Studies on the Wells Turbine for Wave Power Generator*  
(Turbine Characteristics and Design Parameter for Irregular Wave) 

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A Wells turbine operating under an irregular wave condition has been analyzed theoretically. It is shown that the turbine performance depends on a unique parameter, which includes characteristic parameters of irregular waves, turbine speed and dimensions of the turbine and air chamber. In order to obtain the optimum value of this parameter, a model test has been performed in a computer-controlled wind tunnel, which can simulate any kind of oscillating flows based on spectra of irregular waves. From this value, a set of optimum dimensions of the Wells turbine system can be determined. Furthermore, starting and running characteristics of the Wells turbine have been obtained by a computer simulation and compared with the experimental results. It is possible to predict the optimum value of the parameter and behavior of output coefficient by computer simulation.

**Key Words**: Fluid Machine, Wave Power Turbine, Wells Turbine, Ocean Energy, Wave Power Generator, Irregular Wave

1. Introduction

In the past decade, worldwide efforts have been devoted to the development of energy conversion from ocean waves. Various types of wave-energy devices have been proposed. One of the most practically applicable devices is that using the combination of an oscillating water column (OWC) as a primary converter and a Wells turbine as a secondary converter. The schematic layout of the OWC-Wells turbine system is shown in Fig. 1.

The Wells turbine is a self-rectifying axial flow turbine suitable for energy conversion from oscillating air flow. In early investigations\(^{11-17}\), wind-tunnel testing of isolated airfoils with different blade sections and model turbine testing of various rotor geometries were carried out under steady unidirectional flow conditions in order to obtain the basic characteristics of the Wells turbine. From these experimental data, the optimum values of rotor geometrical parameters were suggested\(^{13}\). It is also possible to predict the unsteady starting and running characteristics of the Wells turbine by computer simulation on the basis of the steady characteristics. The simulated results were compared with the experimental ones in the case of a sinusoidally oscillating flow condition\(^{18,19}\). According

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* Received 15th February, 1988. Paper No. 86-1121A  
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to these experimental works, it is possible to design the optimum Wells turbine for the basic flow conditions such as unidirectional or sinusoidally oscillating flow. However, ocean waves are generally irregular, and both wave height and period change from time to time. Therefore, it is important to clarify the turbine performance under irregular wave conditions and to establish the design method of the Wells turbine system for the site where the characteristic parameters of the irregular waves are available.

In the present investigation, a unique design parameter has been derived by theoretically analyzing the behavior of the Wells turbine operating under an irregular wave condition. The parameter includes a significant wave height and mean period of an irregular wave together with turbine speed and representative dimensions of the turbine and air chamber. The optimum value of the parameter depends on the spectrum of irregular waves, and it is to be determined experimentally. This was demonstrated by using a computer-controlled wind tunnel which could simulate any kind of oscillating flow according to the specified spectrum. Furthermore, a numerical simulation of the Wells turbine performance was carried out for the ISSC (International Ship Structure Congress) spectrum(10) and the calculated results were compared with the experimental ones. The simulation is useful to predict the optimum value of design parameters.

2. Nomenclature

\[ A_a : \text{cross-sectional area of air chamber} \]
\[ A_i : \text{flow passage area of turbine} \]
\[ b : \text{blade height} \]
\[ C_a : \text{total enthalpy coefficient} = P_s Q / ((\rho V^2 / 2) + v^2 / 2) \]
\[ C_i, \bar{C}_i : \text{input coefficient, mean input coefficient [Eq. (9)]} \]
\[ C_o, \bar{C}_o : \text{output coefficient, mean output coefficient [Eq. (8)]} \]
\[ C_p : \text{pressure coefficient} = P_s / (\rho V^2 / 2) \]
\[ C_T : \text{torque coefficient} = T / ((\rho / 2) (R_o)^2 + v^2 / 2) b l z v_a \]
\[ f, f^* : \text{frequency, mean frequency of wave motion} \]
\[ f^* : \text{dimensionless frequency} = f / f \]
\[ h(t) : \text{wave height in air chamber} \]
\[ h_{13} : \text{significant wave height} \]
\[ l : \text{moment of inertia of rotor} \]
\[ K : \text{dimensionless period} = R f / V \]
\[ l : \text{blade chord length} \]
\[ m = A_i / A_a \]
\[ P_s : \text{total gauge pressure in air chamber} \]
\[ Q : \text{flow rate} \]
\[ R, R_i : \text{mean radius, tip radius of rotor} \]
\[ S^* : \text{dimensionless spectrum of wave motion [Eq. (11)]:} \]
\[ t : \text{time} \]
\[ t^* : \text{dimensionless time} = t / T \]
\[ T, T_i : \text{output torque, loading torque} \]
\[ T : \text{mean period} \]
\[ V : \text{reference velocity} = h_{13} / (m T) \]
\[ v_a : \text{mean axial velocity} = Q / A_i \]
\[ v^2 : \text{dimensionless axial velocity} = v_a / V \]
\[ X_i : \text{dimensionless moment of inertia} = I / (\pi \rho R^3) \]
\[ X_i : \text{dimensionless loading torque} = T_i / (\pi \rho V^2 R^3) \]
\[ z : \text{number of blades} \]
\[ \alpha : \text{effective angle of attack [Eq. (2)]} \]
\[ \eta : \text{instantaneous turbine efficiency} = P Q / (T \omega) \]
\[ \bar{\eta} : \text{mean turbine efficiency [Eq. (10)]} \]
\[ \nu : \text{hub ratio} \]
\[ \varepsilon : \text{phase of element wave} \]
\[ \rho : \text{density of air} \]
\[ \sigma : \text{solidity of rotor blades} \]
\[ \omega : \text{angular velocity of rotor} \]
\[ \omega^* : \text{dimensionless angular velocity} = \omega / \bar{f} \]

3. Analysis of Turbine Performance

The equation of motion for a rotating system of the Wells turbine can be described in a dimensionless form as

\[ K^2 X_i \frac{d \omega^*}{dt^*} + X_i = C_i(a)(K \omega^*)^2 + v^2 / 2 \frac{4(1 - \nu)}{1 + \nu} \]

where

\[ a = \tan^{-1}(v^2 / (K \omega^*)) \]
\[ K \omega^* = \omega m R T / h_{13} \]
\[ v^* = m T v_a / h_{13} \]

The first and the second terms on the left side of Eq. (1) are inertia and loading terms, respectively, and the right side represents a torque generated by a turbine. It is clear from Eqs. (1) - (4) that the behavior of the turbine can be calculated as a function of \(K \omega^*\) and \(v^*\), when loading characteristics \(X_i(\omega^*)\), torque characteristics \(C_i(a)\) and rotor geometrical parameters \(X_i\), \(\sigma\) and \(\nu\) are specified.

The dimensionless axial velocity defined in Eq. (4) depends on the motion of the OWC. Assuming incompressible flow, \(v^*\) is written as

\[ v^* = \frac{d}{d(t^*)} \frac{d h^*}{d(t^*)} \]

where \(h(t)\) is the height of water level in the air chamber relative to the system. If the dimensionless spectrum \(S^*(f^*)\) referring to significant wave height and mean period of OWC motion is given, the behavior of dimensionless height of water level \(h(t^*)\) can be calculated from the following equation superimposing the element waves:

\[ h(t^*) = \sum \frac{1}{2 \pi} \sqrt{4 \pi S^*(f^*)} A f^* \cos(2 \pi f^* t + \varepsilon) \]
where \( \Delta f^* \) and \( \varepsilon \) indicate divided frequency width and phase shift of the element wave, respectively. Substituting Eq. (6) into Eq. (5), we obtain

\[
v^*_n = -\frac{M}{2\pi} \sqrt{16\pi^2 S^*(f^*)f^*\Delta f^* \sin(2\pi f^* t^* + \varepsilon)}
\]

(7)

It is clear from this equation that \( v^*_n(t^*) \) depends only on the spectrum \( S^*(f^*) \) and is independent of significant wave height and mean period of the OWC motion.

It is mentioned here that characteristics of the irregular motion of OWC are generally different from those of incident waves. The discrepancy is a result of the energy absorbing characteristics of the OWC. On the other hand, the characteristics of the turbine influences the characteristics of the OWC. Therefore, it is necessary to solve Eq. (1) and the equation of motion of OWC simultaneously for a given incident wave \( H(t) \), loading characteristics \( X_c(\omega^*) \) and torque characteristics \( C_t(\omega^*) \), in order to obtain the actual turbine performance (12).

However, the main purpose of the present investigation is to derive a unique design parameter for the Wells turbine operating under irregular wave motion with arbitrary spectrum. Therefore, it is assumed in the following theoretical and experimental investigation that the spectrum \( S^* \) of the OWC motion is known beforehand. Furthermore, the loading torque is adjusted so as to keep the rotational speed constant in the test of running characteristics of the turbine. On the other hand, loading torque is kept constant in the test of starting characteristics.

The performance of the Wells turbine can be obtained from Eqs. (1)–(4) with \( v^*_n \) from Eq. (5). The starting characteristics \( \omega^*(t^*) \) depend on dimensionless parameters \( K \), \( X_i \) and \( X_c \). Similarly, the running characteristics are obtained with rotational speed kept constant. In this case, the time mean output and input coefficient from \( t^*=0 \) to \( t^* \) are given respectively as

\[
C_0 = \frac{1}{t^*} \int_0^{t^*} C_i(\omega^*)^2 + v^*_n^2 \sigma \frac{4(1-\nu)}{1+\nu} \omega^* \, dt^*,
\]

(8)

\[
C_i = \frac{1}{t^*} \int_0^{t^*} C_c(\omega^*)^2 + v^*_n^2 \sigma \frac{4(1-\nu)}{1+\nu} \omega^* \, dt^*.
\]

(9)

Then, mean turbine efficiency is

\[
\eta = C_0/C_i.
\]

(10)

It is obvious from the above relations that the turbine performance depends on a unique parameter \( K\omega^* \) when the turbine operates under an irregular wave condition with a given spectrum \( S^*(f^*) \). Therefore, once an optimum value of \( K\omega^* \) can be determined by either experiment or numerical simulation for a given \( S^*(f^*) \) and dimensionless turbine characteristics, the combination of optimum design values \( (m, R, \omega) \) can be obtained for a site where significant wave height \( H_s \) and mean period \( T \) are known.

4. Test Apparatus and Procedure

The performance test of the Wells turbine under irregular wave conditions was carried out by use of a computer-controlled wind tunnel to simulate an arbitrary wave motion. The outline of the test apparatus is shown in Fig. 2. It is possible to produce both steady and oscillating air flow by controlling the motion of a 1.4 m dia. piston in the tunnel through a microcomputer.

The Wells turbine was set at the exit of the wind tunnel. The details of the turbine are described in Refs. (7)–(8). Two kinds of test rotors were selected for this experiment; they showed relatively good performance in steady and sinusoidally oscillating flow conditions. The geometrical parameters of these two rotors are as follows: rotor N 20-6 (Blade profile: NACA 0020, \( z=6 \), \( \sigma=0.67 \), without guide vanes); N20-7 (Blade profile: NACA0020, \( z=7 \), \( \sigma=0.78 \), with guide vanes).

In the experiment, the irregular wave flow was generated based on the ISSC spectrum (10) which is typical in the field of ocean engineering.

\[
S^*(f^*) = 0.11 f^{-1} \exp(-0.44 f^{-1/3}).
\]

(11)

Figure 3 shows the distribution of the dimensionless ISSC spectrum. Two groups of irregular waves, No. 1 and No. 2 in Fig. 4, were generated for the turbine tests. They were different in their wave shapes but both had the spectrum of Eq. (11).

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**Fig. 2** Outline of wind tunnel producing irregular flow

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Series II, Vol. 31, No. 4, 1988

*JSME International Journal*
In the test of running characteristics, variation of turbine performance with $K\omega^*$ was examined by changing rotational speed step by step from a low speed to a high speed, covering the effective operating range of the Wells turbine. On the other hand, in the test of starting characteristics, behavior of rotational speed from $\omega^*=0$ was observed with loading cut-off, i.e., $X_L=0$. During the test, the turbine output torque, the air flow rate, and the total pressure drop across the rotor were measured; and these data were loaded into a microcomputer memory through transducers and an $A/D$ converter.

5. Experimental Results and Discussion

5.1 Turbine performance under irregular wave condition

Typical performances of rotor N 20-6, namely the variations of the air chamber pressure, input coefficient and output coefficient with time are shown in Figs. 5 (a)-(c), respectively. They are obtained for the test wave No.1 at $K\omega^*=9.31$ near the optimum operating condition for this test wave. Comparing these results with the behavior of the irregular wave in Fig. 4 (a), it is observed that air chamber pressure and input and output coefficients rise sharply when a high wave occurs. The output coefficient becomes negative in intervals of low axial velocity, as shown in Fig. 5 (c), since the rotor blade operates under a small angle of attack and has a negative torque coefficient in that interval. In this experiment, the turbine speed is kept constant by a servomotor generator, which operates either as a power absorber or driver according to the positive or negative turbine output.

Figure 6 shows the variation of mean turbine output coefficient $C_\omega$ defined by Eq. (8). Although $C_\omega$ has a high value in the early period due to a casual existence of a high wave in this test wave, it settles

Fig. 3 Dimensionless ISSC spectrum

(a) Wave No.1

(b) Wave No.2

Fig. 4 Test irregular wave

(a) Variation of air chamber pressure

(b) Variation of input coefficient

(c) Variation of output coefficient

Fig. 5

JSME International Journal
Series II, Vol. 31, No. 4, 1988
gradually after $t^* = 3$.

Figure 7 shows the variation of instantaneous turbine efficiency $\eta$, which is defined as a local mean efficiency averaged for a short time interval of $t^* \pm 0.05$. The point of maximum efficiency indicated by an arrow mark in the figure corresponds to the same marks in Figs. 4(a) and 5. It is clear from these figures that efficiency is not necessarily at a maximum when a high wave occurs, and that there exists an optimum wave according to the operating condition of the turbine. The angle of attack for the rotor blade at the arrow point is about 16°, which almost agrees with the optimum angle of attack for the same rotor in the case of a steady flow condition [5].

Figure 8 shows the variation of mean turbine efficiency $\bar{\eta}$ as defined by Eq. (10). Since $\bar{\eta}$ is a ratio of time mean output to time mean input from $t^* = 0$ to an arbitrary $t^*$, it is expected to approach a certain constant value asymptotically with $t^*$. It is clear from Fig. 8 that the settling time is very short in the present case. Similar results were obtained also in the case of test wave No. 2. In the following discussion, a well-defined performance parameter is necessary to evaluate the Wells turbine operating under an irregular wave condition. For this purpose, the value of $\bar{\eta}$ at $t^* = 15$ is selected as the representative performance parameter.

5.2 Optimum value of $K\omega^*$

In Section 3, a unique parameter $K\omega^*$ was derived theoretically to indicate the operating condition of the Wells turbine under an irregular wave condition. For any given irregular wave or spectrum, the optimum value of $K\omega^*$ can be determined experimentally. In the following discussion, the reciprocal $1/(K\omega^*)$ is used instead of $K\omega^*$ since the former corresponds to the flow coefficient for ordinary fluid machines.

The two solid lines in Fig. 9 show the variation of $\bar{\eta}$ of rotor N20-6 against $1/(K\omega^*)$ for two kinds of irregular waves No. 1 and No. 2. $\bar{\eta}$ increases gradually with $1/(K\omega^*)$, reaches a maximum at about $1/(K\omega^*) = 0.11$, and then decreases. Since $1/(K\omega^*)$ is equivalent to the angle of attack of the rotor blade, the region to the right of the maximum point corresponds to a stall region.

It is obvious from this figure that the behavior of $\bar{\eta} - 1/(K\omega^*)$ characteristics for waves No. 1 and No. 2 are similar and the respective values of maximum efficiency and optimum values of $1/(K\omega^*)$ almost coincide with each other. From these results, it is
concluded that dimensionless performance of the Wells turbine depends only on the spectrum of the irregular wave, being independent of individual groups of irregular waves if they obey a same spectrum.

For the sake of comparison, the turbine performance was obtained for sinusoidally oscillating flow as well. The result is indicated by the two-dot chain line in Fig. 9. In this case, the significant wave height in Eq. (3) was replaced with sinusoidal wave height in calculating $K\omega^*$. The value of maximum efficiency is higher than in the case of the irregular wave, because small waves do not exist in the sinusoidal wave. The optimum value of $1/(K\omega^*)$ is about 0.07, being lower than in the case of the irregular wave, since the effective axial velocity is higher in this case.

6. Numerical Simulation

In this section, the results of numerical simulation of turbine performance for test wave No. 1 are discussed in comparison with the experimental results. The starting and running characteristics were calculated by solving Eqs. (1)-(4) simultaneously, where $C_d(a)$ and $C_l(a)$ characteristics obtained under a steady flow condition were used on the basis of a quasisteady assumption.

Figure 10 shows the starting characteristics of rotor N 20-7 with guide vanes, indicating the variation of dimensionless angular velocity $\omega^*$ from zero to the running condition. The simulated result shows a quick starting and reaches a high-speed running condition at about $t^* = 6$. On the other hand, the experimental acceleration is more gradual and it is not until $t^* = 24$ that self-starting is accomplished. Similar results were observed in the sinusoidally oscillating flow.[10]

The main reason for this discrepancy is presumed to be that the actual starting torque becomes lower than the simulated one in a low-speed range due to the Reynolds number effect. Since the effect of Reynolds number on the torque coefficient is remarkable in the range of high angle of attack[8], the discrepancy is large at starting. However, the difference becomes small in the high-speed range, where the operating Reynolds number is high.

Figure 11 shows a simulated result of variation of input coefficient in a running condition, which corresponds to Fig. 5(c). Although it can simulate an actual behavior qualitatively, the mean output coefficient is higher than the experimental value by about 10% as shown in Fig. 6. This is due to the hysteretic characteristics of $C_d(a)$ observed in an oscillating flow, which leads to a deterioration of generating torque.

The simulated results of mean efficiencies are also indicated in Fig. 9 in the cases of test wave No. 1 and a sinusoidal wave by a broken line and a dot dashed line, respectively. The maximum values of the experimental mean efficiency are lower than the simulated ones by about 10 to 15%. This is caused by the difference of total enthalpy coefficients $C_d(a)$ between the steady and unsteady conditions, as well as the difference of torque coefficients $C_l(a)$ mentioned above. In fact, the simulated result of input coefficient shows a slightly lower value than the experimental one shown in Fig. 5(b). These less accurate predicted values of $C_d(a)$ and $C_l(a)$ in the irregular flow are the main reason for the discrepancy in turbine efficiencies. Accordingly, it is difficult to simulate the turbine efficiency quantitatively at the present stage. Although an exact value of efficiency is not expected, the present numerical simulation is useful to obtain the optimum value of $1/(K\omega^*)$ as a unique parameter for the design of the Wells turbine system under an irregular wave condition.

7. Conclusion

The performance of the Wells turbine was investigated theoretically and experimentally to establish an optimum design method of the turbine under an irregular wave condition. The main conclusions are as follows:

(1) A unique parameter $1/(K\omega^*)$ was derived by theoretically analyzing the Wells turbine performance under an irregular wave condition.

(2) This parameter $1/(K\omega^*)$ includes significant wave height, mean period of irregular wave,
rotational speed of turbine, and dimensions of the turbine and air chamber. The performance of the Wells turbine is a function of these parameters and the dimensionless spectrum of an irregular wave.

(3) The optimum value of $1/(K\omega^*)$ obtained by numerical simulation agrees well with experimental values.

(4) The simulated values of the mean output coefficient and mean turbine efficiency are less than experimental values by 10 to 15%.

Acknowledgement

The authors would like to thank Messrs. T.Nakano, T.Suetsugu, A.Suzuki and H.Koura for their laboratory work. They also gratefully acknowledge the financial support of Grant-in-Aid for Scientific Research from the Ministry of Education, Science & Culture of Japan, and that from the Harada Memorial foundation.

References