Drag Reduction for D-Shape and I-Shape Cylinders*  
(Aerodynamic Mechanism of Reduction of Drag)

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The aerodynamic mechanisms for the reduction of drag for a D-shape cylinder, wherein a front face of a circular cylinder is cut off, and an I-shape cylinder, wherein front and rear faces are cut off, are investigated. For the D-shape and I-shape cylinders with a cutting angle of 50 – 53° and for Reynolds number \(Re > 2.3 \times 10^4\), the shear layer separated from the front edge reattaches on the circular arc of the cylinder, and a transition in the boundary layer as well as turbulent separation occur. As a result, the wake width decreases and the vortex formation region goes downstream. The Strouhal number increases beyond 0.28, the base pressure coefficient rises, and the drag coefficient of the cylinders decreases to half the value for a circular cylinder. The conditions of the above phenomena are clarified.

** Key Words:** Drag Reduction, Pressure Distribution, Strouhal Number, Base Pressure Coefficient, Separation, Transition, Flow Visualization, Oil-Film Method

1. Introduction

It is well known that the reduction of drag for a sphere or a circular cylinder can be achieved by the transition of the boundary layer from laminar to turbulent by means of a tripping wire on the surface of the sphere\(^1\) or the circular cylinder\(^2,3\). To control the flow field around a bluff body, two methods were proposed by Igarashi, et al.\(^4,5\) One is a control rod set in a shear layer separated from the cylinder\(^4\), and the other is a control rod set up-stream of a square prism. Using the latter method, the drag coefficients of the bluff bodies, such as a square prism\(^6\), a circular cylinder\(^7\) and a normal flat plate\(^8\) are reduced by 50% to 75%.

Recently, Aiba et al.\(^9\)–\(^11\) reported that the drag force decreases by 50% and 25% for a circular cylinder and a sphere, respectively, by cutting out at circumferential angle \(\theta_f = 53^\circ\) on the front surface of the bluff bodies. However, the pressure distribution, Strouhal number and flow patterns are not clear. The above method of drag reduction is a unique one compared to others for a bluff body using round edges\(^12\) and cutout at its edges\(^13\).

Based on the above, the present authors investigated the drag reduction of D-shape and I-shape cylinders\(^14\). The pressure distribution, Strouhal number and drag coefficient were measured in the range of \(1.38 \times 10^4 \leq Re \leq 8.0 \times 10^4\). Moreover, flow visualization around both cylinders was conducted and the flow patterns were classified. In the case of the shear layer separating from the front edges that reattaches on the circular arc of the cylinder, the Strouhal number increased beyond 0.28 and the drag coefficient of the cylinder decreased to 0.7. In this paper, the shear layer from both ends of the front surface and its reattachment on the circular arc, the transition to turbulent behavior in the boundary layer and the turbulent separation on the D-shape and I-shape cylinders are investigated. The aerodynamic mechanism of the reduction of drag for the above two cylinders is clarified in the paper. In addition, the appearance of the flow patterns and the associated physical meaning are discussed.

**Nomenclature**

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\begin{align*}
A &: \text{frontal area of the test model cylinder} \\
C_D &: \text{drag coefficient} = D/0.5\rho U^2 \\
C_p &: \text{pressure coefficient} = (p - p_o)/0.5\rho U^2 \\
C_{pb} &: \text{base pressure coefficient} = (p_b - p_o)/0.5\rho U^2 \\
C_{pf} &: \text{fluctuating pressure coefficient} = \Delta p/0.5\rho U^2 \\
D &: \text{drag} \\
d &: \text{diameter of a circular cylinder} \\
f &: \text{vortex shedding frequency} \\
p, p_o, p_b &: \text{pressure, static pressure and base pressure}
\end{align*}
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\[ \Delta p : \text{R.M.S. fluctuating pressure} \]
\[ Re, Re_t : \text{Reynolds number} \]
\[ S : \text{Strouhal number} \]
\[ u, U, U_t : \text{velocity, free stream velocity and transition velocity} \]
\[ \Delta u : \text{R.M.S. of fluctuating velocity} \]
\[ x : \text{stream-wise coordinate from the center of the cylinder} \]
\[ y, y' : \text{coordinate perpendicular to } x \]
\[ \nu, \rho : \text{kinematic viscosity and density of fluid} \]
\[ \theta, \theta_f, \theta_r : \text{circumferential angle on the cylinder} \]
\[ \theta_f, \theta_r : \text{cutting angle of the front and rear surfaces of the cylinder} \]

2. Experimental Apparatus and Procedure

The configuration of the test model and symbols are shown in Fig. 1 (a) and (b). The D-shape cylinder is cut out of the front surface at an angle of \( \theta_f \) on the circular cylinder and the I-shape cylinder is cut out of the front and rear surfaces at angles of \( \theta_f \) and \( \theta_r \). According to the results of Aiba and Hoshino\(^{10} \) for \( d = 20, 30 \) mm, and Igarashi and Shiba\(^{14} \) for \( d = 50 \) mm, the minimum drag for both cylinders is obtained for a cutting angle \( \theta_f = 53^\circ \) and a Reynolds number \( Re = U d / \nu > 2.3 \times 10^4 \). In order to confirm the optimum cutting angle and the universality of the transition Reynolds number, a D-shape cylinder with \( d = 50 \) mm diameter and front cutting angle \( \theta_f = 30 \) and \( 40^\circ \), and D-shape and I-shape cylinders with diameters of 20, 30, 40, 60 and 80 mm and cutting angles \( \theta_f = 53^\circ \) and \( \theta_r = 127^\circ \) were prepared. The experiments were conducted in a low speed 2-D blowing wind tunnel with a measured cross section of 400 mm \( \times \) 150 mm and a length of 800 mm. The uniform flow velocity \( U \) was varied from 4 to 24 m/s, and the range of Reynolds numbers defined by the diameter of the cylinder was \( 1.1 \times 10^5 \leq Re \leq 8.5 \times 10^5 \). The free stream turbulence was about 0.4% in this range.

The Strouhal numbers \( S = f d / U \) of the vortex shedding frequencies \( f \) of the wake and the behavior of the shear layer separated from both cylinders were measured using an I-type hot wire anemometer. Next, the pressure coefficient \( C_p \) and the distribution of R.M.S. fluctuating pressure coefficient \( C_p' \) around both cylinders were measured using an inclined manometer and a semiconductor pressure converter connected to a pressure tap. Moreover, the surface flow on the D-shape cylinder, and the wake flow and vortex formation region behind the cylinder were visualized using a smoke tunnel with a working cross-section of 600 mm \( \times \) 150 mm \( \times \) 800 mm and using an oil-film method.

From the uncertainty analysis presented in the previous paper\(^{14} \), the accuracy of the measurement of the pressure coefficient is \( \pm 1 \) to 5%, and the error in the reading is less than 2%. The uncertainty level of the drag coefficient is in the range of 2 to 5%.

3. Experimental Results and Discussion

3.1 Optimum cutting angle

As shown in Fig. 2(a) and (b), the drag coefficients of D-shape and I-shape cylinder \( C_D \), are drastically reduced from 1.3 to 0.7 at \( Re = 2.6 \times 10^4 \). From previous reports\(^{14} \), the shear layers separated from the front edges reattaches on the circular arc of the cylinder. Then, the wake width of the two cylinders becomes narrowed and its vortex formation region move downstream compared with a circular cylinder.

It is clear that the drag coefficient of bluff bodies is small for a large Strouhal number \( S \) and a small nega-
tive base pressure coefficient \(-C_{pb}\). The effects of the Reynolds number and cutting angle \(\theta_f\) on the Strouhal number and the negative base pressure coefficient \(-C_{pb}\) for the D-shape cylinder are shown in Fig. 3 (a) and (b), respectively. Figure 4 illustrates the correlations between \(\theta_f\) and \(S\), and between \(\theta_f\) and \(-C_{pb}\). The Strouhal number for \(\theta_f = 30^\circ\) and \(40^\circ\), \(S = 0.20 - 0.21\), is nearly equal that for a circular cylinder, and the front cutting angle \(\theta_f\) does not affect the flow around the cylinder. This flow type is called pattern A. In the case of \(\theta_f = 50^\circ\), the value of \(S\) increases drastically from to 0.30 at \(Re > 2.3 \times 10^4\). For \(\theta_f = 53^\circ\), the value of \(S\) is below 0.20 at \(Re < 2.3 \times 10^4\), and increases drastically beyond 0.30 at \(Re > 2.3 \times 10^4\). These figures indicate that the shear layers separated from the two edges of the front flat surface reattach on the circular arc of the cylinder, and the reattachment flow accompanies turbulent separation. This flow type is called pattern B. For \(\theta_f = 60^\circ\), the value of \(S\) is about 0.16, which is lower than that for a circular cylinder. In this case, the shear layers separated from both the front edges do not reattach on the cylinder and the wake width of the shear layers expands considerably. This flow type is called pattern C. From the above descriptions, it can be concluded that the optimum value of angle \(\theta_f\) is in the range from \(50^\circ\) to \(53^\circ\).

3.2 Flow visualization around D-shape cylinder

In order to confirm the above conclusion that the turbulent transition occurs on the D-shape cylinder with a cutting angle of \(\theta_f = 53^\circ\), the flow around the cylinder was visualized using an oil-film method and a smoke tunnel. Figure 5 (a) presents the surface oil-flow patterns on the circular arc of a D-shape cylinder of \(d = 50\) mm at \(Re = 5.33 \times 10^4\). The symbols \(R\), \(S_1\) and \(S_2\) represent the reattachment point of the separated shear layer, the separation point of the boundary layer and the separation point of the reserved flow in a separation bubble, respectively. Figure 5 (b) shows photographs of the wake flow obtained using a long exposure of 1 sec at \(Re = 2.67 \times 10^4\). The photographs were made using smoke from an incense stick embedded in the rear surface of the cylinder. The wake of a circular cylinder and a rectangular cylinder are presented compared with that of a D-shape cylinder in the same figure. The wake width of a circular cylinder is nearly equal to the diameter of the cylinder, because laminar separation...
occurs at $\theta = 78^\circ$. The wake width of a normal rectangular cylinder is 1.5 times as long as the height of the cylinder because the separated shear layers are normal to the flow direction and are deflected by the main flow. In the case of $\theta_f = 53^\circ$, the shear layer that is separated from the edge of the flat front surface reattaches like an impinging jet to the circular arc of the cylinder at $\theta = 72^\circ$. The reattachment flow transits to a turbulent mode and the separation point $\theta_s$ appear at $92^\circ$. In this case, the wake width becomes considerably narrower, and the region of vortex formation moves downstream. As a result, the drag coefficient of the D-shape cylinder decreases due to the decrease in negative base-pressure coefficient. Moreover, in the case of a circular cylinder with a wake splitter\(^{(15)}\), the same wake characteristics appear, but the Strouhal number decreases. This is due to the slowing down of the velocity of the shear layer at the position of the vortex formation region, because it moves downstream due to the presence of the splitter plate.

### 3.3 Transition of flow pattern

For the D-shape and I-shape cylinders, having an optimum cutting angle $\theta_f = 53^\circ$, which yields the minimum drag coefficients for both cylinders, these flow patterns change drastically from a perfect separation type (pattern C) to a turbulent separation type (pattern B) as Reynolds number increases. According to our previous report\(^{(14)}\), the Reynolds number accompanying the change of flow pattern for the D-shape and I-shape cylinders is $Re = 2.4 \times 10^4$ for $d = 50$ mm. From the report by Aiba and Hoshino\(^{(10)}\), the same Reynolds number is $Re = 2.3 \times 10^4$ for $d = 20$ mm. We define the constant Reynolds number accompanying a decrease of drag as transition Reynolds number $Re_t$. In this section, we confirm that the transition Reynolds number is constant.

The variations in the Strouhal number and the base pressure coefficient for the D-shape and I-shape cylinders with cutting angles of $\theta_f = 53^\circ$ and $\theta_f = 53^\circ$, $\theta_c = 127^\circ$ and various diameters $d = 20 \sim 80$ mm as a function of the Reynolds number are shown in Fig. 6(a) and (b) and Fig. 7(a) and (b), respectively. The values of the Strouhal number for the two cylinders are drastically increased from $S \leq 0.15 \sim 0.175$ to $S \geq 0.28 \sim 0.3$ in the narrow range of the Reynolds number of $Re = (2.3 \sim 2.4) \times 10^4$ regardless of the diameters of both cylinders. It is to be noted that the Reynolds numbers, which accompany the drastic change in the flow characteristics and flow patterns of both cylinders, are nearly equal. It is confirmed that the transition in the boundary layers of the D-shape and I-shape cylinders occurs at a constant Reynolds number $Re_t = (2.3 \sim 2.4) \times 10^4$.

### 3.4 Distribution of the fluctuating pressure around the cylinders

The surface pressure around a bluff body fluctuates periodically with the vortex shedding frequencies. Near the separation point, the pressure has maximum amplitude. The value of the R.M.S. fluctuating pressure, $\Delta p$ of the bluff body is proportional to the fluctuating lift and drag acting on the body. The distributions of R.M.S. fluctuating pressure coefficient $C'_p$, defined by $\Delta p/0.5 \rho U^2$ around the D-shape and I-shape cylinders are shown in Fig. 8(a) and (b), respectively. The distribution of $C'_p$ for a circular cylinder is shown in Fig. 8(a) for reference. At $Re = 2.0 \times 10^4$, the pattern of the D-shape cylinder is a perfect separation type (pattern A), the value of $C'_p$ from the front edge to $\theta = 90^\circ$ has exceeded that of the circular cylinder. And the value of $C'_p$ on the rear surface is nearly 0.3. On the other hand, at $Re = 2.67 \times 10^4$ and $Re = 5.3 \times 10^4$, the pattern is the turbulent separation type (pattern C), and the value of $C'_p$ is lower than half that of the circular cylinder. For the I-shape cylinder, the same statements are true for the D-shape cylinder.

### 3.5 Behavior of separated shear layer from the cylinder

The behavior of the shear layer separated from a D-shape cylinder with a cutting angle $\theta_f = 53^\circ$ is investigated at around the transition Reynolds number $Re_t$. The signal traces of the fluctuating velocity measured using the I-type hot-wire in a separated shear layer or in a boundary layer on the circular arc of the D-shape cylinder are...
shown in Fig. 9. For the position of the hot-wire $\theta = 62^\circ$ and $y' = 1.0$ mm shown in the upper part in Fig. 9 (a), the signal traces at $U = 6, 7$ m/s indicate that the amplitude of the fluctuating velocity $u(t)/U$ ranges from 0.2 to 1.8 and the period of low speed are longer and half, respectively. This shows that the location of hot-wire is slightly inside the separated shear layer. At $U = 8$ m/s, the time of low-speed disappears. This shows that the position of hot-wire is outside the separated shear layer. On the other hand, for the position of hot-wire of $\theta = 90^\circ$ and $y' = 0.3$ mm in the neighborhood of the wall, the signal trace at $U = 6, 7$ m/s show that the position of hot-wire is inside the separated shear layer. At $U = 8$ m/s, the signal trace shows that the position of hot-wire is outside the turbulent boundary layer.

### 3.6 Conditions of turbulent separation and perfect separation

We next consider the correlation between the behavior of the shear layer separated from a normal rectangular plate shown in Fig. 5 (b) and the change in flow patterns for the D-shape cylinder with various cutting angles. The
angle of separation $\theta_i$ of the shear layer separated from the normal rectangular cylinder shown in Fig. 10 is nearly 40°. On the other hand, the angle of bend from the frontal surface to the circular arc of the D-shape cylinder is given by $(90° - \theta_f)$. Whether the shear layer separated from the D-shape cylinder can be reattached or not on the circular arc of the cylinder is decided according to whether the difference between the angle of separation $\theta_i$ and the angle of bend $(90° - \theta_f)$ is large or small.

For example, in the case of $\theta_f < 50°$, the shear layer reattaches on the circular arc of the cylinder and then is accompanied by laminar separation. In the case of $\theta_f \geq 56°$, the shear layer does not reattach on the cylinder. In the case of $50° \leq \theta_f \leq 53°$, the shear layer reattaches on the circular arc of the cylinder and then is accompanied by turbulent separation. The flow patterns are classified by the cutting angle $\theta_f$ as shown in Fig. 11. Pattern A occurs in the range of $30° \leq \theta_f \leq 50°$, the shear layers separated from the front edges almost overlap the circular arc of the cylinder, the shear layer reattaches smoothly on the circular arc and the reattached flow is accompanied by laminar separation. The flow patterns mentioned above are identical to those of a circular cylinder with tripping wires. As a result, the D-shape cylinder presents the same flow characteristics as a circular cylinder. In the range of $50° \leq \theta_f \leq 53°$, two flow patterns B and C appear. Pattern B appears at $Re \geq 2.4 \times 10^4$, the shear layer separated from the cutting surface reattaches on the circular arc and accompanies turbulent separation. At $Re < 2.3 \times 10^4$, the pattern is C which is a perfect separation type without reattachment of the separated shear layer on the cylinder. And in the range of $\theta_f > 56°$, the pattern is a perfect separation type regardless of Reynolds number.

4. Conclusions

The aerodynamic mechanisms of the reduction in drag for a D-shape cylinder and an I-shape cylinder were investigated. The following conclusions resulted from the study.

(1) In the range of the front cutting angle of $50° \leq \theta_f \leq 53°$, the flow patterns of both cylinders are classified into three regimes, laminar separation type, turbulent separation type and perfect separation type.

(2) In the range of $50° \leq \theta_f \leq 53°$ and $Re > 23,000$, the flow becomes a turbulent separation type. As a result, the drag coefficient reduces to half that of a circular cylinder.

(3) In the above case, the shear layers separated from the edges of the front flat surface reattach on the circular arc of the cylinder. The reattachment flow is accompanied by turbulent separation.

(4) In the case of $\theta_f < 50°$, the flow from the front surface to the circular arc is smooth and accompanies laminar separation without transition in the boundary layer on the cylinder. There is no effect of the cutting angle on the flow characteristics.

(5) In the case of $\theta_f > 53°$, the flow is a perfect separation type and the shear layers separated from the front edge cannot reattach to the circular arc of the cylinder. The wake width increases considerably and the vortex formation region moves to the rear surface of the cylinder. Thus, the base pressure coefficient decreases, and the drag coefficient increases.

(6) In the range of $50° \leq \theta_f \leq 53°$, the transition Reynolds number from laminar separation or the perfect separation type to the turbulent separation type is $Re_t = 2.3 \times 10^4$. This phenomenon can be explained well with the same mechanism as the transition from laminar to turbulent on the circular cylinder with tripping wires.

References


(3) Igarashi, T., Effect of Tripping Wires on the Flow


