Scattering of Plane Waves by a Rigid Sphere in an Acoustic Quarterspace*  
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The acoustic scattering by a submerged spherical rigid obstacle near an acoustically hard concave corner, which is insonified by plane waves at arbitrary angles of incidence, is studied. The formulation utilizes the appropriate wave-harmonic field expansions and the classical method of images in combination with the translational addition theorems for spherical wave functions to develop a closed-form solution in form of infinite series. The analytical results are illustrated by numerical examples where the spherical object is located near the rigid boundary of a fluid-filled quarterspace and is insonified by plane waves at oblique angles of incidence. Subsequently, the basic acoustic field quantities such as the form function amplitude, the scattered far-field pressure, and the scattered acoustic intensity are evaluated for representative values of the parameters characterizing the system. The limiting case involving a spherical object submerged in an acoustic halfspace is considered and good agreement with a well-known solution is established.

** Key Words:** Acoustic Scattering, Rigid Corner, Addition Theorems

1. Introduction

There exists a vast body of literature on multiple-scattering theory, extending back for more than a century and across the boundaries of many disciplines such as acoustics, electromagnetics, and quantum mechanics. In particular, acoustic scattering by (radiation from) a spherical body (source) positioned near a flat boundary has been the subject of several works during the last 20 years(1)–(6). Seybert and Soenarko(1) applied the boundary integral equation (BIE) method to study general acoustic radiation and scattering in a three-dimensional halfspace. Gaunaurd and Huang(2) employed the translational addition theorems for the spherical wave functions to study acoustic scattering by a hard spherical body near a hard flat boundary. They also considered acoustic scattering by a thin spherical elastic shell near a free surface(3), and by an ideal air-bubble near the sea surface(4). More recently, Hasheminejad(5) examined acoustic radiation from a spherical surface undergoing harmonic modal vibrations near a locally reacting (finite impedance) planar boundary. Shenderov(6) used Helmholtz integral equation (Green’s function) approach to formulate an exact solution for the problem of sound diffraction by an impedance sphere located near a flat impedance boundary. On the other hand, the solutions of acoustic scattering (radiation) problems involving a spherical object (source) in a fluid-filled quarterspace seem to be nonexistent (quite sparse). Only recently, Hasheminejad and Azarpeyvand(7) utilized the translational addition theorems for spherical wave functions to study harmonic acoustic radiation from a modally vibrating spherical source in an acoustic quarterspace. The principal objective of present work is to study acoustic scattering from an impenetrable spherical object that is positioned near the rigid boundary of a fluid-filled quarterspace (Fig. 1). The solution of the problem is generated by systematically analyzing multi-scattering interaction between the object and the rigid corner that can be strong or weak depending on their separation, incident wave frequency, and angle of wave incidence. The current solution could eventually serve to validate those found by numerical schemes.

2. Formulation

The problem can be analyzed by means of the standard methods of theoretical acoustics. The fluid is assumed to be inviscid and ideal compressible that cannot support shear stresses. In view of the fact that the incident plane wave is supposed to be time-harmonic, with frequency \( \omega \), the field equations may conveniently be ex-
pressed in terms of a scalar velocity potential as\(^{(8)}\):

\[
\begin{align*}
(-i\omega)s &= \nabla \phi \\
p &= i\omega \bar{\rho} \Phi \\
\nabla^2 \Phi + \bar{k}^2 \Phi &= 0
\end{align*}
\]

where \(\bar{\rho}\) is the ambient fluid density, \((-i\omega)s\) is the fluid-particle-velocity vector, \(p\) is the acoustic pressure in the inviscid fluid, \(\bar{k} = \omega/c\) is the acoustic wave number, \(c\) is the ideal speed of sound, and we have assumed harmonic time variations throughout with \(e^{-i\omega t}\) dependence suppressed for simplicity.

Undoubtedly, the sound field scattered by an obstacle may often be appreciably affected by a neighbouring surface. Consider a spherical object of radius \(a\) positioned near a concave corner (Fig. 1). A plane sound wave emerging from a distant source is obliquely incident on the object from above with angles of incidence \((\alpha, \beta)\). This incident plane wave will be reflected by the vertical and horizontal boundaries of the corner. The reflected plane waves act as a set of secondary waves that are incident upon the spherical object at different angles. If the vertical and horizontal boundaries are idealized as hard, planar, and of semi-infinite extent, the method of images can be employed to smoothly take their presence into account\(^{(7)}\). Accordingly, the origins \(O_i (i = 1, 2, 3, 4)\) of the spherical coordinate systems \((r, \theta, \varphi)\) are properly placed at the center of each sphere in the real/image system configuration as shown in Fig. 2. These coordinate systems are separated by the distance \(l_{ij} = l_{ij}(i \neq j)\) such that \(l_{i1} = l_{i1} = d_1\), and \(l_{i4} = l_{i4} = d_2\). Furthermore, the corresponding \((x_i, y_i, z_i)\) axes are respectively parallel, i.e., there is no relative rotation of the coordinate systems.

The dynamics of the multi-scattering problem can be expressed in terms of four scalar potentials corresponding to the fields scattered from each sphere in addition to the plane waves incident on these spheres (Fig. 2). The incident waves are written in the coordinate system of each sphere as\(^{(2)}\):

\[
\begin{align*}
\phi_{\text{inc}}^{(1)} &= \exp\{ikr_1[\cos \theta_1 \cos \alpha + \sin \theta_1 \sin \alpha \cos(\varphi_1 - \beta)]\} \\
\phi_{\text{inc}}^{(2)} &= \exp\{ikr_2[\cos \theta_2 \cos(\pi - \alpha) + \sin \theta_2 \sin(\pi - \alpha) \cos(\varphi_2 - \beta)]\} \\
\phi_{\text{inc}}^{(3)} &= \exp\{ikr_3[\cos \theta_3 \cos(\pi - \alpha) + \sin \theta_3 \sin(\pi - \alpha) \cos(\varphi_3 - \pi + \beta)]\} \\
\phi_{\text{inc}}^{(4)} &= \exp\{ikr_4[\cos \theta_4 \cos \alpha + \sin \theta_4 \sin(\varphi_4 - \pi + \beta)]\}
\end{align*}
\]

which may be expanded in terms of the appropriate spherical harmonics as\(^{(9)}\):

\[
\begin{align*}
\phi_{\text{inc}}^{(i)}(r, \theta, \varphi, \omega) &= \\
&= \sum_{q=0}^{\infty} \sum_{p=-q}^{q} (-1)^m A_{pq} J_q(kr_i) P^m_q(\cos \theta_i) e^{i(p\varphi_i - \beta_i)} \\
(i &= 1, 2, 3, 4)
\end{align*}
\]

where \(J_n(x)\) is the spherical Bessel function of the first kind, \(P^m_n(x)\) is the associated Legendre function\(^{(10)}\), \(m_1 = 0\), \(m_2 = p + q\), \(m_3 = q\), \(m_4 = p\) and \(\beta_1 = \beta_2 = \beta_3 = \beta_4 = -\beta\) and

\[
N_{pq} = \frac{2}{2q + 1} \left(\frac{q + p}{q - p}\right)!
\]

Furthermore, the velocity potentials \(\phi_{\text{inc}}^{(2)}, \phi_{\text{inc}}^{(3)}, \phi_{\text{inc}}^{(4)}\), which appear in Eq. (2), can be expressed in the coordinate system of the real sphere as\(^{(2)}\):

\[
\begin{align*}
\phi_{\text{inc}}^{(2)} &= \exp\{ikr_1[\cos \theta_1 \cos(\pi - \alpha) + \sin \theta_1 \sin(\pi - \alpha) \cos(\varphi_1 - \beta)]\} \exp(ikd_1 \cos \alpha) \\
\phi_{\text{inc}}^{(3)} &= \exp\{ikr_2[\cos \theta_2 \cos(\pi - \alpha) + \sin \theta_2 \sin(\pi - \alpha) \cos(\varphi_2 - \beta)]\} \\
\phi_{\text{inc}}^{(4)} &= \exp\{ikr_3[\cos \theta_3 \cos(\pi - \alpha) + \sin \theta_3 \sin(\pi - \alpha) \cos(\varphi_3 - \pi + \beta)]\}
\end{align*}
\]
where \( d_3 = \sqrt{d_1^2 + d_2^2} \), \( \gamma = \tan^{-1}(d_2/d_1) \). Next, the above expressions may advantageously be expanded in the spherical coordinate system associated with the real sphere as \(^{(9)} \)

\[
\phi^{(i)}_{inc}(r_1, \theta_1, \varphi_1, \omega) = e^{ikd_i(x)} \sum_{q=0}^{\infty} \sum_{p=-q}^{q} (-1)^m A_{pq} j_q(kr_1) P^m_q(\cos \theta_1) e^{ip(\varphi_1-\varphi_i)}
\]

(6)

where \( i = 1, 2, 3, 4 \), \( d_2 = ikd_1 \cos \alpha, d_3 = ikd_1 \sin \alpha \cos \beta + \cos \alpha \sin \beta \), and \( d_4 = ikd_1 \sin \alpha \cos \beta \). Similarly, the field scattered by each sphere in its own coordinate system may be expanded in its original coordinate system as \(^{(9)} \)

\[
\phi^{(i)}_{scat}(r_1, \theta_1, \varphi_1, \omega) = e^{ikd_i(x)} \sum_{q=0}^{\infty} \sum_{p=-q}^{q} B^i_{pq}(\omega) h_q(kr_1) P^m_q(\cos \theta_1) e^{ip(\varphi_1-\varphi_i)}
\]

(7)

where \( h_q(x) = j_q(x) + \tilde{j}_q(x) \) is the spherical Hankel function of order \( q \), and \( B^i_{pq}(\omega) \) are the unknown scattering coefficients that are to be determined by imposing the suitable boundary conditions.

Now, utilizing Eqs. (3) and (7), the total acoustic field may be written as \(^{(9)} \)

\[
\Phi = \sum_{i=1}^{4} \left( \phi^{(i)}_{inc} + \phi^{(i)}_{scat} \right)
\]

\[
= \sum_{q=0}^{\infty} \sum_{p=-q}^{q} \left( (-1)^m A_{pq} j_q(kr_1) P^m_q(\cos \theta_1) e^{ip(\varphi_1-\varphi_i)} + B^i_{pq}(\omega) h_q(kr_1) P^m_q(\cos \theta_1) e^{ip(\varphi_1-\varphi_i)} \right)
\]

(8)

where we note that each term represents the spherical wave functions expressed in its original coordinate system. They need to be transformed (translated) to the remaining three coordinate systems before imposing the boundary conditions. This can be achieved through proper application of the general form of translational addition theorem for bi-spherical coordinates. Accordingly, the spherical wave functions of \((r_1, \theta_1, \varphi_1)\) coordinate system can be expanded in terms of spherical wave functions of \((r_1, \theta, \varphi)\) coordinate system as \(^{(9)} \)

\[
z_q(kr_1) P^m_q(\cos \theta_1) e^{ip(\varphi_1-\varphi_i)} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Q_{nm}^{(i)} j_n(kr_1) P^n_m(\cos \theta_1) e^{ip\varphi_1} + \sum_{n=0}^{\infty} \sum_{m=-n}^{n} R_{nm}^{(i)} j_n(kr_1) P^n_m(\cos \theta_1) e^{ip(q_1-\varphi_i)}
\]

(9)

where \( i, j = 1, 2, 3, 4 \), \( z_n(x) \) is one of the spherical Bessel functions of order \( n \), and

\[
Q_{nm}^{(i)} = \frac{2^{m-q}}{N_{mn}^{(i)}} \sum_{\sigma=|m-q|}^{m+q} i^{\sigma} b_{\sigma}^{(i)} z_n(kr_1) P^n_{m+\sigma}(\cos \theta_1) e^{ip(m+\sigma)\varphi_1}
\]

\[
R_{nm}^{(i)} = \frac{2^{m-q}}{N_{mn}^{(i)}} \sum_{\sigma=|m-q|}^{m+q} i^{\sigma} b_{\sigma}^{(i)} j_n(kr_1) P^n_{m+\sigma}(\cos \theta_1) e^{ip(m+\sigma)\varphi_1}
\]

(10)

where \( \theta_1 = \pi + \theta_2 \) (see Fig. 2), and

\[
b_{\sigma}^{(i)} = (-1)^{\sigma} [n_1+m_1]!([n_1+m_1]+1)! [n_1-m_1]!([n_1-m_1]+1)! \frac{2n+1}{(n_2+n_2+1)!} \frac{n_2+n_2-n!}{(n_2+n_2-n)!}
\]

\[
\times \sum_{z=1}^{\infty} \left( (-1)^{\sigma} [(z\cos\theta_1+n_1-n_1)+z\sin\theta_1+n_1-m_1]! [z\cos\theta_1-n_1-m_1]! \right) \right]^{1/2}
\]

(11)

in which Clebsch-Gordan Coefficients are defined as

\[
(b^{(i)}_{nm}) = \sum_{\sigma=|m-q|}^{m+q} \frac{n_2+n_2-n!}{(n_2+n_2-n)!} \left( (-1)^{\sigma} [(z\cos\theta_1+n_1-n_1)+z\sin\theta_1+n_1-m_1]! [z\cos\theta_1-n_1-m_1]! \right)^{1/2}
\]

(12)

where the summation must be performed over the non-negative values of \( z \) that makes all factorials involving \( z \) to be greater than or equal to zero. Subsequently, Incorporating Eqs. (6) and (9) in (8) allows us to express all expansions in terms of the wave functions of the coordinate system centered at the real sphere. Therefore, the total acoustic field with respect to the coordinate system of the real sphere can be written as

\[
\Phi(r_1, \theta, \varphi, \omega) = \sum_{q=0}^{\infty} \sum_{p=-q}^{q} \left( B_{pq}^{(1)} h_q(kr_1) + j_q(kr_1) \right)
\]

\[
\times \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left( B_{nm}^{(2)} Q_{nm}^{(1,2)} + B_{nm}^{(3)} Q_{nm}^{(1,3)} + B_{nm}^{(4)} Q_{nm}^{(1,4)} \right)
\]

\[
+ A_{pq} j_q(kr_1) \left( \exp[-ip\beta] + (-1)^{q+p} \exp[-ip\beta + d_2] \right)
\]

\[
+ (-1)^{q+p} \exp[ip\beta + d_2] + (-1)^{q} \exp[ip\beta + d_1] \right)
\]

\[
\times P^m_q(\cos \theta_1) e^{ip\varphi_1}
\]

(13)

Similar expressions can be obtained for the total acoustic field with respect to the coordinate systems of the image spheres. As a result, Eq. (13) may be generalized in the form:
\[
\sum_{q=0}^{\infty} \sum_{p=-q}^{q} B_{pq}^{(i)} h_{pq}(kr) + j_{pq}(kr) \\
\times \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{4}{\pi} B_{nm}^{(i)} Q_{nm}^{(i)} \\
+ A_{pq} j_{pq}(kr) \sum_{j=1}^{4} (-1)^{j} e^{-j\gamma_{j}} \}
\times \sum_{i=1}^{3} \frac{\partial\Phi(r,\theta,\varphi,\omega)}{\partial r}
\]  
(14)

where \( i = 1, 2, 3, 4 \), \( \gamma_{12} = \gamma_{21} = \gamma_{34} = \gamma_{43} = \frac{\pi}{2}, \) \( \gamma_{13} = \gamma_{31} = \gamma_{24} = \gamma_{42} = \frac{\pi}{2}, \) \( \gamma_{14} = \gamma_{41} = \gamma_{32} = \gamma_{23} = \frac{\pi}{2} \) and \( \gamma_{ii} = 0 \).

The relevant boundary conditions that must be satisfied on the surface of each sphere are simply vanishing of the total radial surface velocity components, i.e.,

\[ (-i\omega)s_{ij}(r_{i},\theta_{i},\varphi_{i},\omega) = \frac{\partial\Phi(r_{i},\theta_{i},\varphi_{i},\omega)}{\partial r_{i}} \Bigg|_{r_{i} = a} = 0 \]
(15)

which lead to the linear system of equations:

\[
\sum_{q=0}^{\infty} \sum_{p=-q}^{q} B_{pq}^{(i)} h_{pq}(ka) \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{4}{\pi} B_{nm}^{(i)} Q_{nm}^{(i)} = \sum_{j=1}^{4} (-1)^{j} \bar{e}^{-j\beta_{j}} \gamma_{j}^{(i)} \\
(\beta_{j} = 1, 2, 3, 4) 
\]
(16)

where we have used the following orthogonality relation for associated Legendre polynomials:

\[
\int_{0}^{\pi} P_{q}^{(i)}(\cos\theta) P_{p}^{(i)}(\cos\theta) \sin\theta d\theta = N_{mn} \delta_{pq} 
\]
(17)

where \( \delta_{pq} \) is Kronecker delta and \( N_{mn} \) is defined in Eq. (4).

The most relevant acoustic field quantities are the scattered normalized far-field pressure magnitude or the scattered intensity. The standard definition of the intensity is given by (8):

\[
I_{scat}(r_{i},\theta_{i},\varphi_{i},\omega) = \lim_{r_{i} \to \infty} \left| \frac{2r_{i}}{\alpha} \sum_{i=1}^{4} \phi_{scat}(r_{i},\theta_{i},\varphi_{i},\omega) \right|
\]
(18)

Likewise, the radial component of the acoustic power flux vector scattered per unit solid angle from the real sphere (scattered acoustic intensity) is given by (8):

\[
J_{scat}(r_{i},\theta_{i},\varphi_{i},\omega) = \frac{1}{2} Re \left\{ p_{scat}(r_{i},\theta_{i},\varphi_{i},\omega) \right\} \\
\times \left| \frac{\partial}{\partial r_{i}} \sum_{i=1}^{4} \phi_{scat}(r_{i},\theta_{i},\varphi_{i},\omega) \right|
\]
(19)

The asterisk denotes a complex conjugate quantity, and

\[
p_{scat}(r_{i},\theta_{i},\varphi_{i},\omega) = i\omega \bar{\theta} \sum_{i=1}^{4} \phi_{scat}^{(i)}(r_{i},\theta_{i},\varphi_{i},\omega)
\]
(20)

This completes the necessary background required for the exact acoustic analysis of a sphere near a rigid corner. Next we consider some numerical examples.

### 3. Numerical Results and Discussion

In order to illustrate the nature and general behaviour of the solution, we consider numerical examples in this section. Realizing the crowd of parameters and the relatively large sized matrices involved here, while keeping in view our computing hardware limitations, we have to confine our attention to a particular model in a specific frequency range. The surrounding ambient fluid is assumed to be water at atmospheric pressure and 300 kelvin (\( \bar{\rho} = 0.997 \text{ g/cm}^{3}, \) \( c = 149700 \text{ cm/s} \)). Accurate computation of the Bessel functions is achieved by employing the MATLAB specialized math functions “besselh” and “besselj.” The precision of the calculated values is checked against the printed tabulations in the handbook by Abramowitz and Stegun (10). Accurate computations for derivatives of spherical Bessel functions are accomplished by utilizing (10.1.19) and (10.1.22) in the latter reference. A MATLAB code is constructed for treating the boundary conditions and to calculate the unknown scattering coefficients and the relevant acoustic field quantities as functions of the nondimensional frequency \( ka \), the angles of plane wave incidence \((\alpha, \beta)\), and the dimensionless distance parameters \((d_{1}/a, d_{2}/a)\). The computations are performed on a Pentium IV personal computer with truncation constants of \( n_{max} = m_{max} = 20 \) so that the largest matrices involved are roughly about \( \times 1800 \). This assures convergence in the high frequency range, and also in case of close proximity of the sphere to the quadrupole boundary.

Figure 3 displays the variation of the backward-scattered normalized far-field pressure magnitude or the form function amplitude (i.e., \( |f_{scat}(r,\theta,\varphi,\omega)| \)) with the nondimensional frequency in case of \( \alpha = \frac{\pi}{2}, \beta = 0, \pi/4 \) incidence upon the rigid sphere for several increasing values of distance parameters \((d_{1}/a = 2, d_{2}/a = 3, 5, 20)\). This (complete) form function exactly accounts for the contributions of the real sphere and the incident wave as well as that of the waves reflected from the rigid corner. We also show the corresponding plot of the standard form function for a single rigid sphere submerged in a boundless acoustic medium. Furthermore, we have generated the form function amplitude results for a rigid sphere submerged at a distance \( d = d_{1} = 2a \) from a rigid flat boundary by using an independently developed MATLAB code. Examination of these plots leads to the following important observations. In case of the sphere in the acoustic half-space, there are strong oscillations in the form-function plot (triangular markers) that seem to be centered at a mean value of two with an oscillating amplitude of value near two (2). This, as one would expect, is twice the value for a single sphere (circular markers) which would be the prediction offered by Born approximation in this case (4). In particular, the amplitude maxima of the form functions
extend to over four times that of a (single) rigid sphere in a boundless medium. The form function plots associated with the acoustic quarterspace are quite different from the halfspace results. In fact, the overall amplitudes of these form functions are roughly an order of magnitude larger than the corresponding halfspace amplitudes, especially for normal wave incidence ($\beta = 0$) at all three separations. These somewhat unexpected large amplitudes clearly demonstrate that the multiple-scattering interactions extremely amplify in case of submersion near a rigid corner even at relatively large separations. As the separation grows, the form function peaks become sharper and more densely packed. This behaviour may be clarified as follows:\(^{(12)}\). The successive interactions among the various reflected and creeping waves in the acoustic quarterspace are much more complex than in the infinite fluid medium case since there are a variety of repeatedly incident waves acting on the spherical scatterer from different directions and with different phases. Accordingly, the increasing rapid oscillations observed in figures are associated with the fact that the phase difference between the scattering object and the rigid boundary oscillate increasingly faster as the separation grows. More specifically, the oscillating nature of the phase factors $e^{ikd_{\text{inc}}\cos \alpha}$, $h_{\sigma}(kd_i)$, and $j_{\sigma}(kd_i)$ present in $\phi_{(i\text{nc})}^{(0)}$, $Q_{(i\text{mp})}^{(0)}$, and $R_{(i\text{mp})}^{(0)}$ ($i = 1, 2, 3, 4; i \neq j$) imply different arrival times for these waves. This effect forms an oscillation pattern in the latter terms at large $kd_i$.

Figure 4 displays the influence of the distance parameters ($d_1/a$, $d_2/a$) on angular distribution of the scattered far-field pressure for a unit amplitude plane wave obliquely incident ($\alpha = \pi/4$, $\beta = 0$) upon the spherical obstacle located near a rigid corner at selected nondimensional frequencies ($ka = 1, 5, 10$). The far-field value of the radial coordinate in each case is simply chosen by making several computer runs while seeking for the convergence of the scattered pressure directivity patterns. The choice of $r_{\infty} = 100a$ is found to be adequate for all cases. Furthermore, a maximum number of twenty modes are included in all summations in order to assure convergence at the higher frequencies and also close proximity of the sphere to the rigid boundary. It is very interesting to notice the change in directionality of the scattered waves as the frequency, and the distance parameters are varied. First,
we note that as the sphere is moved away from either the vertical or the horizontal boundary, the scattered far-field pressure directionality remarkably increases, especially at the higher frequencies (i.e., $ka = 5, 10$). At the lowest frequency considered ($ka = 1$), the pressure patterns are very uniform as they show a very low directionality with a relatively large backward scattering (i.e., at $\theta = \alpha + \pi$) for all distance parameters. However, the overall scattered pressure magnitudes noticeably increase with decreasing the distance parameters. At the intermediate frequency ($ka = 5$), increasing the distance parameters leads to a radical increase in the pressure directivities. In other words, when the sphere is located relatively close to the half-space boundary (i.e., $d_1/a = 2, d_2/a = 3, 5$) the pressure patterns become very non-directional with appearance of relatively strong backward scattered main (side) lobes. A very similar behaviour is observed at the highest frequency ($ka = 10$), except that the overall scattered far-field pressure magnitudes are now distinctively larger.

To further assess the effect of rigid corner on the scattered acoustic field, the radial acoustic intensity distribution, $I_{rad}(r_1, \theta_1, \varphi_1, \omega)$, at selected nondimensional frequencies and distance parameters in the neighbourhood of the rigid sphere under oblique wave incidence is illustrated in Fig. 5. It is interesting to study the change in strength and directionality of the scattered energy as the incident wave frequency and the distance parameters are changed. At the lowest frequency ($ka = 0.1$), the extremely small amplitude scattered sound energy is almost evenly distributed around the rigid sphere for both distance parameters. The proximity to the vertical wall does not seem to be very influential here. At the intermediate frequency of $ka = 1$, the presence of the vertical wall begins to show its effect as the scattered energy directionality somewhat enhances while its overall magnitude drastically increases. At $ka = 10$, the sound intensity magnitudes strongly amplify and there is a strong lobe in the specular direction (i.e., $\theta = 135^\circ$), with fairly small diffraction side-lobes in the neighbouring directions. Furthermore, the symmetry around the specular direction is destroyed due to the phase difference that exists between the wave directly incident on the rigid sphere and the secondary wave also incident on the sphere after reflection from the rigid corner. As noted earlier, such phase-difference is due to the fact that the direct and secondary waves do not hit the sphere simultaneously, but with a relative delay. Finally, in order to check overall validity of the work we have computed the form function amplitude versus nondimensional frequency for a rigid sphere immersed near a hard flat boundary by selecting in turn ($d_1/a = 2, 4, 10; d_2/a = 100$) and ($d_1/a = 100; d_2/a = 2, 4, 10$) in our general MATLAB code. The numerical results, as shown in Fig. 6, accurately reproduce the curves displayed in Fig. 3 of Ref.(2).

4. Conclusions

This work presents analytical solutions as well as numerical results for the fundamental boundary value problem concerning the general interaction of a plane sound wave with an impenetrable sphere immersed in an acoustic quarterspace. The solution is based on the classical method of images and the translational addition theorems for spherical wave functions. The scattered far-field pressure directivity, the form function amplitude, and the scattered acoustic intensity are illustrated for a selected range.
of frequencies and separations. The numerical results reveal the central effects of the distance parameters, the incident wave frequency, and the angle of wave incidence on amplitude and directionality of the scattered field in comparison with the acoustic halfspace problem. The presented exact solution can serve as the benchmark for comparison to other solutions obtained by strictly numerical or asymptotic approaches. It can also be extended to sound scattering by arrays of rigid particles suspended near a rigid corner.

References


(9) Ivanov, Y.A., Diffraction of Electromagnetic Waves on Two Bodies, (1970), National Aeronautics and Space Administration, Washington, D.C.

