Theoretical and Experimental Studies on Transient Heat Transfer for Forced Convection Flow of Helium Gas over a Horizontal Cylinder*

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Forced convection transient heat transfer for helium gas at various periods of exponential increase of heat input ($Q_0 \exp(t/\tau)$) to a horizontal cylinder (heater) was theoretically and experimentally studied. In the theoretical study, transient heat transfer was numerically solved based on a turbulent flow model. It was clarified that the surface superheat and heat flux increase exponentially as the heat generation rate increases with the exponential function. The values of numerical solution for surface temperature and heat flux agree well with the experimental data for the cylinder diameter of 1 mm. In the experimental studies, the authors measured heat flux, surface temperature, and transient heat transfer coefficients for forced convection flow of helium gas over horizontal cylinders under wide experimental conditions. The platinum cylinders with diameters of 1.0 mm, 0.7 mm, and 2.0 mm were used as test heaters. The gas flow velocities ranged from 2 to 10 m/s, and the periods ranged from 50 ms to 15 s. It was clarified that the heat transfer coefficient approaches the quasi-steady-state one for the period $\tau$ longer than about 1 s, and it becomes higher for the period shorter than around 1 s. The heat transfer shifts to the quasi-steady-state heat transfer for longer periods and shifts to the transient heat transfer for shorter periods. The transient heat transfer coefficients show significant dependence on cylinder diameters, there are higher for smaller cylinder diameters. The empirical correlations for quasi-steady-state heat transfer and transient heat transfer were obtained based on the experimental data.

Key Words: Transient Heat Transfer, Forced Convection, Helium Gas, Exponentially Increasing Heat Input, Period, Numerical Solution, Turbulent Flow, Horizontal Cylinder, Diameter

1. Introduction

Transient forced convection heat transfer process is important for safety assessment of a high temperature gas cooled reactor (HTGR) at the accident of excess reactivity(1)--(3).

Concerning the problem of transient heat transfer with exponentially increasing heat generation rate ($Q = Q_0 \exp(t/\tau)$), here, $Q$ is heat generation rate, $Q_0$ is initial heat generation rate, $t$ is time, and $\tau$ is period of heat generation rate), there are only a few analytical and experimental works as far as the authors know. Soliman et al.(4) analytically obtained a temperature change in plate by taking into account the turbulent boundary layer around the plate. However, the solution of heat transfer coefficient for water is 50% higher than their experimental data. Kataoka et al.(5) conducted the transient experiment of water which flows in parallel to a cylinder, and obtained an empirical correlation for the ratios between the transient heat transfer coefficient and steady state one in term of one nondimensional parameter composed of period, velocity, and heater length. Liu and Fukuda(2),(3) obtained the experimental data and correlation for both parallel flow and cross-flow of helium gas over a horizontal cylinder. However, the experimental data were limited to a single diameter of cylinder for the parallel flow, and limited to a low-Reynolds number region for the cross flow.
The above previous researches have not resulted in a correlation with reliability based on theoretical solution. Moreover, there is almost no experimental research on transient forced convection heat transfer process for helium gas flowing over various diameters of heaters at a high region of Reynolds number, and there are no detailed knowledge on the effects of the period of heat generation rate, the flow velocity, the gas temperature, and the diameter on the transient heat transfer for helium gas.

In this study, transient heat transfer for forced convection flow of helium gas flowing over single horizontal cylinders was first numerically solved based on turbulent flow model. Then, it was studied experimentally using a forced convection heat transfer apparatus under wide experimental conditions. The surface superheat and heat flux were numerically obtained at various periods of heat generation rate, velocity, temperature, and diameter of heater. The values of numerical solution were compared with the measured experimental data. The empirical correlations for quasi-steady-state heat transfer and transient heat transfer were obtained based on the experimental data.

### Nomenclature

- \( a \): thermal diffusivity, \( m^2/s \)
- \( C_1 \): constant in the e equation
- \( C_2 \): constant in the e equation
- \( C_e \): constant in the eddy diffusivity
- \( c \): specific heat, \( J/(kg\cdot K) \)
- \( D \): heater diameter, m
- \( h \): heat transfer coefficient, \( W/m^2\cdot K \)
- \( L \): effective length of heater, m
- \( N_u \): Nusselt number, \( hL/\lambda \)
- \( P \): Pressure, Pa
- \( Pr \): Prandtl number
- \( Q \): heat generation rate per unit volume, \( W/m^3 \)
- \( Q_0 \): initial heat generation rate per unit volume, \( W/m^3 \)
- \( q \): heat flux, \( W/m^2 \)
- \( R_0 \): radius of circular channel, m
- \( r \): coordinate normal to central axis of cylinder, m
- \( r_0 \): radius of cylinder, m
- \( Re \): Reynolds number, \( UL/\nu \)
- \( T \): temperature, K
- \( \Delta T_s \): temperature difference between wall and gas, K
- \( t \): time, s
- \( U \): velocity of gas, m/s
- \( u \): x component velocity, m/s
- \( v \): r component velocity, m/s
- \( x \): coordinate along the central axis of cylinder, m
- \( k \): turbulence kinetic energy
- \( \rho \): density, \( kg/m^3 \)
- \( \lambda \): thermal conductivity, \( W/m\cdot K \)
- \( \nu \): kinematic viscosity, \( m^2/s \)
- \( \sigma_k \): effective Prandtl number for \( k \)
- \( \sigma_T \): turbulent Prandtl number
- \( \sigma_e \): effective Prandtl number for \( e \)
- \( \tau \): period of heat generation rate, s
- \( \tau^* = \frac{\tau U}{L} \): nondimensional parameter

### Subscripts

- \( h \): cylinder (test heater)
- \( f \): helium gas
- \( s \): surface of cylinder
- \( st \): quasi-steady-state
- \( t \): turbulence
- \( tr \): transient
- \( w \): inner wall of circular channel
- \( \infty \): free stream

### 2. Numerical Solution

#### 2.1 Physical model

Figure 1 shows the physical model in this numerical solution. The heater was mounted horizontally along the center axis of the circular channel, which is 20 mm in diameter. Cylinders with diameters of 0.7 mm, 1.0 mm, and 2.0 mm were used as the test heaters. The test heater is 80 mm in length. Fluid and heater are helium gas and platinum cylinder, respectively. Forced convection transient heat transfer for helium gas flowing over single horizontal cylinders at various periods of exponentially increasing heat input \( (Q = Q_0 \exp(t/\tau)) \) were numerically calculated.

Following assumptions were made to obtain the numerical solution.

1) Two-dimensional problem \((x, r)\).
2) Non-compressible helium gas is considered.

#### 2.2 Basic conservation equations

For the fluid (helium gas), mass, momentum and energy conservation equations take the following forms, respectively.

\[
\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (ru) = 0 \tag{1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \]
\[
= \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( (\nu + \nu_t) \frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r(\nu + \nu_t) \frac{\partial u}{\partial r} \right) \right) \tag{2}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial r} \]
\[
= \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\partial}{\partial x} \left( (\nu + \nu_t) \frac{\partial v}{\partial x} + \frac{\partial}{\partial r} \left( r(\nu + \nu_t) \frac{\partial v}{\partial r} \right) \right) \tag{3}
\]
\[
\begin{align*}
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial r} &= \frac{\partial}{\partial x} \left( \alpha + \frac{\nu_1}{\sigma'_T} \right) \frac{\partial T}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \frac{\nu_1}{\sigma'_T} \right) \frac{\partial T}{\partial r} \right) \\
&= \frac{\partial}{\partial x} \left( \alpha + \frac{\nu_1}{\sigma'_T} \right) \frac{\partial T}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \frac{\nu_1}{\sigma'_T} \right) \frac{\partial T}{\partial r} \right) + G - \varepsilon \\
&= \frac{\partial}{\partial x} \left( \alpha + \frac{\nu_1}{\sigma'_T} \right) \frac{\partial T}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \frac{\nu_1}{\sigma'_T} \right) \frac{\partial T}{\partial r} \right) + \left( C_1 G - C_2 \varepsilon \right) \frac{\varepsilon}{k} \\
&= \nu_1 \left( \frac{\partial T}{\partial x} \right)^2 + 2 \left( \frac{\partial T}{\partial r} \right)^2 + \left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial r} \right)^2 + 2 \left( \frac{\partial T}{\partial r} \right) \\
&= \frac{\partial T}{\partial t} + \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) + \frac{\dot{Q}}{\rho \cdot c_h} \\
\end{align*}
\]

For the cylinder, the energy conservation equation takes the following form.

Boundary Conditions:
- at \( r = r_0 \) (cylinder surface),
  \[ u = v = 0 \]
- \( T_s \) at solid boundary
  \[ T_s = T_{fluid} \]
- \( q_s \) at solid boundary
  \[ q_s = q_{fluid} \]

at \( r = R_0 \) (circular channel)
- \( u = v = 0 \)
- \( T_e = T_\infty \)

CFD2000 program was used in this numerical solution. The above basic conservation equations were discretized using finite volume method (FVM). The convection term was calculated using second-order upwind difference scheme, and the diffusion term was calculated using arithmetic mean method. Collocation grid was used to save the computer memory. PISO (Pressure Implicit with Splitting of Operation) algorithm was used to calculate the pressure distribution.

Figure 2 shows the calculation regions of the upper parts of the cylinder and the fluid. J1K2 means the region of heater (cylinder), and the others are the field of flowing fluid. The lengths of the regions of J1K2 and J2K2 are the same with that of the cylinder in the experiments (80 mm). The radiiuses of the regions of J1 are 0.35 mm, 0.5 mm, and 1.0 mm, and the lengths of the regions of J2 are 9.65 mm, 9.5 mm, and 9.0 mm for the cylinder diameters of 0.7 mm, 1.0 mm, and 2.0 mm, respectively. The lengths of regions along the flowing direction are 1 cm, 8 cm, and 1 cm for the K1, K2, and K3, respectively.

The numbers of meshes in J-direction are 3 for J1, and 60 for J2 for the cylinder diameter of 1.0 mm, and those in K-direction are 5 for K1, 40 for K2, and 5 for K3. Table 1 shows the numbers of meshes at various cylinder diameters. Table 2 shows the conditions of numerical solution.

### 3. Results of Numerical Solution

#### 3.1 Numerical results of velocity and temperature distributions

Figure 3 shows velocity vector over the cylinder with a diameter of 1.0 mm at velocity of 10 m/s and period of 397.5 ms. The length of the cylinder is 80 mm. The velocity shows a relatively flat distribution in the free stream flowing, and it decreases to zero at the wall of the cylinder and the inner wall of the stainless steel test section. Figure 4 shows the temperature distributions over a 1.0 mm-dia. cylinder at surface superheats of 10 K, 100 K, 200 K, 300 K, and 400 K. The calculation condition for velocity and period are 10 m/s and 85.8 ms. It can be seen from this figure, the surface temperature increases as the heat generation rate increases exponentially, then the temperature within the boundary layer also increases with the increase of the surface temperature. It is understood that the gradient of the temperature distribution near the wall of the cylinder is higher at a higher surface superheat.

#### 3.2 Numerical solution of surface superheat and heat flux

Figure 5 shows time-dependence of heat generation rate (\( \dot{Q} \)), surface superheat (\( \Delta T_s = T_s - T_\infty \)), and heat...
Fig. 3  Velocity distribution over a 1.0 mm-diameter cylinder at a velocity of 10 m/s and a period of 397.5 ms.

Fig. 4  Temperature distribution over a 1.0 mm-diameter cylinder at a velocity of 10 m/s and a period of 85.8 ms under various surface superheats.

Fig. 5  The relation of $\dot{Q}$, $q$, and $\Delta T_s$ with $t/\tau$ at the period of 166.1 ms.

Fig. 6  The relation of $\dot{Q}$, $q$, and $\Delta T_s$ with $t/\tau$ at the period of 397.5 ms.

Fig. 7  The relation of $\dot{Q}$, $q$, and $\Delta T_s$ with $t/\tau$ at the period of 1.5 s.

3.3 Effects of velocity, and temperature on transient heat transfer

Figures 8 shows the time-dependence of $\Delta T_s$ and $q$ at velocities of 10 m/s and 6 m/s. As shown in the figure, the temperature increases almost with the same rate. However, the heat flux increases higher than that at a small velocity. It is understood that the heat transfer coefficient is higher as the velocity increased. Figure 9 is the relationship of the $q$, $\Delta T_s$ with $t/\tau$ at the period of 772 ms and the temperatures of 313 K and 353 K. As can be seen from the figure, the gas temperature shows little effect on the numerical solution of heat flux ($q$) and surface superheat ($\Delta T_s$). Because the heat transfer coefficient ($h$) is obtained by dividing the heat flux with the surface superheat ($\Delta T_s$).
3.4 Heat transfer coefficients
Heat transfer coefficient, $h$, is defined as shown in the following equation.

$$ h = \frac{q}{\Delta T_s} \quad (15) $$

Figure 10 shows the heat transfer coefficient at velocity of 10 m/s and period of 395 ms for the diameter of 1.0 mm cylinder. The symbol of circle shows the experimental data, and the solid line is the numerical solution. It can be seen that the heat transfer coefficient of numerical solution approaches immediately from a higher value to an asymptotic value. However, the experimental data shows the asymptotic value at a time of about 3 times of the period. The asymptotic values will be used as the heat transfer coefficients at transient (or unsteady-state) heat transfer process. As seen from the figure, the asymptotic values agree within about 5%.

Figure 11 shows the heat transfer coefficient at various periods for the cylinder diameters of 0.7 mm, 1.0 mm, and 2.0 mm. The heat transfer coefficients are higher for smaller cylinder diameters. The heat transfer coefficients show some increases as the period becomes extremely smaller than 0.1 s. However, the increasing rate is smaller compared with author’s experimental data shown in Fig. 13. 

4. Experimental Study
4.1 Experimental apparatus and method
Experiment apparatus was reported in previous papers(2), (3). Figure 12 shows the schematic diagram of the experiment apparatus. The experiment apparatus is composed of gas compressor (2), flow meter (5), test section (6), surge tank (3), (8), cooler (7), the heat input control system, and the data measurement and processing system. The gas was circulated by a compressor, and the fluctuations of gas flowing and pressure due to compressor were removed with the surge tanks. Moreover, the gas temperature inside the loop was heated to the desired temperature level by a preheater, and cooled by a cooler before the gas flows into the compressor. Flowing rate in the test section was measured with the turbine meter, and the pressure was
measured with the pressure transducer. The temperature of the turbine meter exit and the temperature near test section heater were measured by thermocouples with a precision of ±1 K. Helium gas with a high purity of 99.9999% was used as the test fluid.

The platinum test heater was heated by direct current from a power source. The heat generation rates of the heater were controlled and measured by a heat input control system. The average temperature of test heater was measured by resistance thermometry using a double bridge circuit including the test heater as a branch. The test heater was annealed and its electrical resistance versus temperature relation was calibrated in water, washed with a trichloroethylene liquid before using it in the experiment. The heat flux of the heater is calculated by the energy equation in the cylinder. The test heater surface temperature can be calculated from the measured average temperature and heat generation rate by solving unsteady state heat conduction equation in the test heater by assuming the surface temperature around the test heater to be uniform. Experimental error was estimated to be ±1 K in the heater surface temperature and ±2% in the heat flux.

4.2 Experimental results and discussion

4.2.1 Experiment conditions

The experimental conditions are almost the same with those shown in Table 2. However, the period and gas temperature are ranged from 50 ms to 15 s, and from 303 K to 353 K, respectively. The corresponding Reynolds numbers ranged from $7.3 \times 10^3$ to $1.9 \times 10^4$.

4.2.2 Experimental results

The symbols in Figs. 5–7 show typical experimental data of the time-dependence of heat generation rate, surface superheat, and heat flux at the heat generation rate increasing periods of 166.1 ms, 397.5 ms, and 1.5 s under flow velocity of 10 m/s (Reynolds number 1.9×10^4) and gas temperature of 313 K, respectively. It is understood that surface superheat and heat flux increase exponentially as the heat generation rate increases with the exponential function.

As shown in Fig. 10 mentioned before, the heat transfer coefficients approach constant values from higher initial values when the time passes over a certain time of about 3 times of the period ($t/\tau > 3$). It was confirmed that the heat transfer coefficients approach asymptotic values similarly at all periods, velocities, diameters, and gas temperatures. These asymptotic values were used as the heat transfer coefficients at the transient (or unsteady-state) heat transfer process.

Figure 13 shows the relation between the heat transfer coefficient and the period of heat generation rate at a gas temperature of 303 K. The heat transfer coefficient, $h$, becomes to approach asymptotic value at every velocity when $\tau$ is longer than about 1 s. The heat transfer process in this region transmits heat as well as a usual convective heat transfer through the thermal boundary layer influenced by the flow of helium gas. It is called the quasi-steady-state heat transfer here. On the other hand, when the period $\tau$ is shorter than about 1 s, $h$ increases as $\tau$ shortens. This shows that the heat transfer process is in the unsteady state, and the heat transfer in this region is significantly affected by the influence of the temperature gradient within the thermal boundary layer around the test heater. Especially, in the region of $\tau$ shorter than 200 ms, the thermal boundary layer becomes very thin, then the conductive heat transfer near the heater comes to govern the heat transfer process, and the heat transfer coefficient increases greatly with shorter period in this region. It was clarified that the heat transfer phenomenon was divided into a quasi-steady-state heat transfer and a transient heat transfer on the boundary of around 1 s. The coefficient of heat transfer increases with the flow velocity as shown in the Fig. 13.

4.2.3 Correlations for quasi-steady-state and transient (unsteady state) heat transfer at various diameters

Figure 14 shows the relation between the
Nusselt number and the Reynolds number for the periods ranging from 69 ms to 13.5 s. They are shown on $N_{st}$ versus $Re$ graphs. As shown in the figures, for the periods longer than about 1 s, the Nusselt numbers are not influenced by the period, but increase with flow velocity. They can be correlated by the following empirical equation by the method of least squares. It was shown by the dashed lines to compare with the experimental data.

$$N_{st} = 2.2Re^{0.5}Pr^{0.4} \quad \text{for } D = 1 \text{ mm} \quad (16)$$

Where, $N_{st} = hL/\lambda_f$, $Re = UL/\nu_f$, $h$ (W/m$^2$K) is heat transfer coefficient, $L$ (m) is effective length of the heater, $\lambda_f$ (W/mK) is thermal conductivity of helium gas, $U$ (m/s) is flow velocity, and $\nu_f$ (m$^2$/s) is kinematic viscosity of helium gas. The Prandtl number $Pr$ is about 0.68 in the range of this experiment.

On the other hand, for the periods under about 1 s, the Nusselt numbers are affected both by the period and the flow velocity. They approach asymptotic values in the quasi-steady-state heat transfer for the periods longer than about 1 s. The effect of flow velocity becomes weak for shorter periods by decreasing the gradient of the data in the graphs. The solid lines are the values by correlations for each period if they are correlated by the method of least squares.

Figure 15 shows the quasi-steady-state heat transfer at diameters of 0.7 mm, 1.0 mm, and 2.0 mm. The Nusselt number decreases with the increase in diameter. Following correlation for the diameters ranged from 0.7 mm to 2.0 mm were obtained.

$$N_{st} = 2.2Re^{0.5}Pr^{0.4}(D/D_0)^{-0.5} \quad (17)$$

Where, $D$ is the cylinder diameter, and $D_0$ is 1 mm.

As mentioned in the Introduction, Liu et al.\(^{(3)}\) carried out an experiment on the transient heat transfer of helium gas flowing across a horizontal cylinder at low Reynolds number region, and obtained an empirical correlation of the ratio of transient Nusselt number to the quasi-steady-state one using a dimensionless period of $\tau^*$ ($\tau^* = \tau U/L$, $U$ is flow velocity, and $L$ is characteristic length).

The present experimental data can be also correlated using the dimensionless period, $\tau U/L$, as shown in Fig. 16, and we obtained the following correlation. It can be seen from Fig. 16, the transient heat transfer approaches quasi-steady-state one for the nondimensional period larger than about 300. The heat transfer shifts to the quasi-steady-state heat transfer for longer period and shifts to the transient heat transfer for shorter period at the same flow velocity. The transient heat transfer approaches the quasi-steady-state one for higher flow velocity.

$$N_{tr} = N_{st}[1 + 2.0(\tau U/L)^{-0.8}] \quad (18)$$
5. Conclusions

Forced convection transient heat transfer at various periods of exponential increase of heat input to a horizontal cylinder was theoretically and experimentally studied. Following results were obtained.

(1) The values of numerical solution for surface temperature and heat flux agree well with the experimental data for the cylinder diameter of 1 mm.

(2) The gradient of temperature distribution near the cylinder becomes larger as the surface temperature increases.

(3) The measured heat transfer coefficient approaches the quasi-steady-state one for the period $\tau$ over about 1 s, and it becomes higher for the period of $\tau$ shorter than about 1 s.

(4) The conductive heat transfer becomes predominant for the period less than about 1 s, especially in the region of less than 200 ms, though the transient heat transfer is influenced by both convection and the conductive heat transfer in the quasi-steady-state heat transfer region for the period larger than about 1 s.

(5) The transient heat transfer coefficients show significant dependence on cylinder diameters, they are higher for smaller cylinder diameters.

(6) The correlation for steady-state heat transfer in a parallel flow is quiet different from that in a cross-flow (Liu and Fukuda, 2002)\(^3\). The empirical correlation of the transient forced convection heat transfer was obtained by the quasi-steady-state heat transfer and the dimensionless parameter, $\tau U/L$, based on the experiment data.

References


