Modelling Pollen Distribution by Wind through a Forest Canopy*

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Pollen released from trees within a forest is transported by the wind through the canopy. Some is trapped by the foliage as it is advected and dispersed while falling under gravity. Taking into account these three mechanical processes, the quantitative model presented here is based on principles of conservation of mass. It is assumed that the pollen particles, being small, quickly reach their terminal velocity with respect to the mean air flow, and are mechanically dispersed by the turbulence generated by the air flow through the foliage. The rate of removal of the pollen from the flow, the so-called “trapping rate”, is assumed proportional to the volumetric concentration of the pollen mass in the air. The aim is to obtain analytic solutions to the resulting advection-dispersion-trapping (convection-diffusion-decay) equations. Some examples are presented to illustrate the effects of various parameters.

Key Words: Mathematical Modelling, Forest Canopy, Pollen, Advection, Dispersion, Trapping

1. Introduction

This paper explores some aspects of the problem of modelling the motion of small pollen particles through a forest canopy. Pollen that is released from a tree may be blown by the wind through the foliage, while falling to the ground. The foliage (branches, twigs, leaves) induces turbulence in the air flow, and this spreads the particles through a process of mechanical dispersion. Some pollen particles may contact the foliage and adhere, and this is referred to as “trapping”. The rest of the pollen continues to fall to the ground. The aim here is to set up a reasonable mathematical model for the process and to find, if possible, analytical solutions, so that the resulting distribution of trapped pollen throughout the canopy, as well as that of the “fall-out” on the ground can be readily calculated. The effects of parameter variation can then be efficiently explored. The model could also be applied to the transport of other particles such as aerosols or dust through a windbreak or an orchard.

The dispersion of heat, gases and small particles through a forest canopy or crop foliage is of vital interest to the agricultural, horticultural and forestry industries. While fluid-mechanical models may be applied, some of the parameters (e.g. dispersion and diffusion coefficients, trapping rate parameters for the foliage) are only available experimentally.

In particular, attention has been paid to the scale and structure of air turbulence within forests. Amiro & Davis (1988)[1] reported the existence of large-scale high-velocity eddies within the canopy of a spruce forest. Much more recent interpretation of spatially-distributed airflow velocity and temperature variations is reported by Morimoto et al. (2006)[2], where coherent eddy structures are inferred; these penetrate into the canopy from the free stream above the foliage, and so are a boundary phenomenon. By making measurements in a deciduous forest in various seasons, Morimoto et al. also deduce that the eddy diffusion coefficient is strongly influenced by the leaf density.

Model crops and forests have been investigated in laboratory-scale models. Seginer et al. (1976)[3] used a wind-tunnel model built of slender cylindrical rods to simulate the canopy. Their measurements imply an exponential wind profile, and constant turbulence intensity, skewness and mixing length along the height of the “canopy”. The turbulence structure was also investigated using a model canopy by Raupach et al. (1986)[4].

Related phenomena in crops such as rice and wheat has resulted in studies of the wave-like movement of the
vertical stalks. Travelling wave-type structures, known as *honami* (ho = ear of rice, wheat, etc.), *nami* = wave] are observed on windy days. Measurements of air velocities and pressures in waving wheat have been reported by many authors, e.g. Finnigan (1979)(5),(6).

For the mathematical model discussed in this paper, the turbulence is supposed to be of uniform strength and scale throughout the canopy. While the wind profile is supposed uniform here, developments of the current model are underway to incorporate height-varying wind speed and turbulence length scales, including anisotropy of the dispersion lengths to allow for the non-uniform and anisotropic structure of the foliage, different trapping rates in various layers in the canopy (e.g. light foliage, dense foliage) as well as the free air flow above the canopy.

This work extends previous analysis of particle transport by the atmosphere (e.g. McKibbin, 2003; McKibbin et al., 2005)(7),(8) by including a trapping process to remove material from the flow.

### 2. The Model

It is assumed that the air is of uniform density and that its motion is not affected by the small total volume fraction of particles. Because the pollen particles are small, they quickly assume their terminal velocity with respect to the air, moving horizontally with the wind and downwards at their settling speed. Some are removed from the flow by being trapped on the foliage; here, it is assumed that the trapping rate is proportional to the pollen concentration; it also depends on the vegetation type and the density of the foliage. A catalogue of “interception” (trapping) characteristics for various canopy types has recently been provided by Teske & Thistle (2004)(9).

The mass concentration of particles per unit volume of the atmosphere is denoted \( c(x,y,z,t) \) [with dimensions \( \text{kg m}^{-3} \)], where the Cartesian coordinate system is arranged so that the \( x-y \) plane is the ground and the \( z \)-axis points vertically upwards. The ground is assumed to be horizontal, although this may not be a significant restriction, since the mean airflow, even if not horizontal, is generally likely to be parallel to the ground provided wind speeds are not large. The principle of conservation of mass applied to the pollen particles yields the equation

\[
\frac{\partial c}{\partial t} = m - \nabla \cdot q - Tc
\]  

where \( m \) is a mass source term (mass release rate per unit volume \( \text{[kg s}^{-1} \text{m}^{-3}] \)), \( q \) is the pollen mass flux per unit cross-sectional area of air \( \text{[kg s}^{-1} \text{m}^{-2}] \), and \( T \) \( \text{[s}^{-1}] \) is a trapping rate parameter. The parameter \( T \) depends on the foliage type, and, as other parameters (wind speed, turbulence length scales) do, may vary with height within the canopy. Above the canopy, \( T = 0 \). The mass flux term includes advective, dispersive and settling mass flows, and is of the form

\[
q = cu - D \otimes \nabla c - cS k,
\]

where \( u = (U,V,0) \) is the mean wind velocity vector, with wind speed \( W = \sqrt{U^2 + V^2} \), \( S \) is the pollen settling speed in the downward direction (\( k \) is a unit vector in the \( z \)-direction) and \( D =威尔 \) is the dispersion tensor written in terms of wind speed and a dispersion length tensor. This tensor \( L \) reflects the dominant length scales in the turbulent air flow through the canopy. In general, it is likely to be non-isotropic, and also dependent on the forest type and level within the canopy.

For the simple case of a particle release of mass \( Q \) at time \( t = 0 \) from the point \( (X,Y,Z) \) within the canopy, \( m \) may be expressed in terms of Dirac delta functions in the form

\[
m = Q \delta(x-X) \delta(y-Y) \delta(z-Z) \delta(t).
\]

Other mass releases may be then be constructed using the results for this basic case (see below). Substitution of expressions (2) and (3) into Eq. (1) and some rearrangement gives

\[
\frac{\partial c}{\partial t} + \nabla \cdot (cu - D \otimes \nabla c - cS k) + Tc = Q \delta(x-X) \delta(y-Y) \delta(z-Z) \delta(t).
\]

The dispersion tensor is of the form

\[
D = \begin{bmatrix}
D_{xx} & D_{xy} & 0 \\
D_{yx} & D_{yy} & 0 \\
0 & 0 & D_{zz}
\end{bmatrix},
\]

which is a symmetric second-order tensor. If the wind direction is at angle \( \theta \) to the \( x \)-axis (i.e. \( U/W = \cos \theta \), \( V/W = \sin \theta \)), and the longitudinal (downwind), transverse (crosswind) and vertical dispersion coefficients are \( D_L \), \( D_T \) and \( D_V \) respectively, then the dispersion tensor has the form

\[
D = \begin{bmatrix}
D_L \cos^2 \theta + D_T \sin^2 \theta & (D_L - D_T) \sin \theta \cos \theta & 0 \\
(D_L - D_T) \sin \theta \cos \theta & D_L \sin^2 \theta + D_T \cos^2 \theta & 0 \\
0 & 0 & D_V
\end{bmatrix}.
\]

The dispersion coefficients are of the form \( (D_L, D_T, D_V) = W(L_L, L_T, L_V) \) where the \( L \) values are the dominant turbulence length scales in the downwind, crosswind and vertical directions, and may be found from in situ measurements [e.g. as in experiments carried out near the Kakuma campus, Kanazawa University, Japan; see Morimoto et al. (2006)(2)]. In general, it may be assumed that the settling speed, the wind speed and direction, the turbulent dispersion and the trapping rate parameter all vary with elevation. Then \( S = S(z) \), \( U = U(z) \), \( V = V(z) \), \( D = D(z) \) and \( T = T(z) \), and Eq. (4) can be written

\[
\frac{\partial c}{\partial t} + U(z) \frac{\partial c}{\partial x} + V(z) \frac{\partial c}{\partial y} - S(z) \frac{\partial c}{\partial z} = \frac{dS}{dz} c + T(z)c
\]

\[
= D_L(z) \frac{\partial^2 c}{\partial x^2} + 2D_T(z) \frac{\partial^2 c}{\partial x \partial y} + D_V(z) \frac{\partial^2 c}{\partial y^2} + D_L(z) \frac{\partial^2 c}{\partial x^2} + D_L(z) \frac{\partial^2 c}{\partial z^2} + Q \delta(x-X) \delta(y-Y) \delta(z-Z) \delta(t)
\]
This equation is difficult to solve analytically for even the simplest continuously-varying wind and particle parameters. One way to simplify the model is to assume piecewise constant approximations to the continuous functions for the parameters in Eq. (7). The forest canopy and the wind distribution are then divided into contiguous layers parallel to the ground; the wind speed, dispersion and trapping parameters are assumed constant within each layer, while continuity conditions are used to match the solutions within each layer across the layer interfaces. This method also allows for a foliage-free region above the canopy, where pollen that escapes upwards can then be transported downwind before re-entering the foliage further downwind.

To illustrate the method, the uniform case is discussed in the next section. It is assumed that all parameters are constant and, in the first instance, that the vertical dispersion is negligible. The latter assumption allows consideration of downwind and downward movement only; the complications associated with above-canopy flows can then be ignored.

3. Uniform Wind + Negligible Vertical Dispersion

The simplest example is when a mass $Q$ of pollen particles with constant settling speed $S$ is released at time $t = 0$ from a point in the canopy at height $Z$ above the ground into a steady, uniform, horizontal wind of speed $W$ (a constant) flowing through the foliage. A set of Cartesian coordinate axes $(x, y, z)$ is aligned so that the wind is in the positive $x$-direction [$\theta = 0$ in Eq. (6) above and $(x, y, z) = (W, 0, 0)$]. The origin is on the ground directly below the point of release, i.e. $X = Y = 0$. If the downwind, or longitudinal, dispersion coefficient (in the $x$-direction) is $D_L$, and the crosswind, or transverse, dispersion coefficient (in the $y$-direction) is $D_T$ (each a constant) and the vertical dispersion is relatively small enough to be regarded as negligible (i.e. $D_H = 0$), then the concentration $c(x, y, z, t)$ at time $t$ after release is given by the solution to a simplified version of Eq. (7), viz.,

$$
\frac{\partial c}{\partial t} + W \frac{\partial c}{\partial x} - S \frac{\partial c}{\partial z} + T_c = D_L \frac{\partial^2 c}{\partial x^2} + D_T \frac{\partial^2 c}{\partial y^2} + Q \delta(x) \delta(y) \delta(z) \delta(t). \quad (8)
$$

The solution is

$$
c(x, y, z, t) = \frac{Q}{4\pi \sqrt{D_L D_T T_T}} \exp \left[ -\frac{(x-Wt)^2}{4D_L t} - \frac{y^2}{4D_T t} - T_t \right] \times \delta(z-Z-S t). \quad (9)
$$

The mass released spreads out in the downwind and crosswind directions while falling with speed $S$. The concentration is zero except at height $z = Z - S t$.

3.1 Trapped pollen

The total mass of pollen per unit volume, $M_T$, trapped at any point $(x, y, z)$, $0 < z < Z$ in the canopy after the release is found by integrating the trapping rate $T_c$ with respect to time:

$$
M_T(x, y, z) = \int_0^\infty T_c(x, y, z, t) dt. \quad (10)
$$

Note from Eq. (9) that the pollen concentration is non-zero only at the time taken to fall from the release point to the sample level, $t = t_c = (Z - z)/S$. This means that, for this model, no pollen moves upwards from the release point, and so $M_T(x, y, z) = 0$ for $z > Z$. The trapped amount [in kg m$^{-3}$] for $0 < z < Z$ is

$$
M_T(x, y, z) = \frac{TQ}{4\pi \sqrt{D_L D_T T_T}} \frac{1}{S} \exp \left[ -\frac{(x-Wt_c)^2}{4D_L t_c} - \frac{y^2}{4D_T t_c} - T_t \right] = \frac{TQ}{4\pi \sqrt{D_L D_T} Z-z} \frac{1}{S} \exp \left[ -\frac{x^2}{4D_L (Z-z) / S} - \frac{y^2}{4D_T (Z-z) / S} - \frac{S}{4D_L t_c} - \frac{T_t}{S} \right] \quad (11)
$$

3.2 Total trapped pollen

The total amount of pollen trapped throughout the foliage from the point release is given by

$$
Q_{TT} = \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty M_T(x, y, z) dx \ dy \ dz = Q \left[ 1 - e^{-\frac{z Z}{S}} \right] \quad (12)
$$

which leaves a total amount $Q_D = Q - Q_{TT} = Q e^{-\frac{z Z}{S}}$ deposited on the ground. Both $Q_{TT}$ and $Q_D$ depend only on the total release $Q$ and the dimensionless parameter $TZ/S$. Note that they do not depend on the wind speed or turbulent dispersion parameters. The dependence of $Q_{TT}/Q$ on $TZ/S$ is shown in Fig. 1(a). As expected, the foliage-trapped amount increases with increasing trapping rate parameter $T$ and release height $Z$, while it is reduced by high values of the settling speed $S$.

If the mass source $Q$ is distributed uniformly over a vertical line from the point $(0, 0, 0)$ on the ground to point $(0, 0, H)$, which may represent a thin tree of height $H$, then the total pollen trapped is given by integrating the effect of the distributed source elements $(Q/H)dz$ over the line:

$$
Q_{TT} = \int_0^H \frac{Q}{H} \left( 1 - e^{-\frac{z Z}{S}} \right) dz = Q \left[ 1 - \frac{S}{TH} \left( 1 - e^{-\frac{z Z}{S}} \right) \right] \quad (13)
$$

with a corresponding total ground deposit $Q_D = Q - Q_{TT}$. Again, these values depend only on the total release $Q$ and the dimensionless combination $TH/S$, and the behaviour is similar to that described above for the point source. The relationship is shown in Fig. 1(b).
3.3 Spatial distribution – horizontal line source

The spatial distribution of the trapped and deposited pollen is of interest. For a point source, Eq. (11) describes a 3-dimensional distribution which is not easy to show as a 2-dimensional figure. Instead, a horizontal line source perpendicular to the wind is used for illustration here. If a release of mass per unit length \( Q \) [kg m\(^{-1}\)] occurs along the line \( (x = 0, -\infty < y < \infty, z = Z) \) at \( t = 0 \), the subsequent pollen concentration is independent of \( y \) and the lateral dispersion \( D_L \) and is given by solving Eq. (4) with a source term \( Q \delta(x)\delta(z-Z)\delta(t) \), to give

\[
c(x,y,z,t) = \frac{Q}{2\sqrt{\pi D_L}} \exp \left[ -\frac{(x-Wt)^2}{4D_L t} \right] \delta(z-(Z-St)),
\]

The amount that is trapped in the foliage [kg m\(^{-3}\)] is calculated, using Eq. (10), to be

\[
M_T(x,y,z) = \frac{TQ}{2\sqrt{\pi D_L}} \exp \left[ -\frac{(x-Wt)^2}{4D_L t} - Tt \right] \delta(z-(Z-St)),
\]

where \( t = (Z-z)/S \) as before. Figure 2 shows iso-surfaces for \( M_T/Q \) for the parameter set: \( Z = 3 \) m, \( W = 0.5 \) m s\(^{-1}\), \( L_d = 0.5 \) m \((D_L = W/L_d = 0.25 \) m\(^2\) s\(^{-1}\)), \( S = 0.2 \) m s\(^{-1}\), \( T = 0.1 \) s\(^{-1}\). The position of the line source is marked with an asterisk; the value of \( M_T \) is infinite at that (singular) point.

The total trapped amount per unit width of canopy in the \( y \)-direction is

\[
\bar{M}_T = \frac{Q}{2\sqrt{\pi D_L}} \exp \left[ -\frac{(x-Wt)^2}{4D_L t} \right]
\]

and, as expected, is similar in form to that for a point source, given by Eq. (12).

3.4 Spatial distribution – vertical surface source

The result in Eq. (15) for a line source can be extended to that of a perpendicular plane source of height \( H \) and of infinite extent in the \( y \)-direction. This could correspond to a line of trees releasing pollen uniformly throughout their height. Integration of the line solution for \( M_T \) given above, for a distribution of line sources of strength \( (Q/H)dZ \), in the form

\[
M_T(x,y,z) = \int_0^H \frac{TQ}{2\sqrt{\pi D_L S}} \exp \left[ -\frac{(x-W(Z-z)/S)^2}{4D_L (Z-z)/S} \right] dZ
\]

leads to a distribution of trapped pollen as shown in Fig. 3. Note that some pollen is moved in the upstream direction close to the “trees”, which are placed at \( x = 0 \), but none is...
moved above the sources.

4. Uniform Wind + Vertical Dispersion

When non-zero vertical dispersion $D_v$ is included, with the assumption that pollen release and flight is completely within the canopy, the solution for a mass release $Q$ from the point $(0,0,Z)$ at time $t=0$ is given by

$$c(x,y,z,t) = \frac{Q}{8\sqrt{\pi^3 D_L D_T D_v} t^3} \exp\left[\frac{(x-Wt)^2}{4D_L t} - \frac{y^2}{4D_T t}\right] - \frac{|z-(Z-St)|^2}{4D_v t} - T.$$  \hspace{1cm} (18)

[Note: The solution for zero trapping ($T = 0$) is given in McKibbin (2003)(7)]. Although the solution (18) does not strictly satisfy the boundary condition requirement for zero vertical dispersive flux at the ground, it represents a close approximation for most parameter sets [see comments in McKibbin (2003)(7)]. The amount trapped at any point in the foliage is given by

$$M_T(x,y,z) = \int_0^\infty Tc(x,y,z,t)dt$$

$$ = \frac{1}{8\pi \sqrt{D_L D_T D_v} \alpha} \exp\left\{\frac{1}{2} \frac{Wx}{D_L} - S(z-Z) - 2\alpha \beta\right\}$$

where

$$\alpha = \frac{1}{2} \sqrt{\frac{x^2}{D_L} + \frac{y^2}{D_T} + \frac{(z-Z)^2}{D_v}}$$

and

$$\beta = \frac{1}{2} \sqrt{\frac{W^2}{D_L} + \frac{S^2}{D_v} + 4T}$$

The total deposition on the ground is found by integrating the mass flux, calculated from Eq. (18), through the surface $z=0$, which gives

$$Q_D = \int_0^\infty -q_{z=0} dt$$

$$ = \int_0^\infty S c + D_v \frac{\partial c}{\partial z} \bigg|_{z=0} dt$$

$$ = \frac{Q}{2} \left(1 + \frac{1}{\sqrt{1+4D_v T/S^2}}\right) e^{-\frac{S^2}{4D_v T/S^2}}$$  \hspace{1cm} (20)

and the trapped amount is $Q_{TT} = Q - Q_D$. Note that for $T = 0$, $Q_D = Q$ (i.e. no pollen is trapped), and that as $T \to \infty$, $Q_D \to 0$, as expected. Also, these quantities are independent of the wind speed and horizontal turbulent dispersion, but do depend on the vertical dispersion. As $D_v \to 0$, the results become those in section 3 above (Eq. (12)). The dependence of $Q_{TT}/Q$ on $Tz/S$ for several values of $D_v T/S^2$ is shown in Fig. 4.

As before for the zero vertical dispersion case, results for a line source at $x=0$, $z=Z$ give

$$M_T(x,y,z) = \frac{TQ}{2\pi \sqrt{D_L D_v}} \exp\left\{\frac{1}{2} \frac{Wx}{D_L} - S(z-Z) - 2\alpha \beta\right\} K_0(2\alpha \beta)$$  \hspace{1cm} (21)

where

$$\alpha = \frac{1}{2} \sqrt{\frac{x^2}{D_L} + \frac{(z-Z)^2}{D_v}}$$

and

$$\beta = \frac{1}{2} \sqrt{\frac{W^2}{D_L} + \frac{S^2}{D_v} + 4T}$$

and $K_0$ is the modified Bessel Function of the Second Kind, order 0. With parameters as in the examples given above, and setting $L_v = 0.2$ m, with $D_v = W L_v = 0.1$ m$^2$ s$^{-1}$, Fig. 5 shows iso-surfaces of trapped pollen.

As in the example with no vertical dispersion, the line source solution above is extended to that of a perpendicular-
lar plane source (line of trees) of height $H$ and of infinite extent in the $y$-direction. Some iso-surfaces for the density of the trapped pollen are shown in Fig. 6.

Comparison between the two different models (neglecting or including vertical dispersion) shows that the main effect is in the quantity of pollen modelled as rising above the release point(s). As the typical dispersion length scale in the vertical direction is reduced, the second model predictions approach those of the first. This is shown in Fig. 7, where $L_V = 0.001$ m; comparison with Fig. 3, where the analysis assumes ab initio that $L_V = 0$ shows close agreement between the shape of the contours of trapped pollen.

5. Summary and Conclusions

Some results from models of pollen particle trapping in plant foliage have been presented. The models are linear, and analytical solutions allow fast calculation of distributions. The formulae given can be used to more readily investigate effects of parameter variation than fully numerical solution procedures.

Neglect of vertical dispersion may not be serious if the typical turbulence length scale in the vertical direction is small. However, the dispersion of pollen up into higher levels of foliage is not predicted by such a model.

Continuing work on development of models that are composed of layers of different foliage densities, including free air above the canopy and low foliage levels near the ground, is currently being carried out. The approach is to consider that a new layer is defined wherever there is a significant change in mean wind speed, mean wind direction, any dominant turbulence length scale, foliage type (trapping characteristic) or settling speed (perhaps caused by particle agglomeration).

In addition, any form of pollen or seed release can be modelled by adding together the effects of releases of particles (perhaps of different sizes and shapes) from points that are distributed spatially through a forest or which occur either at different instants or continuously over specified time periods. Some preliminary results for atmospheric flows have been reported in McKibbin (2006)(10).

Acknowledgement

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References

(9) Teske, M.E. and Thistle, H.W., A Library of Forest