Comparison of Near Wake-Flow Structure behind a Solid Cap with an Attached Bubble and a Solid Counterpart

Akira T. TOKUHIRO**, Hidekazu NO***, Michel CALL*** and Koichi HISHIDA****

An experiment to study the near-wake flow structure behind an air bubble attached to a cap and separately, a solid equivalent was conducted. The objective was to elucidate the near-wake characteristics of the “cap-bubble” relative to the solid; in particular, to elucidate the role of the “moving tail”. Experiments were performed in 80 × 80 mm² × 1 m tall channel with each object suspended in downward flow of water. Both the cap-bubble and solid had an approximate equivalent diameter, \( D_{eq} \sim 11.5 \text{ mm} \) (volume \( \sim 0.8 \text{ mL} \)). The average downward flow velocity \( (U \sim 25 \text{ cm/s} \sim \text{rise velocity of bubble}) \) defined Reynolds number was, \( 2450 < Re_{Deq} < 2890 \). The Eötvös and Weber numbers were, \( 17.8 < E\ddot{o} < 17.85 \), \( 6.04 < We < 6.06 \). Particle Image Velocimetry using a cross-correlation method was used to generate velocity data; vorticity, turbulent kinetic energy (TKE) and other parameters were then calculated. Graphic and numerical comparisons between the objects led to the following: 1) for the cap-bubble, the influence on the flow structure due to motion of the tail is relatively minor, 2) tail motion contributes to transverse fluctuations and uniformity in TKE distribution and 3) object deformation, oscillation and gyration localizes the flow structure in contrast to a solid object. Results are discussed.

Key Words: Bubble, Bubble Motion, Near-Wake, Particle Image Velocimetry, Flow Structure

1. Introduction

Multiphase, multi-component natural and engineered flows often contain (gas) bubbles, liquid droplets, and solid particles. Some well known examples, such as boilers, bubble columns, oil-water separators and others involve solid-liquid, gas-liquid, solid-gas and solid-gas-liquid flows. It is self-evident that convective heat and mass transfer spanning local-to-global transport scales are critical to many natural, industrial and environmental flows. A thorough understanding of the basic phenomena that drive the complex interactions between the dispersed and continuous phase, although key, remains elusive, as well as an ability to computationally simulate these flows at relevant dimensionless scales. Various aspects of dispersed multiphase flows have studied and Levich (1962)(1), Clift, Grace and Weber (1978)(2) are just one of several classic texts (that describe phenomena between bubbles, drops, and particles and the continuous phase).

Recent developments in numerical simulations of the dispersed multiphase flow by Bunner and Tryggvason (1999)(3) and Tomiyama (1998)(4) are still limited in scope and Reynolds number \( (Re \sim O(100)) \) but provide experiment-like simulations and challenges for a validation/verification methodology. These simulations presume that these problems can be tackled numerically by working with the governing equations in integral form, which subsequently lead to conservative finite difference schemes and thus, avoid assumptions about the differentiability of the solution. However, some of the documented approaches do not yield detailed gas-liquid flow structure nor insights into mutual interaction (between phases) and modeling of bubble dynamics. While studies by Kariyasaki (1987)(5), Takagi and Matsumoto (1995)(6) and Hao and Prosperetti (1999)(7) yield interesting results,
studies by Tokuhiro et al. (1998, 1999)(8), (9), Fujiwara et al. (1998, 2000)(10), (11) and Fujiwara (2002)(12) indicate that understanding the coupling of local to global flow structure, linking few to many bubbles (or others), is important if not in detail, then with respect to the level of detail required.

In the present work we applied Particle Image Velocimetry (PIV) to the measurement of the flow around a bubble-shaped solid and a “capped” bubble in order to study the effect of a moving posterior to the near-wake flow dynamics. Ultrasonic Doppler Velocimetry (UDV) was also used in a complementary role. Our objective was to determine the differences between the solid model and capped bubble, particularly in the solid’s and the capped bubble’s near-wake flow structure.

2. Materials and Methods

The experimental apparatus, shown in Fig. 1, consists of upper and lower rectangular tanks (148 liters each) connected by an acrylic vertical channel, 80×80 mm in cross-section and 1 m in length. Water flows downward through the channel. A flow rectifier, comprised of small-diameter tubes, was installed at the top of the channel. The solid model and capped bubble were suspended from above by stiff piano wire in the middle of the channel. In some cases the model (or cap) was further held in place by weighted lead “shot”. The solid model, shaped like an ellipsoid, was made of polyamine epoxy, while the capped bubble consisted of a solid ellipsoid-like “cap” (or shell) underneath which an air bubble was attached. A single air bubbles was injected into the inverted cap at nearly the bottom of the channel (see Fig. 1); the cap was then inverted to release the bubble. Bubbles were released until one was captured by the cap. An image of each model is shown in Fig. 3.

Measurements were performed using a synchronized PIV system. The measurement system consists of the following: 1) a NEW WAVE Solo PIV3-15 Nd-YAG laser (λ = 532 nm, maximum laser power 880 W), 2) a Charge-Coupled Device (CCD) camera to record the PIV images (KODAK ES-1.0, 1 018×1 008 pixel, 29 fps frame rate), 3) a Pulse Generator to synchronize the laser and camera (Quantum Composers Model 9318). Light from the laser passed through a cylindrical lens, creating a light sheet (~1 mm thick) that illuminated nylon seeding particles (Expancel DU-80; 18 – 24 μm dia.; ρ ~ 17 kg/m^3 density). Although one cannot know absolutely, inspection of some near interface images (for reflections) indicated that the seed particles did not have a tendency to attach to the interface. The dynamic motion of the bubble in particular maintained a gas-liquid interface. Figure 2 is a schematic representation of the PIV system. The same cross-correlation technique described in Fujiwara et al. was applied to obtain the velocity vector field data from the images. Associated dynamic parameters such as vorticity were estimated from these data; a notable work on PIV is that by Brückner (1998)(13), while No (2003)(14) provides specific details of the present set-up.

The UDV system (UVP by Met-Flow SA; 2001(15)) was used to measure the free stream velocity upstream of the suspended object. This facilitated flow rate adjustments to assure a downward flow velocity corresponding to the bubble’s terminal rise velocity in a stationary pool.

3. Results

Table 1 summarizes the single object near-wake stud-
ies of relevance to this study. The studies by Tokuhiro, Fujiwara, Hishida and co-workers have compared the following: 1) single rising air bubble, 2) solid ellipsoid, 3) hollow cap with bubble attached underneath, 4) solid cap with flat bottom, 5) capped oil droplet and 6) oil droplet, all in vertical channel flow of water. Here the bubble and all listed variants assume an approximate oblate ellipsoidal shape as dictated by Bhaga and Weber’s (1981)\(^ {16}\) flow regime map of buoyant rising particles. The Reynolds number based on an equivalent bubble diameter, \(D_{eq} \sim 11.5\) mm, and volume, \(V \sim 0.8\) mL, with an free rising velocity of \(U \sim 25\) cm/s spans 2 450 \(< Re < 2 890\). The corresponding Eötvös and Weber numbers were: 17.8 \(< Eo < 17.85, 6.04 < We < 6.06\). We noted however, that the flow in the channel is downward in order to maintain the object in the field-of-view of the PIV CCD camera. Figure 3 shows a raw PIV image with the laser sheet entering from left and a caricature of the solid and hollowcap with air bubble models inserted. The “hollow cap with bubble tail” results from attaching an bubble to the underside of an ellipsoidal shell. The near-wake structure of a cap-bubble isolates contributions from motion of its tail\(^1\). Overall, the present set-up and PIV method employed were akin to Fujiwara and co-workers. For brevity, we refer you to No, Call and Tokuhiro (2003)\(^ {18}\) and No for additional details.

Figure 4 shows the average vector field plot with the magnitude of the U- and V-components superimposed on the vector field in a color contour. In both figures, (a) represents a solid (filled) with an approximately ellipsoidal top and flat bottom and (b) represents the similarly-shaped (hollow) cap “with an air bubble skirt” (simply referred to as “cap”). The origin of the \(x - y\) axes is located at the approximate center of the object and distances are normalized by the equivalent diameter \((D = D_{eq})\). The average here is based on about 650 vector fields. The apparent time-averaged axisymmetry in the depicted structure was not confirmed since our objective was to note semi-quantitative differences. Figure 5 shows the corresponding \(U_{rms}\) and \(V_{rms}\) (rms-root-mean-square) in color, again first for the solid and then the cap. Subsequently in Fig. 6, we show the calculated vorticity and Reynolds shear stress (with density) iso-contour for the solid and cap respectively. In Fig. 7 we show iso-contours of the calculated turbulent kinetic energy (TKE) distribution and the production term for the solid and cap. The equations as used are given in the Appendix. Lastly in Fig. 8, we show the transverse profiles of \(V_{rms}\) and TKE, respectively for the solid (filled circle) and cap (open circle) in intervals of \(x/D = 0.2\) starting from \(x/D = 1.0\) to 2.0. The figures and corresponding discussion below is mostly based on two-dimensional PIV measurements. An earlier work by Fujiwara and co-workers confirmed that the measured dynamics are similar in the plane perpendicular \((x-z\) plane) to that presented here \((x-y\) plane).

\(^1\) Though possible, flow between the attached bubble and the (underside of) shell nor the influence of the solid-liquid-gas interface at the tip of the shell, on the near-wake structure could not be measured in our experiment. Thus the composite phenomena is discussed.

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Table 1 Summary of single object near-wake studies of relevance to this study

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Purpose</th>
<th>Investigator(s)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble vs. solid</td>
<td>Solid: non-slip, no deformation, no oscillation and no tail motion</td>
<td>Tokuhiro (1990), Fujiwara et al. (1998; 2000; 2002)</td>
<td>Q: difference in turbulent production (v'v'\ \delta V/\delta y); was this due to tail, oscillation or deformation?</td>
</tr>
<tr>
<td>Capped bubble vs. solid</td>
<td>Isolate influence of tail motion</td>
<td>No (2003), present study</td>
<td>The influence of the tail motion was minor; the bubble’s motion was very different from that of capped bubble and solid</td>
</tr>
<tr>
<td>Capped bubble vs. bubble</td>
<td>Study influence of oscillation, deformation and some differences in tail motion</td>
<td>No (2003), present study</td>
<td></td>
</tr>
<tr>
<td>Capped bubble vs. capped oil</td>
<td>Study different density, viscosity and surface tension of the tail fluid</td>
<td>No (2003), present study</td>
<td>Motion of the tail contributes little to the near-wake flow structure</td>
</tr>
<tr>
<td>Bubble vs. oil droplet</td>
<td>Study differences in slip condition, gyration, oscillation and deformation</td>
<td>No (2003), present study</td>
<td>Due to the oil droplet’s gyration, the near-wake structure is different than the bubble</td>
</tr>
<tr>
<td>Bubble, solid and capped bubble arranged in square &amp; narrow channel</td>
<td>Study wall effect in bubble dynamics</td>
<td>Call (2003)</td>
<td>Narrow channel accentuates axial flow structure</td>
</tr>
</tbody>
</table>
Fig. 4 Average vector field of the U (upper) and V-components; color-scale velocities are [m/s]. The solid and cap with bubble "tail" are respectively left (a) and right (b) unless otherwise indicated. Coordinate axes shown in upper, "(a)"

4. Discussion

The rising motion of buoyant bubbles and solid objects in liquid pools can be characterized by key dimensionless groups: the Re-, Eö- and We-numbers (or derived Morton). Owing to the investigators’ original work on rising air bubbles, the flow regime of interest based on Re, Eö and We-numbers (as above) is the oblate ellipsoid (OE) region. Here the bubble equivalent diameter is $D_{eq} \sim 1$ cm and free rise velocity, $U \sim 25$ cm/s. Generally, the OE-bubble gyrates, oscillates and shows a zigzag motion as it rises. In order to understand the balance of forces that dictate the rising motion, flow visualization, as well as notable works by Fan and Tsuchiya (1990)\textsuperscript{19}, Davies (1976)\textsuperscript{20} and Ongoren and Rockwell (1988)\textsuperscript{21} provide some guiding insights. These are briefly summarized as follows and via reference to Fig. 10.

The external flow impinging the buoyant object (a solid) generates vorticity at the separation point and vortex sheets that “roll-up” into spirals are conveyed as vorticity along the free shear layer (Fig. 10, "(1)"). Vorticity is continuously supplied into the near-wake and a discernable circular vortex is formed (2). The vortices of $+/−$ sign grow along both sides (planar perspective) and eventually entrain a shear layer and external irrotational flow into the near-wake (3). This entrained flow leads to reversed flow in the wake and periodic transverse flow “cuts-off” the vorticity. A vortex is thus shed and terminates a half cycle (4). The object may tilt due to a change in the balance of forces. The transverse stream may cut across the wake such that it defines the demarcation between the near-wake and downstream where the shed vortex is subject to viscous decaying forces (5). This sequence of events, presented qualitatively, is common to an object (solid, bubble or similar) subject to an impinging flow. The differences noted in the near-wake structure is however, due to
the surface boundary condition (slip vs. non-slip) and
dynamics of the body itself (imposed vs. free oscillation).
The frequency of vortex shedding and separately oscil-
laboration of the bubble (or object) is often described by a
Strouhal-number (St). The two frequencies synchronize
("lock-in") in the case of the bubble in order to mini-
imize its energy loss; that is, via minimization of the spa-
tial extent of the wake. Davies investigated the near-wake
flow past a D-shaped cylinder with the semicircular part
of the “D” facing upstream. In comparisons of a fixed D-
cylinder to one oscillating back-and-forth, Davies found
that when the shedding and oscillation frequencies were
locked ($7 \times 10^4 < St < 4 \times 10^5$), the vortices in near-wake
showed 35% more circulation than vortices in the far-
wake. They were confined along the free shear layer and
rolled-up closer to the cylinder, thus minimizing its spa-
tial distribution. Ongoren and Rockwell investigated flow
past a cylinder that moved transverse to the channel flow at
controlled frequencies and amplitudes. They found that as
the vortex shedding and oscillation frequency approached
each other, the near-wake flow structure assumed a min-
imal spatial extent in contrast to a fixed, non-oscillating
cylinder.

With respect to the zigzag rising motion of the bub-
ble, the periodic shedding of vortices at the (planar) left
and right of the bubble, as well as observed deformation,
oscillation and gyration (DOG), strongly suggests that the
balance of forces on the bubble is complex but periodic.
Since the simultaneous shedding and DOG-motions are
also influenced by the slip/non-slip interfacial condition
of a bubble or (model) solid equivalent, it is of interest to
isolate each of the dynamic contributions to the near-wake
flow structure. Understanding the influence of the bubble
“tail” motion on the near-wake, as isolated and contrasted
for the three bodies of interest, bubble, cap with bubble tail
attached and solid equivalent sheds light on phenomeno-
logical differences, the details of which may lead to an understanding of the balance of forces. That is, the bubble has slip at its interface and features DOG-motions, including free motion of its tail, whereas the solid equivalent has no slip at its interface and no other (interfacial) motion. Finally, the cap with bubble is like a solid but does have free motion of its tail. The investigators here have reported on results in this regard and herein describe new findings below.

Average vector field plots of the near-wake flow structure for both the solid cap and cap with air bubble tail as shown in Fig. 4 depict an overall similarity in features. The solid has a slightly narrower wake and a slightly smaller distance between the centers of the vortices, in contrast to the cap with bubble tail. However, we cannot with confidence attribute differences to the posterior boundary condition. The difference in the respective boundary condition is, as mentioned, existence of a dynamic bubble tail beneath the cap and the nonexistence of such in the solid. We thus seek notable differences in the near-wake flow structure that may be attributable to the dynamic interaction between the moving tail and flow field.

Previously, Fujiwara and co-workers investigated and compared the near-wake flow structure between a single air bubble and a solid, ellipsoidal equivalent under very similar flow conditions. The major conclusion of this work was that a bubble’s gyration/oscillations\(^*\) uniformly distribute the turbulent kinetic energy (TKE) in the near-wake, in contrast to the solid. In particular, Fujiwara et al. noted that the \(-\overline{\omega v} dV/dy\)-term of turbulent production is significantly different when comparing the solid and bubble. This figure is shown in Fig. 9. In other words, there is evidence that the \(-\overline{\omega u} dU/dx, -\overline{\omega v} dU/dy\), and \(*\) Gyration is here understood as the largely three-dimensional precessional motion; oscillation is understood as the largely two-dimensional back-and-forth rocking motion.
\(-u'v'dU/dx\), and \(-\) terms of turbulent production are associated with the similarity in “form” between the bubble and solid, while the transverse, turbulent fluctuation’s exchange of energy with the gradient of the mean, transverse flow is, indeed associated with the DOG attributable bubble motion. However, this study did not determined whether the dispersion of TKE originated from that generated by the separation points of the bubble (the form), accompanied by deformation and oscillatory motion, and interfacial slip condition and/or as contributed by the motion of the tail itself. In comparing the near-wake structure of the solid and cap, we sought to address the significance of the relative contribution of the tail motion itself.

Thus in the present study we sought to isolate the influence of the motion of the tail itself to the near-wake flow structure. Semi-quantitative comparisons of the flow field contours of the solid and solid-cap (with bubble tail) for \(U\) (Fig. 4), \(U_{rms}\) (Fig. 5), vorticity (Fig. 6), Reynolds shear stress (Fig. 6) and turbulent kinetic energy (normalized by the square of the average velocity) and also production (Fig. 7), do not show significant differences in magnitude nor distribution\(^3\). Essentially, the flow is dominated by the downstream quantities, whether this is \(U\), \(U_{rms}\) or vorticity.

One of the observed differences between the solid and cap is the distribution of \(V_{rms}\). In Fig. 5 we see that the \(V_{rms}\) iso-contours appear closer to the cap-bubble’s posterior than that for the cap. In other words, in the very near wake of the cap-bubble, there is a relative lack of \(V_{rms}\). This is further substantiated by Fig. 8, where one sees that \(V_{rms}\) for the cap is consistently smaller than the solid up to \(x/D \approx 2.0\). This result appears to indicate that deformation

\(^3\) The \(x\)-origin for solid and cap-bubble do not exactly coincide in the figures nor is the PIV data so precise that we can discern very local phenomena. Thus the next best approach, a semi-quantitative comparison is made.
of the bubble’s tail and its associated slip condition reduces flow fluctuations in the near-wake. Subsequently, a $V_{rms}$ magnitude of the same order as the solid only appears further downstream.

The other notable difference between the solid and cap is the near-wake distribution of the turbulent kinetic energy (TKE) shown in Fig. 7. In Fig. 6, both the solid and cap show similar distributions and magnitudes. In contrast to the solid and consistent with the noted difference in the $V_{rms}$ distribution, the TKE distribution of the cap is smaller in the near-wake ($x/D < 2.0$) and only assumes magnitudes similar to the solid further downstream. This observation is supported by Fig. 7 that depicts this contour-wise, while Fig. 8 shows profiles in $x/D = 0.2$ increments, starting at $x/D = 1.0$. Further as both the solid and caps have identical, non-slip boundary condition along the ellipsoidal surface, the generation of TKE along the free shear layer, at left and right, is as expect common to both.

Interestingly enough, via the work of Fujiwara et al. and our present work, we have learned that the existence of a dynamic bubble tail alone does not generate significant differences in the turbulent production distribution observed between an air bubble and a solid equivalent. That is, from this study we see an overall similarity with only a slightly less dispersed distribution of the production term between the solid and cap. This result is consistent and similar to the solid ellipsoid model of Fujiwara et al. It thus appears that the sum total of gyration/oscillation of an air bubble, but relatively little from the motion of the tail itself, contributes to the marked difference in $\overline{u'v'dV/dy}$-term for a bubble as compared to the solid equivalent.

5. Conclusion

An experimental study was conducted to elucidate differences in the near-wake flow structure between a hollow, ellipsoidal cap with an air bubble attached underneath it and a similarly-shaped solid cap with a flat tail. We sought to understand the contribution to the near-wake flow structure as induced by motion of the object’s posterior tail. Characterization of the tail motion-induced near-wake flow structure facilitates relative to that due to bubble’s gyration, oscillation and form drag, facilitates our understanding of freely rising gas bubbles.

The experiment was performed in a square channel, $80 \times 80 \text{mm}^2$ in cross section and 1000 mm long. The cap with bubble and solid cap were separately suspended in downward flow of purified water. Both the “solid” and “cap” had an approximate volume of 0.8 mL. The Reynolds number for the flow, based on the objects’ equivalent diameter and average downward flow velocity ($U = 25 \text{ cm/s}$), was $2450 < Re < 2890$. Velocity measurements were taken using Particle Image Velocimetry. The cross-correlation deduced vector fields were further processed to yield root-mean-square, vorticity, Reynolds stress, turbulent kinetic energy, and turbulent production distributions.

Our results to date indicate that for similarly-shaped caps, one with a nearly flat, solid bottom and the other with an air-filled tail, the oscillatory motion of this tail,
though constrained by the hollow cap, contributes mainly to the transverse fluctuations in velocity and also uniform distribution of the turbulent kinetic energy. Thus, influence of the motion of the tail itself (to the flow structure) is minor relative to the near-wake flow structure induced by a free bubble’s upstream form, oscillatory and gyrating motion. Comparison to a previous work (by the lead author) thus revealed that the oscillatory/gyration motion, as well as the upstream form slip condition generates significant differences in the $v'v'dV/dy$-term of turbulent production. This term in particular contrasts an air bubble and a similarly-shaped solid equivalent.

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Appendix

1) Weber number
$\text{We} = \frac{\rho U_b^2 d_e}{\sigma}$

2) Eötvös number
$E_o = \frac{g \rho d_e^2}{\sigma}$

3) Reynolds shear stress
$\sigma_{u'v'} = -\frac{\rho (u'u')}{U_o^2}$

4) Vorticity (given a velocity vector centered at “pixel $i, j$”)
$\omega_{i,j} = \frac{\Delta x (u_{i-1,j-1} + 2u_{i,j-1} + u_{i+1,j-1} - u_{i+1,j+1} - 2u_{i,j+1} - u_{i-1,j+1}) + \Delta y (v_{i+1,j-1} + 2v_{i+1,j} + v_{i+1,j+1} - v_{i-1,j+1} - 2v_{i-1,j} - v_{i-1,j-1})}{8\Delta x \Delta y}$
Fig. 10 Qualitative schematic of incremental flow structures for flow past a buoyant object

5) Turbulent Kinetic Energy (since PIV is planar and on the assumption that \( v' \sim w' \))

\[
\text{TKE} = \frac{u'^2 + 2v'^2}{2U_o^2}
\]

6) Production term of turbulent production

\[
P = -\left\{ \langle u'u' \rangle \frac{\partial U}{\partial x} + \langle u'v' \rangle \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) + \langle v'v' \rangle \frac{\partial V}{\partial y} \right\}
\]

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