3000$) are slightly different from those of Coleman *et al.*⁽¹⁾, possibly due to the difference in Prandtl number and temperature dependence of viscosity.

Next, we investigate the reason for the difference in turbulence statistics between compressible and incompressible flows at $y^* = 15$. Possible reasons may be as follows: The production term, $P_{11} = -2\langle\rho\rangle\{u''_1 u''_2\}\partial\{u_1\}/\partial x_2$, for the compressible turbulent flow is smaller compared with the corresponding incompressible flow, as are the pressure-strain correlation terms ϕ_{11} , ϕ_{22} and ϕ_{33} . Note that the contribution of the compressibility terms to the energy transfer is negligible and is not the main factor. The production term of streamwise velocity fluctuation normalized by semi-local variables P_{11}^* for the compressible turbulent flow almost agrees with that of the corresponding incompressible flow (see Fig. 5). Thus, the production

Fig. 5 Dependence of production term of streamwise normal stress on Re_τ^* at $y^* = 15$

term of the streamwise normal stress is hardly related to the difference in turbulence intensities between compressible and incompressible flows. Hence, we shall examine the pressure-strain correlation terms of Reynolds stress ($\langle\rho\rangle\{u''_i u''_j\}$) equation, $\phi_{ij} = \langle p' \partial u'_i / \partial x_j \rangle + \langle p' \partial u'_j / \partial x_i \rangle$, where p is pressure.

3.2 Pressure-strain correlation terms

Figure 6(a) and (b) show profiles of the pressure-strain correlation terms in streamwise, wall-normal and

Fig. 6 Profiles of RMS velocity fluctuations: (a) Compressible flow and (b) Incompressible flow

Fig. 7 Dependence of pressure-strain correlation terms on Re_τ^* at $y^* = 15$: (a) ϕ_{11}^* , (b) ϕ_{22}^* and (c) ϕ_{33}^*

Fig. 8 Dependence of pressure-strain correlation terms on Re_τ^* at $y^* = 45$: (a) ϕ_{11}^* , (b) ϕ_{22}^* and (c) ϕ_{33}^*

spanwise normal stress transport equations, ϕ_{11}^* , ϕ_{22}^* and ϕ_{33}^* for the incompressible flow, which are normalized by semi-local variables, and ϕ_{11}^+ , ϕ_{22}^+ and ϕ_{33}^+ for the incompressible flow, respectively. The values of ϕ_{11}^* , ϕ_{22}^* and ϕ_{33}^* in relation to Re_τ^* at $y^* = 15$ and 45 are shown in Figs. 7 and 8, respectively. The absolute pressure-strain correlation terms of the compressible turbulent flow increase at $y^* = 15$ and 45 with the increase in Re_τ^* , which is similar to the dependence on Re_τ for the corresponding incom-

pressible turbulent flow, except that the value of ϕ_{22}^* is almost zero at $y^* = 15$. The absolute pressure-strain correlation terms of the compressible turbulent flow at $y^* = 15$ are smaller than those of the incompressible flow, whereas they are almost the same at $y^* = 45$. Hence, the energy transfer due to the pressure-strain correlation terms diminishes, thereby suppressing the redistribution from streamwise turbulence energy to the other components. As a result, the streamwise turbulence intensity of the compress-

ible flow is larger than that of the corresponding incompressible flow, whereas the wall-normal and spanwise turbulence intensities are smaller (see Fig. 3). This mechanism is supported by the investigation of Foysi et al.^{(3),(4)} at higher Mach numbers. On the other hand, the pressure-strain correlation terms between compressible and incompressible flows are almost the same as one another at $y^* = 45$, making the same turbulence intensities (see Fig. 4).

Next, we investigate the reason why the absolute values of ϕ_{11}^* , ϕ_{22}^* and ϕ_{33}^* of the compressible turbulent flow at $y^* = 15$ are smaller than those of the corresponding incompressible flow. Three possible reasons for this are: (1) The RMS pressure fluctuation $(p')_{rms}^* = \langle p'^2 \rangle^{1/2} / (\langle \rho \rangle u_{\tau*}^2)$ diminishes; (2) the RMS velocity-derivative, $(\partial u'_\alpha / \partial x_\alpha)_{rms}^* = \langle (\partial u'_\alpha / \partial x_\alpha)^2 \rangle^{1/2} / (u_{\tau*} / \delta_{v*})$, becomes smaller; and (3) the absolute correlation between pressure and velocity-derivative fluctuations, $R_{p' \partial u'_\alpha / \partial x_\alpha} = \langle p' \partial u'_\alpha / \partial x_\alpha \rangle / [(p')_{rms} (\partial u'_\alpha / \partial x_\alpha)_{rms}]$, decreases (see Hamba⁽¹²⁾). First, the difference in the RMS pressure fluctuation between compressible and incompressible flows does not appear at $y^* = 15$ (see Fig. 9). Note that Sarkar et al.⁽⁵⁾ showed the attenuation for the RMS pressure fluctuation scaled by the wall shear stress,

not for $(p')_{rms}^*$. Second, the values of $(\partial u'_\alpha / \partial x_\alpha)_{rms}^*$ ($\alpha = 1, 2, 3$) of the compressible flow at $y^* = 15$ are smaller than those for the corresponding incompressible flow (see Fig. 10). Third, the values of $R_{p' \partial u'_\alpha / \partial x_\alpha}$ ($\alpha = 1, 2, 3$) at $y^* = 15$ for compressible turbulent flow are lower than those of the incompressible flow (see Fig. 11). Therefore, the reason for smaller pressure-strain correlation terms at the low-Reynolds and Mach numbers is attributed to the smaller RMS velocity-derivatives as well as the lower absolute correlation between pressure and velocity-derivative fluctuations. Note that the mechanism underlying the wall-bounded compressible turbulent flow is different from those of the compressible mixing layer and homogeneous shear flow, which are due to less RMS pressure fluctuation.

4. Conclusion

The differences in turbulence statistics between compressible and incompressible turbulent channel flows were investigated using semi-local scaling, while taking account of the effect of the semi-local friction Reynolds number Re_τ^* . DNS of the compressible turbulent channel flows between isothermal walls at $(M, Re) = (1.0, 2000)$, $(1.25, 2000)$, $(1.5, 2000)$, $(1.5, 2500)$, and $(1.5, 3000)$, and the DNS of the incompressible turbulent channel flows at $Re_\tau = 100, 150$, and 300 were performed. We chose semi-local wall units $y^* = 15$ and 45 as the optimal locations to investigate the dependence of Reynolds stresses and pressure-strain correlation terms on Re_τ^* , where $y^* = 15$ and 45 corresponded to the locations where streamwise and wall-normal intensities were maximum, respectively. Compared to incompressible flows, the reduction in the pressure-strain correlation term for compressible flows at $y^* = 15$, which was not observed at $y^* = 45$, was due to the reductions in the RMS velocity-derivative fluctuations and in the absolute correlation between pressure and velocity-derivative fluctuations.

Fig. 9 Dependence of RMS pressure fluctuation on Re_τ^* at $y^* = 15$

Fig. 10 Dependence of RMS velocity-derivative fluctuations on Re_τ^* at $y^* = 15$: (a) $(\partial u'_1 / \partial x_1)_{rms}^*$, (b) $(\partial u'_2 / \partial x_2)_{rms}^*$ and (c) $(\partial u'_3 / \partial x_3)_{rms}^*$

Fig. 11 Dependence of correlation coefficients between pressure and velocity-derivative fluctuations on Re_τ^* at $y^* = 15$: (a) $R_{p' \partial u_1 / \partial x_1}$, (b) $R_{p' \partial u_2 / \partial x_2}$ and (c) $R_{p' \partial u_3 / \partial x_3}$

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