Heat Transfer and Flow Around Elliptic Cylinders in Tandem Arrangement*

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An experimental investigation has been conducted to clarify heat-transfer characteristics and flow behaviors around four elliptic cylinders having an axis ratio 1:2. They were placed in tandem arrangement with a constant angle of attack to the upstream uniform flow. The Reynolds number based on the major axis length, c, ranged from about 15,000 to 70,000. The angle of attack was varied from 0° to 90° at 30° intervals and the nondimensional cylinder spacing l/c from 1.25 to 4.0, where l denotes the streamwise distance between the neighboring cylinder centers. It has been found that the heat-transfer features vary drastically with the angle of attack and also with the cylinder spacing, corresponding to a great change in the flow features. Further, the heat-transfer capability of the present elliptic cylinders at narrower cylinder spacings and smaller angles of attack is shown to be comparable to that of the in-line circular cylinders.


1. Introduction

The exploration of compact and high-performance heat exchangers for conserving energy is a very important and urgent problem. Among the many types of heat exchangers, those made of circular tubes are used in many industries. Flow around the tubes, however, is not always normal to the tube axis. In such a situation, the cross section of the tube in the flow direction becomes an ellipse. An elliptic cylinder is a basic and general shape, which can become a flat plate in a special case and also a circular cylinder depending on its axis ratio. Furthermore, it is well known that its fluid dynamic drag at a small angle of attack is lower than that of a circular cylinder. This may be an advantageous feature when using elliptic tubes as a heat-transfer surface element, since the pumping power required may be reduced and the heat exchanger may be more compact. Recently, a few experimental studies have been published on tube banks of lenticular or oval-shaped tubes to reduce the drop in pressure.

As far as forced convection heat-transfer characteristics of the elliptic cylinders are concerned, only a few investigations are available. From this standpoint, the present authors have conducted experimental studies of forced convection heat transfer and flow around one and two elliptic cylinders. In order to employ elliptic tubes as a heat-transfer surface element of heat exchangers, it is basically important to examine the heat-transfer characteristics of elliptic tube banks and to explore an optimum arrangement.

This paper aims to clarify the local and mean heat-transfer characteristics of four elliptic cylinders having an axis ratio 1:2 arranged tandem in a uniform flow of air. They are functions of the Reynolds number, the angle of attack and the cylinder spacing. Furthermore, correlations between their local heat-transfer features and the flow features around each
cylinder are discussed, and a comparison is made for
the mean heat-transfer coefficient of the present
elliptic cylinders with that of the in-line circular cyli-
ders\(^{(1)}\).

2. Nomenclature

\(c\) : length of the major axis of an elliptic cylinder
\(d\) : diameter of the circular cylinder of equal
circumferential length
\(f\) : vortex shedding frequency
\(h\) : local heat-transfer coefficient = \(q/(T_u - T_w)\)
\(l\) : streamwise distance between the neighboring
cylinder centers
\(Nu\) : local Nusselt number = \(h \cdot c/\lambda\)
\(Nu_m\) : mean Nusselt number = \(h_m \cdot c/\lambda\)
\(Nu_{max}\) : arithmetic mean Nusselt number of four
elliptic cylinders
\(q\) : heat flux per unit area and unit time
\(Re\) : Reynolds number = \(U_m \cdot c/\nu\)
\(Re_e\) : Reynolds number = \(U_m \cdot d/\nu\)
\(s\) : surface distance from the leading edge, taken
as positive along the upper side
\(St\) : Strouhal number = \(f \cdot c/U_m\)
\(T_w\) : wall temperature
\(T_u, U_u\) : temperature and velocity of the upstream
uniform flow
\(U, u\) : streamwise mean and turbulent fluctuating
velocities
\(x\) : distance from the center of the first elliptic
cylinder in the direction of upstream uniform
flow
\(y\) : distance normal to the \(x\)-axis
\(\alpha\) : angle of attack
\(\lambda, \nu\) : thermal conductivity and kinematic viscosity
of air at \(T_w\)

3. Experimental Apparatus and Technique

The arrangement of elliptic cylinders and the
coordinate system are shown in Fig. 1. That is, a line
through the centers of four elliptic cylinders is aligned
with the direction of upstream uniform flow, the incli-
nation angles between the major axes and the uniform
flow direction are the same.

The open wind tunnel used in the present study is

![Fig. 1 Arrangement of four elliptic cylinders and the
coordinate system](image)

the same as that employed in the previous work\(^{(4)}\).
The nozzle contraction ratio is 13.5 to 1. The test
section at the tunnel exit is 1200 mm long, 400 mm
high and 150 mm wide. One of the side walls of the test
section was changed in order to place the four elliptic
cylinders and to change their spacings.

The four elliptic cylinders examined have an axis
ratio of 1:2, the major axis being 50 mm and the
spanwise length 150 mm. They were made of fiber-
reinforced plastics by the same procedure as in the
previous work\(^{(4)}\). Heating of the cylinder was conduct-
by means of an electric current to a stainless steel
sheet of 0.059 mm thick, 30 mm wide and 455 mm long,
which was wound helically around the cylinder and
adhered to the surface. Forty-nine alumel-chromel
thermocouples of 0.07 mm in diameter were embedded
on the cylinder surface in order to measure the wall
temperature. Heat loss by conduction and radiation
was neglected in the following results.

Although it was possible to heat all four elliptic
cylinders at the same time, for practical purposes,
only the measuring cylinder was heated under the
condition of constant heat flux in the present experi-
ments since thermal interactions among each cylinder
may lead to a difficulty in estimating the temperature
of the oncoming flow, especially for the downstream
cylinders.

The free-stream velocity \(U_u\) ranged from about 5
m/s to 21 m/s and a corresponding Reynolds number
\(Re\) based on the major axis length, \(c\), of the elliptic
cylinder ranged from about 15 000 to 70 000. Even the
lowest velocity was sufficiently high to neglect effects
of natural convection. The free-stream turbulence
intensity was about 0.3% in the Reynolds number
range described above. The cylinder spacing \(l\), defined
as the distance between centers of two neighboring
cylinders, was varied from 1.25 to 4 times of the major
axis length, and the angle of attack from 0° to 90° at
30° intervals. The tunnel blockage ratio varies from
0.062 5 at \(\alpha=0°\) to 0.125 at \(\alpha=90°\) and the aspect ratio
of the present elliptic cylinders was 3. The present
data, however, are uncorrected for the tunnel wall
effects. It is to be noted that the heat-transfer charac-
teristics seem to be sensitive to the experimental
conditions such as the temperature difference between
the free-stream flow and the room, the turbulence
intensity of the free-stream flow and the accuracy of
the arrangement of the elliptic cylinders.

A flow visualization study was made with a water
channel and floating aluminum powders in order to
investigate flow features for each cylinder. The
Reynolds number examined was about 5 000, which
was much lower than those in the heat-transfer study.
Further, the mean and turbulent fluctuating velocities
in the near wake were measured under the nonheated condition with a constant temperature hot-wire anemometer. A FFT analyzer was also used to detect a vortex shedding frequency from each cylinder. Various difficulties were encountered in using a hot-wire anemometer in the wake region with low velocity and high-turbulence intensity. However, even in such flow fields, the hot-wire anemometer may be useful to examine qualitatively the correlation between the heat-transfer features and the flow features.

4. Experimental Results and Discussions

4.1 Flow features

In order to investigate the local heat-transfer characteristics of the four elliptic cylinders in relation to the flow behaviors around them, the flow characteristics are clarified.

The flow visualization photographs are shown for $l/c = 1.25, 2.0, 2.5$ at $a = 0^\circ$ in Fig. 2. At $l/c = 1.25$, all the separated shear layers from the upstream cylinders attach to the immediate downstream cylinders. The wake width formed behind the 2nd cylinder becomes narrower than that behind the 1st one, due to the downstream shift of the separation point on the 2nd cylinder. The flow between each cylinder is relatively stagnant and pairs of vortices are formed. At $l/c = 2.0$, the flow pattern between the 1st and 2nd cylinders is especially unstable. A slight increase of $l/c$ beyond 2.5 makes all the separated shear layers roll up upstream of each cylinder.

The flow visualization photographs are shown for $l/c = 1.25, 2.0, 2.5$ at $a = 60^\circ$ in Fig. 3. The wake increases its width causing a violent motion of fluid compared to that for $a = 0^\circ$, and a relatively fast flow through the downstream neighboring two cylinders can be observed at narrow cylinder spacing. The local and overall heat-transfer characteristics of elliptic cylinders are remarkably affected by the flow patterns.

Fig. 2  Flow patterns at $a=0^\circ$ and $Re = 5000$
described later.

The distributions of streamwise mean and turbulent fluctuating velocities between each cylinder for $a = 0^\circ$ and $Re = 50,000$ are shown in Fig. 4. At the smallest cylinder spacing $l/c = 1.25$, the wake width, which is defined in the present paper as a transverse distance between two maximum points of the turbulence intensity, is the largest behind the 1st cylinder and decreases for the downstream cylinders. The shear layers separated from the upstream cylinder are considered to attach to the downstream cylinders as shown in Fig. 2. It is very clear that the oncoming flow to the 2nd cylinder obtains a very high peak of turbulence intensity compared to that for the 3rd and 4th cylinders. These flow behaviors bring about the local heat-transfer characteristics demonstrated in the following.

At $l/c = 2.0$, the distribution of the turbulence intensity possesses definite peaks only in the oncoming flow to the 2nd cylinder and becomes relatively uniform in that to the 3rd and 4th cylinders. It may be considered that two shear layers separated from the 2nd and 3rd cylinders roll up upstream of the 3rd and 4th cylinders, respectively, and the oncoming flow having a relatively uniform velocity approaches them. At $l/c = 2.5$, the cylinder spacing is wide enough for two separated shear layers to roll up upstream of the downstream cylinder. Accordingly, all the oncoming flows to the 2nd, 3rd and 4th cylinders are similar in the velocity and also turbulence intensity distributions. This may result in almost the same heat-transfer profile for the 2nd, 3rd and 4th cylinders as shown later.

Figure 5 shows the Strouhal number of all the cylinders for $l/c = 1.25, 2.0$ and $2.5$ at $a = 0^\circ$. At $l/c = 1.25$, there exists no dominant frequency in the velocity fluctuation in the wake behind the 1st, 2nd and 3rd cylinders, since two shear layers separated from
the upstream cylinder attach onto the neighboring downstream one as shown in Fig. 4. However, as for the 4th cylinder, there is no cylinder downstream of it and the two separated shear layers roll up forming a vortex street. Accordingly, the dominant frequency in the velocity fluctuation and the Strouhal number can be found as shown in the figure. The dominant frequency decreases as $Re$ increases.

At $U/c = 2.0$, the Strouhal number for the 1st cylinder appears first at about $Re = 30,000$, and that for the 2nd cylinder changes discontinuously at almost the same Reynolds number. It may be presumed that this is similar to the so-called jumping phenomenon well-known for two in-line circular cylinders. The Strouhal numbers for the 3rd and 4th cylinders are relatively low and show a similar trend. At $U/c = 2.5$, there exists a long streamwise distance between two neighboring cylinders and the vortex shedding exists for all four cylinders. Their characteristic variations are found to be roughly similar to each other.

The distributions of the mean and turbulent

![Fig. 5 Strouhal number at $a=0^\circ$](image)

![Fig. 6 Streamwise mean and turbulent fluctuating velocities among each cylinder: $a=60^\circ$ and $Re=50,000$](image)
fluctuating velocities at $\alpha=60^\circ$ are demonstrated in Fig. 6 and the corresponding Strouhal number variations in Fig. 7, respectively. As for the case of $\alpha=60^\circ$ and $l/c=1.25$, the wake width is wide, which can be clearly seen behind the 1st cylinder, and the turbulence intensity increases for the farther downstream cylinders. Generally the distributions of mean and turbulent fluctuating velocities at $l/c=2.0$ and 2.5 are similar to each other. However, at $l/c=2.5$ inside the wake behind the 1st cylinder, the mean velocity decreases slightly while the turbulence intensity decreases by nearly half compared to those at $l/c=2.0$. This may result in a sudden decrease of $Nu$ at $l/c=2.5$, as described later. On the other hand, the Strouhal number shows no essential variation under such a flow variation as found in Fig. 7. It is clearly shown that the Strouhal number is nearly constant for all four cylinders, although there exists no dominant frequency only for the 1st cylinder at $l/c=1.25$.

### 4.2 Local heat-transfer characteristics

Typical examples of the local Nusselt number distribution of the four cylinders for various cylinder spacings at $\alpha=0^\circ$ are presented in Fig. 8. It is noted here that general characteristic features of the local Nusselt number distribution exhibit no essential dependency on the Reynolds number, except at low Reynolds numbers. In accordance with this fact, the results are presented only for $Re=50,000$ in the following figures. The local Nusselt number distribution for the 1st cylinder shows no essential variation with the cylinder spacing, which is very similar to that for the single cylinder, though $Nu$ in the wake region increases as $l/c$ increases from 1.5 to 2.0. On the contrary, the $Nu$ distributions for the 2nd, 3rd and 4th cylinders vary drastically depending on whether or not the two shear layers separated from the upstream cylinder attach to the downstream cylinder.

At the smallest cylinder spacing $l/c=1.25$, two shear layers separated from the upstream cylinder attach to the immediate downstream cylinder. Then $Nu$ attains maxima at two attachment points on each cylinder. The flow inside the wake region bounded by two neighboring cylinders and two separated shear layers is stagnant and thus $Nu$ is low. Downstream of the attachment point, $Nu$ decreases with the surface distance because a boundary layer develops. $Nu$ reaches a minimum near the separation point and then increases slightly in the separated flow region. The flow approaching the attachment region on the 2nd cylinder is highly turbulent and the boundary layer developing on its surface has the ability to flow against an unfavorable pressure gradient on the rear side of the cylinder. This results in a downstream shift of the separation point compared to that on the 1st cylinder. Accordingly, the wake width formed behind the 2nd cylinder becomes narrower and the distance between the separation point on the 2nd cylinder and the attachment point on the 3rd cylinder becomes quite small. Such a situation is the same for the 4th cylinder, as shown clearly in the figure. At $l/c=2.0$, the wall temperature near the leading edge of the 2nd
cylinder was observed to fluctuate severely since the flow pattern around the cylinder changes intermittently from that found at \( \text{Re} = 1.25 \) and 1.5 to that at \( \text{Re} \geq 2.5 \) as mentioned in Fig. 2 previously. Therefore, the results shown in the figure are the time-averaged mean values, resulting in a data scatter near the leading edge.

As for the 3rd and the 4th cylinders, however, a considerable variation of the Nusselt number distribution occurs. That is, \( \text{Nu} \) attains a maximum at the leading edge, decreases with the surface distance, and reaches a minimum. Subsequently \( \text{Nu} \) increases in the wake region. Such a drastic variation of \( \text{Nu} \) may be due to a change of the flow pattern around the 3rd and 4th cylinders. That is, the shear layers separated from the 2nd and 3rd cylinders roll up upstream of the 3rd and 4th cylinders respectively. Accordingly, the main flow at free-stream temperature is entrained onto the front surface of the 3rd and 4th cylinders. Further, a slight increase of \( \text{Re} \) from 2.0 to 2.5 brings about a great variation in the Nusselt number distribution only for the 2nd cylinder, whose profile is almost the same as those for the 3rd and 4th cylinders. At the largest cylinder spacing in the present study \( \text{Re} = 4.0 \), the profiles of \( \text{Nu} \) for all four cylinders are almost similar to those at \( \text{Re} = 2.5 \), and 3.0, though a variation rate of \( \text{Nu} \) on the front side of the 2nd, 3rd and 4th cylinders at \( \text{Re} = 4.0 \) is steeper than that at \( \text{Re} = 2.5 \), due to the highly turbulent oncoming flow.

Represented in Fig. 9 are the local Nusselt number distributions of all four cylinders for various cylinder spacings at \( \angle = 60^\circ \). The oncoming flow to the 1st cylinder is decelerated upstream of the cylinder and the wake produced behind it is wide. In accordance with such a flow feature around the 1st cylinder, the local Nusselt number distribution is quite different from that at \( \angle = 0^\circ \) shown in Fig. 8. \( \text{Nu} \) in the separated flow region of the 1st cylinder is strongly affected by the cylinder spacing. That is, when \( \text{Re} \) increases from 1.25 to 2.0, \( \text{Nu} \) increases slightly. However it exhibits a sudden decrease as \( \text{Re} \) increases from 2.0 to 2.5. The Nusselt number distribution for the 2nd cylinder exhibits a large variation with the cylinder spacing, since the flow feature between the 1st and 2nd cylinders varies greatly. On the other hand, the heat-transfer behavior for the 3rd and 4th cylinders is affected very little by \( \text{Re} \).

4.3 Mean heat-transfer characteristics

In general, \( \text{Nu} \) can be expressed as a function of \( \text{Re} \) as follows:

\[
\text{Nu} = A \text{Re}^n
\]  

(1)

\( A \) and \( n \) in Eq. (1) depend on both the angle of attack and the cylinder spacing. Summarized in Table 1 are their values for all the cylinders, which were

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<th>( \angle )</th>
<th>1st C.</th>
<th>2nd C.</th>
<th>3rd C.</th>
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Fig. 9 Local Nusselt number distribution, \( \angle = 60^\circ \) and \( \text{Re} = 50000 \)


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determined with the least-squares method. They are naturally valid only in the Reynolds number range examined in the present experiments, $15\ 000 \leq Re \leq 70\ 000$. When all four elliptic cylinders are heated, it is naturally presumed that the value of $Nu_\alpha$ for each cylinder decreases slightly compared with the present one. There exists a paper estimating the reduction rate of mean heat transfer for each cylinder\cite{11}.

Exemplified in Figs.10 and 11 are characteristic variations of the mean Nusselt number. Fig.10 represents variations of $Nu_\alpha$ with $\alpha$ at $l/c=1.25, 1.5, 2.0, 2.5, 3.0$ and $4.0$ for the four cylinders. At cylinder spacings $l/c$ less than 2.0, $Nu_\alpha$ for the 1st cylinder decreases slightly with an increase of $\alpha$, becomes minimum around $\alpha=30^\circ$, and subsequently increases between $\alpha =30^\circ$ and $90^\circ$. However, at $l/c$ greater than 2.5, these characteristic trends change drastically. That is, at $l/c=2.5$, $Nu_\alpha$ is approximately independent of $\alpha$. Further, at $l/c$ greater than 3.0, $Nu_\alpha$ is almost constant between $\alpha=0^\circ$ and $30^\circ$, and decreases rather abruptly between $\alpha=30^\circ$ and $60^\circ$ at Reynolds numbers higher than 40 000. This is originated from the attachment of the separated shear layers from the 1st cylinder to the 2nd one, and that the vortex motion in the wake between them depends strongly upon the cylinder spacing. It may be considered that these flow features change drastically around $l/c=2.5$ as illustrated in Figs.6 and 9.

As far as the 2nd cylinder is concerned, $Nu_\alpha$ is generally higher than that for the 1st cylinder. It is

Fig. 10 Variation of $Nu_\alpha$ with $\alpha$ for each cylinder

Fig. 11 Variation of $Nu_\alpha$ with $l/c$ for each cylinder
clear that \( Nt_u \) at \( l/c = 1.25 \) attains a minimum around \( \alpha = 60^\circ \) since the 2nd cylinder is considered to be located wholly inside the wake region having low velocity between the 1st and 3rd cylinders. On the other hand, \( Nt_u \) at \( l/c = 1.5 \) and 2.0 becomes minimum at about \( \alpha = 30^\circ \), whose behavior is rather similar to that for the 1st cylinder. However at cylinder spacings greater than \( l/c = 2.5 \), \( Nt_u \) increases slightly with \( \alpha \), though \( Nt_u \) at \( l/c = 4.0 \) exhibits a somewhat different trend.

The characteristic variations of \( Nt_u \) with \( \alpha \) for the 3rd and 4th cylinders are very similar and \( Nt_u \)’s dependency upon \( \alpha \) is small for all \( Re \) and \( l/c \) as found in Fig. 10. Such heat-transfer features may be originated from the flow features. That is, the 3rd and 4th cylinders are located in a highly turbulent and wide wake flow and their characteristics exhibit no essential variations with the cylinder spacing or Reynolds number.

Represented in Fig. 11 are variations of \( Nt_u \) with \( l/c \) at \( \alpha = 0^\circ, 30^\circ, 60^\circ \) and \( 90^\circ \) for all four cylinders. It is to be noted that the abscissa is the reciprocal \( c/l \) of the nondimensional cylinder spacing. The results at \( l/c = 0 \) are the previous data for the single cylinder \(^{110}\). \( Nt_u \) for the 1st cylinder does not show a large variation with \( l/c \) at \( \alpha = 0^\circ \) and \( 30^\circ \). However, this is not the case at \( \alpha = 60^\circ \) and \( 90^\circ \). As \( l/c \) increases beyond 2.0 \((c/l < 0.5)\), \( Nt_u \) decreases almost discontinuously and its value is much lower than that for the single cylinder \((c/l = 0)\), even at \( l/c = 4.0 \). Its approach to that for the single cylinder seems to be very slow since the wake is quite wide and its streamwise variation is very slow.

The variation of \( Nt_u \) for the 2nd cylinder is very complicated. In the case of \( \alpha = 0^\circ \), \( Nt_u \) increases with an increase of \( l/c \), becomes a maximum around \( l/c = 2.0 \) \((c/l = 0.5 \text{ in the figure})\) at Reynolds numbers higher than about 40,000. However its further increase brings about a sudden decrease of \( Nt_u \), which corresponds to a similar variation of the Strouhal number represented in Fig. 5. Even at \( l/c = 4.0 \), \( Nt_u \) is much higher than that for the single cylinder. \( Nt_u \) at \( \alpha = 30^\circ \) shows a peculiar feature. That is, it attains a minimum around \( l/c = 1.5 \) at high Reynolds numbers. On the other hand, when \( l/c \) increases from 1.25, \( Nt_u \) at \( \alpha = 60^\circ \) and \( 90^\circ \) increases rapidly, reaches a maximum between \( l/c = 1.5 \) and 2.0 and subsequently approaches gradually to the value for the single cylinder.

The results for the 3rd and 4th cylinders show qualitatively similar behaviors to each other. \( Nt_u \) for both of them decreases rather monotonically as \( l/c \) increases and approaches that for the single cylinder.

In order to use the elliptic tubes in heat exchangers, it is very important to estimate accurately the overall heat-transfer rate of the elliptic tube banks. As a preliminary examination, an arithmetic mean Nusselt number \( Nt_{\text{mea}} \) of \( Nt_u \) for all four elliptic cylinders is evaluated in the present study. The reference length \( d \) in \( Nt_{\text{mea}} \) is a diameter of a circular cylinder whose circumferential length is equal to that of the present elliptic cylinder.

Figure 12 shows a variation of \( Nt_{\text{mea}} \) with \( l/d \) for \( \alpha = 0^\circ, 30^\circ, 60^\circ \) and \( 90^\circ \) at \( Re_d = 15,400, 41,000, 54,000 \), comparing with that of in-line four circular cylinders at \( Re_d = 41,000 \), although all four cylinders were heated \(^{111}\). At all the Reynolds numbers examined, \( Nt_{\text{mea}} \) shows only a small change with \( l/d \), which is different from that of the circular cylinders. However, the present result increases a little at small cylinder spacing, whose value is comparable to that for the circular cylinders. From these results, it is not unreasonable to note that the cylinder spacing and the angle of attack of the elliptic cylinders should be arranged as small as possible for the compactness and the drag reduction of the heat exchangers while attaining a relatively high heat-transfer capability.

5. Conclusions

Local and overall heat-transfer characteristics of four elliptic cylinders having an axis ratio 1:2 in various tandem arrangements were clarified experimentally in relation to the flow characteristics around them in the present investigation. The main points obtained are summarized as follows:

(1) The local heat-transfer coefficient depends strongly upon the angle of attack and the cylinder spacing for all four cylinders, although \( Nt_u \) on the front surface of the 1st cylinder exhibits no essential change with the cylinder spacing. There exists a
critical cylinder spacing, at which the local heat-transfer coefficient varies suddenly for all the cylinders corresponding to the change of the flow pattern. Its variation is notable especially for the rear side of the 1st cylinder at large angles of attack, and also for the 2nd cylinder at low ones.

(2) The overall heat-transfer coefficients were estimated as functions of the angle of attack, the cylinder spacing, and the Reynolds number for all the cylinders. It was found that the mean Nusselt number, especially for the 1st and 2nd cylinders, shows a great dependency upon those parameters. At the critical cylinder spacing, $N_u_{\text{cr}}$ suddenly changes especially for the 1st cylinder. The overall heat-transfer capability of elliptic cylinders at small cylinder spacings and at small angles of attack is shown to be comparable to that of the in-line circular cylinders.

(3) The correlations between the local heat-transfer characteristics and the flow features around the cylinders were made clear for the Reynolds number, the angle of attack and the cylinder spacing by measuring the Strouhal number, the width of the wake and the mean and turbulent fluctuating velocities in the near wake, and by the flow visualization study.

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