Influence of Polymer Additives on Transitional Plane Poiseuille Flow*

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Transition in dilute polymer solutions was investigated in the same manner as outlined in the previous report for Newtonian fluids. First, we investigated the natural transition arising from a minimum critical Reynolds number. The results suggested that the structural alteration of a turbulent spot resulted in polymer drag reduction. Next, we maintained laminar flow above the minimum critical Reynolds number and visualized the growth of spots artificially created in the laminar flow. The results showed that polymer additives reduce the spreading angle of a spot. Striped streak patterns of ink were observed around the spots. This phenomenon is supposedly caused by the instability of polymer solutions and instability in the background flow.

Key Words: Non-Newtonian Fluid, Drag Reduction, Transition, Turbulent Spot, Flow Visualization

1. Introduction

Transition in dilute polymer solutions has been investigated for pipe flow^{(1)~(3)} and boundary layer flow along concave walls.⁽⁴⁾ So far, there have been few investigations of this phenomenon. Although the stability theory which has been developed is mostly applicable to plane Poiseuille flow, very few experiments have been carried out on this flow. It is not yet clear whether polymer additives stabilize Poiseuille flow. Transitional flow has some coherent structures which are similar to turbulent flow. It is easier to detect the coherent structures in transitional flow. Hence, the results of the experiments is useful to investigate the coherent structures in turbulent flow.

We previously reported⁽⁵⁾ on the transitional plane Poiseuille flow with water as the Newtonian fluid. The results were as follows. The intermittency factor attained 1 at a Reynolds number of about 2 000

when a natural transition arose from a minimum critical Reynolds number, although the profiles of turbulent spots were still observable. The flow was fully developed turbulence for a Reynolds number of greater than 3 000. Less turbulence in the inlet flow raised the critical Reynolds number. Under these conditions, another experiment was conducted on the spots artificially created in the laminar flow over the range of $2\,000 < Re < 3\,000$. The spreading half angle increased from 2° to 10° with increases in the Reynolds number from 2 000 to 3 000. The propagation velocity of the leading edge also increased from $0.5u_{\max}$ to $0.6u_{\max}$. This report presents the results for dilute polymer solutions and makes a comparison with those for water.

2. Nomenclature

H : distance between parallel plates
 n, K : constants in a power-law model
 q : volume of fluid issued from a wall hole
 Re : Reynolds number $= HV/\nu$
 Re^* : generalized Reynolds number, eq. (2)
 T : time from initial excitation
 t : period while fluid issues from a wall hole

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- (opening time of a solenoidal valve)
 $u_{\max}$: maximum velocity in centerline
 u_r : frictional velocity
 V : mean velocity determined from a flow rate
 x : downstream distance from a wall hole
 y : perpendicular distance from a wall surface
 λ : pipe frictional coefficient
 ν : kinematic viscosity

3. Experimental Equipment and Procedure

3.1 Dilute polymer solutions

The polymeric additive is a polyacrylamide (Separan AP-30) with a concentration of 200 wppm. A concentration of a few wppm results in a turbulent drag reduction. We added a higher concentration of additive to cancel the mechanical degradation caused by a pump. The rate of degradation approached zero after the solutions were pumped for about 30 min. We then performed the measurements. The viscosity in our dilute polymer solutions was constant over the shear rates higher than 10^3 s^{-1} . However the viscosity was not constant over the lower shear rates. If we apply a power-law model

$$\tau = K(du/dy)^n, \quad (1)$$

n is about 0.8 below a shear rate of 10^3 s^{-1} . We used a generalized Reynolds number

$$Re^* = \left(\frac{n}{2n+1} \right)^n \frac{6 V^{2-n} (H/2)^n}{K} \quad (2)$$

for our dilute polymer solutions.

3.2 Experiment on natural transition

The rectangular channel is 300 mm in width and 15 mm in height. The inlet is a round mouth with a radius of 82 mm. A honeycomb and screens allocated at the positions upstream from the inlet. The pressure drop is much less in the test section than in a valve. Hence, the flow rate was nearly constant even in transitional intermittent flow.

We measured the instantaneous velocity and the wall shear rate by hot-film velocimeters, and visualized the flow by ink streak lines. We measured the velocity by a conical probe whose position was 3.67 m (245 H) from the inlet and was in the middle of the lateral width. The wall shear rate was measured by a flush mounting probe. We visualized the flow by ink fed through twenty-four holes each separated by 10 mm in the width direction. The diameter of the holes was 0.5 mm and the distance from the inlet was 3.3 m.

3.3 Experiment on growth of spots

Figure 1 shows the test channel. This is the same channel as that mentioned in the previous section, except it includes an inlet contraction. This inlet contraction reduces the inflow turbulence and so maintains laminar flow above a minimum critical Reynolds number. We artificially created a spot in this

laminar flow with an initial disturbance by a jet from a 0.5 mm wall hole. There were five wall holes over a region of from 0.88 m to 2.59 m from the channel inlet. Each hole was in the middle of the lateral width. The initial disturbances were intermittently created by opening and closing a solenoidal valve. We could adjust the strength of the initial disturbances by a needle valve, which was located between the solenoidal valve and the head tank containing the issuing fluid. We did not measure the strength of the initial disturbance directly. We varied the ratio of the mean velocity in the jet to the mean velocity in the Poiseuille flow to adjust the strength. Furthermore, we adjusted the opening time, t , of the solenoidal valve. This time was determined by the equation, $Vt/H = 6.5$.

The flow was visualized downstream from the ink outlet located at a distance of 2.23 m or 3.3 m from the inlet. We appropriately combined the five holes and the two ink outlets to investigate the growth of spots. The upstream ink outlet consisted of holes with a shorter lateral distance of 5 mm. This outlet was available for high-speed flow because the ink streak lines diffused to cover the whole test section for low-speed flow. We used black ink as the dye.

4. Experimental Results and Considerations

4.1 Natural transition

The minimum critical Reynolds number is about 1 300 and is the same as that for water. When Reynolds number increases above slightly lower value than 1 300, turbulence appears at the channel inlet in dilute polymer solutions as water. Furthermore, when Reynolds number increases above 1 300, the turbulence at the inlet does not disappear downstream and, instead, builds up a spot.

Figure 2 shows the relation between the intermittency factor and the Reynolds number. This relation was obtained by the same method as used previously⁽⁵⁾ and is the same as that for water. Figure 3

Fig. 1 Test channel (Experiment on growth of a turbulent flow.)

gives the frictional coefficients. Our dilute polymer solutions exhibit drag reduction in the transitional region. We can consider as the possible causes of drag reduction (1) the lower frequency of spot generation and (2) the alteration of spot structure. However, since the polymer additives do not affect the relation between the intermittency factor and the Reynolds number, the alteration of spot structure is the likely cause of drag reduction.

Figure 4 shows the velocity profiles. The broken lines represent the laminar velocity profiles for a power-law fluid,

$$u = \frac{2n+1}{n+1} \left(1 - \left| 1 - \frac{y}{H/2} \right|^{(n+1)/n} \right). \quad (3)$$

A log layer begins to appear in the velocity profile and the intermittency factor attains 1 at a Reynolds number of about 2 000, for which the profiles of spots are still observable. The flow is not fully developed turbulence. Using water, we showed in the previous report that the flow became fully developed turbulent for Reynolds numbers greater than 3 000. In Fig. 4, the velocity profile at a Reynolds number of 2 600 exhibits

Fig. 2 Intermittency factor

Fig. 3 Pipe friction coefficient

the same features as those under turbulent drag reduction. That is, the buffer layer is thicker and the Kármán's constant is the same, 0.4, as water. The profiles of spots are unobservable for Reynolds numbers greater than 3 000 in visualization. Therefore, the flow develops into a fully turbulent one at the same Reynolds number, 3 000, as water.

We shall now explain how to determine the viscosity in the abscissa of Fig. 3. The dilute polymer solutions are non-Newtonian fluids, as described in chapter 3. The viscosity changed slightly with changes in the shear rate. It has yet to be established how to make the y coordinates nondimensional so as to represent the log profiles for non-Newtonian fluids. The non-Newtonian characteristics in viscosity hardly cause drag reduction for dilute polymer solutions and so is not serious. Our dilute polymer solutions exhibit constant viscosity for shear rates higher than 10^3 s^{-1} . This constant viscosity was used when the Reynolds numbers were 1 900 and 2 600. At lower Reynolds numbers, the viscosity was determined by the wall shear stress. We nondimensionalized the y coordinates using these values of viscosity.

Figure 5 shows the ratio of the maximum velocity in the centerline to the mean velocity. In laminar flow, u_{max}/V is equal equal to 1.44 by Eq. (3) and is a little less than 1.5 for water.

The change of ratio is similar to that for water in the transitional region.

Fig. 4 Velocity profile

The results in Figs. 2 to 5 show that the transition does not differ much from that in water except for the alteration of spot structure. However, a dilute polymer solution exhibited a singular phenomenon in visualization. That is, striped patterns instantaneously appeared on the ink streak lines along a wall after the spots passed through. Figure 6 shows an example of the phenomenon. We discuss the cause of the striped streak patterns in section 4.2.

4.2 Growth of spot

The reduction of inflow turbulence raised the critical Reynolds number to 3 500, which is equal to that for water. We conducted the experiments under three conditions. The first was for a high Reynolds number of about 3 000 and a strong initial disturbance. The second was for the same Reynolds number and a weak initial disturbance. The third was for a low Reynolds number of about 2 000.

Figure 7 shows the growth of spots under the first condition. The initial disturbance immediately generates a pair of oblique waves (Fig. 7-1). These waves are stronger than those in water and become complicated downstream (Fig. 7-2). Figure 8 shows the incipient spots in water. While a spot originates from between the pair of waves in water, a spot is formed by the oblique waves in a dilute polymer solution. The spot begins to split into two spots in the same way as it would in water.

We observed the same striped pattern around the spanwise tips of a spot as those described in section 4.1 (Fig. 7-7, and 8). These patterns are enlarged and in Fig. 9 are compared with the results for water under similar conditions. The phenomenon raises two questions. The first is whether or not the polymer additives stabilize the flow. The second is how a spot modifies the background flow. We⁽⁶⁾ visualized the flow disturbed by a trip wire in water and a dilute polymer solution. The result suggests for the first question that the dilute polymer solution is more unstable than water for a two-dimensional disturbance. Wygnanski, Haritonidis & Kaplan⁽⁷⁾ and Carlson, Widnal & Peeters⁽⁸⁾ observed oblique waves

Fig. 5 Ratio of maximum velocity to mean velocity

around the spanwise tips of a spot in Newtonian fluids. While we can find the oblique waves in both Fig. 9-1 and Fig. 9-2, we can not find the striped streak patterns in water. The striped streak patterns appeared at the place where the oblique waves disturb the flow. The waves rub and extend the ink streak lines on a wall (Fig. 9-2). The wave number of the oblique waves is about one order of magnitude lower than that of the disturbance, which causes the striped streak patterns. Nishioka, Asai & Iida⁽⁹⁾ showed in transitional air flow that the high-frequency disturbance was rapidly amplified when the velocity profile formed an inflectional local shear layer. This phenomenon, called secondary instability, has some features which are similar to ours. We can interpret our phenomenon according to the concept of secondary instability as follows. The spot causes the background flow to become unstable, which causes high-frequency disturbances, and these disturbances form the striped streak patterns. If we conduct the experiment for a higher Reynolds number in water, we may find the same phenomenon under conditions of less stability.

Figure 10 shows the results with a weak initial disturbance. This initial disturbance caused weaker oblique waves than those shown in Fig. 7. Figure 11 shows the growth of spots at a lower Reynolds number. We had to make the initial disturbance stronger for the generation of a spot in this case. This disturbance similarly caused oblique waves. The striped streak patterns appeared at the same places (Fig. 11-3, 5) as those for a higher Reynolds number (Fig. 7 and Fig. 9).

We previously reported for water that the oblique waves sometimes appeared in front of a spot, as shown in Fig. 12. These oblique waves propagated, broke down, and formed part of the spot. The waves were one of the causes for a spot to grow, although the presence of oblique waves in front of a spot for dilute

Fig. 6 Striped streak patterns. $Re^* = 1\,400$

Fig. 7 Growth of a turbulent spot (A high Reynolds number and a strong initial disturbance.) $Re^* = 2600$, $t = 222$ ms, $q = 100$ mm³

Fig. 8 Oblique waves by an initial disturbance and incipient spots in water

polymer solutions was not found. However, the visualized region is restricted to the region near the wall surface. We are planning to investigate this wave phenomenon by a different method unrestricted to the wall region.

We obtained the propagation velocity of the leading edge and the spreading angle from photographs. The gradient of the lines in Fig. 13 equals the propagation velocity of the leading edge. The ratio of the propagation velocity to the centerline velocity increased from 0.5 to 0.6 by increasing the Reynolds

Fig. 10 Growth of a turbulent spot (A high Reynolds number and a weak initial disturbance.) $Re^* = 2700$, $t = 224 \text{ ms}$, $q = 60 \text{ mm}^3$

Fig. 9 Oblique waves around the spanwise tips of a turbulent spot

Fig. 11 Growth of a turbulent spot. (A low Reynolds number.) $Re^*=1700$, $t=317\text{ms}$, $q=130\text{ mm}^3$

Fig. 12 Oblique waves in front of a spot in water (The arrows point to crests of oblique waves.) $Re=3000$, $t=481\text{ms}$, $q=101\text{mm}^3$

number from 2 000 to 3 000 for water. The ratio for a dilute polymer solution is nearly equal to that for water.

The spots spread their own lateral width downstream, as shown in Fig. 14. There is no difference between the results for both fluids at a low Reynolds number. However, the dilute polymer solution has smaller spreading angles than water at a high

Reynolds number. This tendency is significant for an incipient spot ($25H < x < 50H$), with the spreading angles being suppressed to about 60% of that for water. This suppression may be caused by the stability for a three-dimensional disturbance and may be related to the turbulent drag reduction. We shall pursue our studies on this suppression of the spreading angle.

5. Conclusions

We investigated the natural transition arising from a minimum critical Reynolds number and obtained the following conclusions.

(1) The relation between an intermittency factor and Reynolds number was the same as that for water. The flow was fully developed turbulent for Reynolds numbers greater than 3 000. The drag reduction was caused by the alteration of spot structure.

(2) The striped streak patterns were visualized after the spots passed through. These patterns are supposedly caused by the instability of a dilute polymer solution and the instability in the background flow.

We maintained laminar flow above a minimum critical Reynolds number and investigated the growth of spots artificially created in the laminar flow. The conclusions are as follows.

(3) The initial disturbances caused oblique waves which were stronger than those in water. These waves formed the spots when the initial disturbance

Fig. 13 Propagation of a leading edge

Fig. 14 Spreading angle of a turbulent spot

was of sufficient strength.

(4) The ratio of the propagation velocity to the centerline velocity is nearly equal to that for water. The ratio increased from 0.5 to 0.6 by increasing the Reynolds number from 2 000 to 3 000 for water.

(5) The spreading half angle increased from 2° to 7° by increasing the Reynolds number from 2 000 to 3 000. A dilute polymer solution has smaller spreading angles than water at high Reynolds numbers. This tendency is significant for an incipient spot ($25H < x < 50H$), with the spreading angles for a dilute polymer solution being about 60% of that for water.

(6) The striped streak patterns appeared around the spanwise tips of a spot. These streak patterns are similar to those described in (2) and supposedly appear due to the same cause.

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