Growing Process and Structure of Axisymmetric Vortex Breakdown*

(Experiments on the Bubble Growing Process)

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This study investigates the developmental process of an axisymmetric (bubble type) vortex breakdown. In the preceding reports, the growing process of the bubble was discussed, based on the inviscid linear theory of the internal wave. In this study, experiments on the unsteady swirling pipe flow are conducted to confirm the theoretical results concerning the growing process. The generation of a group of internal waves by the abrupt change of the upstream boundary condition, its propagation and dispersion are measured by a Laser-Doppler-Velocimeter, and the obtained data are compared with the results of the inviscid theory. The growing process of a bubble is clearly visualized by the method of intermittent injections of dye, and also examined by numerical analysis of the Navier-Stokes equations. The obtained results of the growing process showed a good agreement with the theory.

Key Words: Unsteady Flow, Fluid Vibration, Vortex Breakdown, Internal Wave, Swirling Flow, Flow Visualization, Numerical Analysis, Navier-Stokes Equations

1. Introduction

The vortex breakdown phenomena appear in a swirling flow with an abrupt change of the flow structure. Many studies on the phenomena have been made such as the flow instability (Ludwig and Flugwiss)⁽¹⁾, the finite transition analogous to the hydraulic jump (Benjamin)⁽²⁾ and the solitary wave (Leibovich and Randall)⁽³⁾. Torrance and Kopecky⁽⁴⁾, Grabowski and Berger⁽⁵⁾ carried out the numerical analysis of the Navier-Stokes (N-S) equations to examine the stationary vortex breakdown phenomena.

The authors have pointed out in the previous reports⁽⁶⁾⁻⁽⁹⁾ that the phenomena can be explained as due to the existence of internal waves, and many types of breakdown phenomena are distinguished by their azimuthal wave modes. This study investigates the

axisymmetric type (bubble type) vortex breakdown with emphasis on the birth and growing process of a bubble. In the preceding report⁽¹⁰⁾, a theoretical model based on the inviscid linear theory was proposed. In this report, the theoretical results are examined by both the numerical analysis of the N-S equations and the experiments of the unsteady swirling flow. The propagation and dispersion of the internal wave due to the change of the upstream boundary condition are experimentally confirmed. To obtain detailed information of the unsteady phenomena, the flow fields are observed by the streaklines which are visualized by the intermittently injected dye. Furthermore, the traveling mechanism of a bubble is clarified with relation to the upstream condition.

2. Experimental Apparatus and Method

The experiments are conducted on the birth and growing process of an axisymmetric vortex breakdown in a straight pipe. Figure 1 (a) shows a schematic view of an experimental apparatus. Water, which is supplied to the upper constant head tank A, enters the adjustable guide vanes C to get the azimuthal velocity component. Then, passing through the test

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pipe D (60 mm inside diameter, 1 000 mm in length) and the flow control vanes K and I, water is discharged to the cistern. The angle Ψ of the vanes can be changed arbitrarily by a speed-controlled motor M [Fig. 1 (b)]. The flow field is visualized by dye injection through a small tube J [Fig. 1 (a)]. In order to obtain detailed information on the unsteady phenomena, dye is injected intermittently with little disturbance to the flow field [Fig. 1 (c)]. Axial and azimuthal velocity components of the flow field are measured with a Laser-Doppler-Velocimeter (LDV). In a steady flow, the velocity profile along the diameter is recorded by an X-Y recorder. In a transient process, the velocity change at a fixed position is recorded by an oscillograph.

The experiment was conducted as follows. A steady flow was established initially with a fixed vane angle Ψ , constant flow rate Q and an appropriate suction rate q through a thin pipe H [Fig. 1 (a)]. Then the vane angle Ψ was changed to some extent at a specified speed of motor M driven by a microcomputer, while a flow rate Q and a suction flow rate q were kept constant. The transition of the flow field induced by this change of the upstream boundary condition was observed by the visualized streaklines, and the corresponding axial and azimuthal velocities were measured with LDV. The vane angle was settled with the precision of $1/50^\circ$, so that good reproducibility of the experiment was obtained.

(a) Experimental apparatus

(b) Detail of movable vanes

(c) Detail of dye injecting mechanism

Fig. 1

3. Experimental Results

First, the experimental results on the steady flow are shown, and then the results on the unsteady flow are mentioned.

3.1 Steady flow

The axial and azimuthal velocity profiles at a fixed axial position $z/a=4$ (a is pipe radius) are shown in Fig. 2 for several vane angles. Figure 3, on the other hand, shows the axial and azimuthal velocity profiles at several axial positions with fixed vane angle. For the case of no swirl, $\Psi=0$ in Fig. 2, the axial velocity is almost uniformly distributed over 75% of the radius. For larger vane angle, the axial velocity profile takes a peak near the pipe axis and becomes flat gradually toward downstream (Fig. 3). The azimuthal velocity increases as the vane angle becomes large. Its magnitude decreases gradually toward downstream while its shape is almost similar (Fig. 2 and 3).

Now, the Rossby number in the neighborhood of the pipe axis is defined as follows,

$$R_{oc} = w_c / (a \Omega_c)$$

where w_c is the axial velocity at the pipe axis and Ω_c

Fig. 2 Axial and azimuthal velocity profile for several vane angles

Fig. 3 Axial and azimuthal velocity profile at several axial positions

is the angular velocity near the axis, i. e., the gradient of the azimuthal velocity at the axis. Figure 4 shows the Rossby number along the axis for several vane angles. The Rossby number increases in the axial direction for small values of Ψ , but keeps almost constant for a large value ($\Psi=25^\circ$). The axisymmetric breakdown phenomena occurs with the vane angle Ψ larger than 35° in the present experiment. The above result ensures the assumption in the analysis in section 5 that the Rossby number of the primary flow is constant in the axial direction.

Figure 5 shows the axial velocity along the pipe axis for the vane angles $\Psi=28^\circ$ and 30° . In the measurement, the suction flow q is 19% of the total flow rate and the location of the suction tube is $z_0/a=17$. The axial velocity along the z axis has a minimum at $z/a=11$ for $\Psi=28^\circ$ and at $z/a=9$ for $\Psi=30^\circ$. As mentioned in the previous paper⁽⁹⁾, the bubble appears when the minimum value of the axial velocity becomes negative. The influence of the suction is examined by experiments with a suction tube at several locations. Influence is restricted within a distance of $3a$ upstream from the inlet of the tube. In general, nonaxisymmetric breakdown, such as the spiral or the double helix breakdown, occurs more downstream than the axisymmetric one. Hence, the location of the suction tube is determined so as to suppress the nonax-

Fig. 4 Rossby number along the axis for several vane angles

Fig. 5 Axial velocity distribution along the axis

isymmetric breakdown but not to effect the occurrence of the axisymmetric breakdown.

3.2 Birth and growing process of the bubble

Figures 6 (a)~(c) show the sequential photographs of the birth and growing process of axisymmetric breakdown with Reynolds number=1780, where the vane angle is changed from 30° to 35° during the period of 30 s. The period of dye injection is 0.5s. In the stationary state, the intervals of dye segments are all the same [Fig. 6 (a)]. After a change of the vane angle, a stagnation point appears in the flow field [Figs. 6 (b), (c)], and a bubble is being formed [Figs. 6 (d), (e)]. In these figures, the arrow shows the dye segment which remains at almost the same position throughout the growing process. From these figures, it is found that the dye segments which were injected after the arrow indicated a segment reverse upstream toward the stagnation point, but the segments injected before the arrow indicated one go far downstream. The streaklines obtained from the theoretical model are shown in Fig. 7. The primary flow in the calculation is assumed to be a rigidly rotating flow with

Fig. 6 Photographs of the growing process of a bubble visualized by intermittent dye injection

uniform axial velocity ($R_o=0.48$), and a step change with the nondimensional amplitude of 0.6 is applied to the azimuthal velocity at the entrance. In the comparison between Figs. 6 and 7, the theoretical model gives a qualitatively good explanation for the growing process of the bubble.

In the following, the transition of the bubble location is discussed. Figure 8 shows the variation of the bubble location during the growing process for several values of t_ψ , which is the interval when the vane angle is changed from 30° to 35° . In the experiment, the Reynolds number is 1740 and the suction flow rate is $q/Q=0.2$. The abscissa is the normalized axial location, and the ordinate is the time τ after the vane angle began to change. When the vane angle is changed rather slowly ($t_\psi=30$ or 20 s), the bubble appears relatively downstream and moves upstream toward the stationary position. When the vane angle is changed faster, on the other hand, the bubble appears

Fig. 7 Behavior of streak surfaces obtained from the theoretical model ($R_o=0.48$)

upstream, moves downstream for a while, and then turns toward the steady position. The meaning of these bubble movements is explained in section 5.

4. Numerical Analysis on Navier-Stokes Equations

In this section, the unsteady growing process of a bubble is examined numerically, and these results are compared with theoretical ones. Now, the basic equations for incompressible and viscous fluid are the Navier-Stokes (N-S) equation and the equation of continuity. The velocity components in the directions r , θ and z of cylindrical coordinates are denoted by u , v and w , respectively. All quantities are normalized by a pipe radius a , a mean axial velocity w_m and the representative time $t_0=2a/w_m$ as follows,

$$\left. \begin{aligned} U &= u/w_m, V = v/w_m, W = w/w_m, R = r/a, Z = z/a, \\ T &= t/t_0, \text{ Reynolds number } Re = 2aw_m/\nu \end{aligned} \right\} \quad (1)$$

We use the stream function Ψ and the vorticity η defined as

$$U = \frac{1}{R} \frac{\partial \Psi}{\partial Z}, W = -\frac{1}{R} \frac{\partial \Psi}{\partial R}, \eta = \frac{\partial U}{\partial Z} - \frac{\partial W}{\partial R} \quad (2)$$

Eliminating the pressure terms from the N-S equation, we obtain the basic three equations specified with variables Ψ , η and V :

$$\left. \begin{aligned} \frac{1}{2} \frac{\partial \eta}{\partial T} + \frac{1}{R} \frac{\partial \Psi}{\partial Z} \frac{\partial \eta}{\partial R} - \frac{1}{R} \frac{\partial \Psi}{\partial R} \frac{\partial \eta}{\partial Z} - \frac{\eta}{R^2} \frac{\partial \Psi}{\partial Z} - \frac{2V}{R} \frac{\partial V}{\partial Z} \\ = \frac{2}{Re} \left(\frac{\partial^2 \eta}{\partial R^2} + \frac{1}{R} \frac{\partial \eta}{\partial R} + \frac{\partial^2 \eta}{\partial Z^2} - \frac{\eta}{R^2} \right) \\ \frac{1}{2} \frac{\partial V}{\partial T} + \frac{1}{R} \frac{\partial \Psi}{\partial Z} \frac{\partial V}{\partial R} - \frac{1}{R} \frac{\partial \Psi}{\partial R} \frac{\partial V}{\partial Z} + \frac{V}{R^2} \frac{\partial \Psi}{\partial Z} \\ = \frac{2}{Re} \left(\frac{\partial^2 V}{\partial R^2} + \frac{1}{R} \frac{\partial V}{\partial R} + \frac{\partial^2 V}{\partial Z^2} - \frac{V}{R^2} \right) \\ \eta = \frac{1}{R} \left(\frac{\partial^2 \Psi}{\partial R^2} - \frac{1}{R} \frac{\partial \Psi}{\partial R} + \frac{\partial^2 \Psi}{\partial Z^2} \right) \end{aligned} \right\} \quad (3)$$

Fig. 8 Behavior of a bubble location

The numerical solutions of these equations are obtained by the method of finite differences (SOR). As the upstream boundary condition for Ψ and V , Leibovich's experimental formula⁽¹¹⁾ is applied for $0 \leq R \leq R_b$, and quadratic profile, which vanishes at the wall, is connected at $R = R_b$.

$$W = \begin{cases} W_1 + W_2 e^{-qR^2} & 0 \leq R \leq R_b \\ aR^2 + bR - a - b & R_b \leq R \leq 1 \end{cases} \quad (4)$$

$$V = \begin{cases} K(1 - e^{-qR^2})/R & 0 \leq R \leq R_b \\ a'R^2 + b'R - a' - b' & R_b \leq R \leq 1 \end{cases}$$

Furthermore, the next condition is applied to vorticity.

$$\frac{\partial U}{\partial Z} = CR(1 - R), \quad C > 0 \quad (5)$$

In the calculation, the azimuthal velocity at the entrance was increased gradually to some intensity, with which the occurrence of the bubble is expected. Namely,

$$K = \begin{cases} 0.95 + 0.6T & \text{for } 0 \leq T \leq 0.5 \\ 1.25 & \text{for } 0.5 \leq T \end{cases} \quad (6)$$

Figure 9 (a) shows the experimental result of a bubble, Fig. 9 (b) is the numerical result of the streak surface ($Re=100$, $T=2.7$) and Fig. 9 (c) an analytical result of the streak surface based on the inviscid linear theory ($T=105$). In comparing these results, the inviscid theoretical model explains the growing process of a bubble qualitatively.

5. Inviscid Linear Solution

In this section the results of the inviscid linear theory are compared with the experimental results.

5.1 Expression of internal wave

The inviscid linear model for axisymmetric breakdown in the former report⁽¹⁰⁾ is summarized here. The following assumptions are made: (i) Fluid is incompressible and inviscid. (ii) The steady flow without the disturbance (primary flow) is a uniform flow with rigid rotation. (iii) The disturbance is small as compared with the primary flow. According to the assumption (i), the basic equations are the Euler equation and the continuity equation. These equations are linearized with respect to the disturbance. Now, the solution of axisymmetric disturbance is put in the following complex form.

$$\begin{cases} \bar{u} = U(r)e^{\lambda t - imz}, \bar{v} = V(r)e^{\lambda t - imz} \\ \bar{w} = W(r)e^{\lambda t - imz}, \bar{p} = P(r)e^{\lambda t - imz} \end{cases} \quad (7)$$

where λ : eigenvalue ($\lambda = \alpha + in$, n : angular velocity), m : wave number in the axial direction ($m = \gamma + i\varepsilon$) and U , V , W and P are eigenfunctions of the radius r . From the assumption (iii), the primary flow is represented as,

$$\bar{u} = 0, \bar{v} = \Omega r, \bar{w} = w_m, \bar{p} = p_0 + \rho(\Omega^2/2)r^2 \quad (8)$$

where Ω is the angular velocity of the primary flow. In

the following, all quantities are represented in a dimensionless form:

$$\left. \begin{aligned} T &= \Omega t, R = r/a, Z = Z/a, u_n = u/(a\Omega) \\ v_n &= v/(a\Omega), w_n = w/(a\Omega), M = am, \Lambda = \lambda/\Omega \\ N &= n/\Omega, \text{ Rossby number } Ro = w_m/(a\Omega) \end{aligned} \right\} \quad (9)$$

The complex solutions are analytically obtained in a closed form as⁽⁶⁾

$$\left. \begin{aligned} \bar{u}_n &= C J_1(\sigma R) \cos(NT - MZ - \phi) \\ \bar{v}_n &= -C \{2/(N - MR_o)\} J_1(\sigma R) \sin(NT - MZ - \phi) \\ \bar{w}_n &= C(\sigma/M) J_0(\sigma R) \sin(NT - MZ - \phi) \end{aligned} \right\} \quad (10)$$

where

$$\sigma = M \{-1 + 4/(N - MR_o)^2\}^{1/2}$$

In the expression, C and ϕ are arbitrary constants for the amplitude and the phase of the disturbance. The value of σ is determined as the zeros of the Bessel function from the boundary condition at a pipe wall. Although there exists an infinite number of σ , we limit to the least-order value: $\sigma = \sigma_1 = 3.832$, because most phenomena appear in the lower mode. When the axial wave number M is a real number, eigenvalue is purely imaginary, $\lambda = iN$. The angular frequency N of disturbance is

(a) Photographs of a bubble

(b) Numerical streak surface ($Re=100$, $T=2.7$)

(c) Theoretical streak surface ($Ro=0.48$, $T=105$)

Fig. 9

$$N = MR_o \pm 2\sqrt{M^2/(\sigma^2 + M^2)} \quad (11)$$

The phase velocity C_p and group velocity C_g of disturbance are given by

$$\left. \begin{aligned} C_p &= N/M = R_o \pm 2/(\sigma^2 + M^2)^{1/2} \\ C_g &= \partial N/\partial M = R_o \pm 2\sigma^2/(\sigma^2 + M^2)^{3/2} \end{aligned} \right\} \quad (12)$$

As mentioned in the preceding paper⁽¹⁰⁾, the steady axisymmetric vortex breakdown corresponds to the stationary wave, whose phase velocity $C_p=0$ and the wave packet of the stationary wave due to the change of upstream boundary condition propagates with the group velocity C_g . Figure 10 shows the relationship between the wave number M and C_p or C_g . There are two different kinds of waves for the same value of M . The stationary wave exist in the lower mode.

In the following, the expressions for the unsteady flow field ($Z>0$) are introduced under the condition that an arbitrary change of azimuthal velocity is given at the upstream boundary. As the first step, we consider the wave propagation due to an impulsive velocity change at the upstream boundary. This flow field is represented in the form of superposing the internal wave. Next, the time T is divided into small intervals ΔT , and the amplitude of an impulse is approximated as the pulse with the constant amplitude during the interval ΔT . Then, superposing the pulse disturbance, the velocity at the time T_k is described as follows

$$\begin{aligned} \bar{v}_n(T_k, Z) &= \sum_{i=0}^{k-1} \frac{C_i}{2\pi} J_1(\sigma_1 R) \int_0^\infty \left[\frac{2}{N + MR_o} \sin\left\{ (N + MR_o) \frac{\Delta T}{2} \right\} \cos\left\{ (N + MR_o) \left(T_k - T_i - \frac{\Delta T}{2} \right) - MZ \right\} \right. \\ &\quad \left. + \frac{2}{N - MR_o} \sin\left\{ (N - MR_o) \frac{\Delta T}{2} \right\} \cos\left\{ (N - MR_o) \left(T_k - T_i - \frac{\Delta T}{2} \right) + MZ \right\} \right] dM \end{aligned} \quad (13)$$

where the amplitude C_i of the pulse applied during the interval ΔT is successively determined to satisfy the upstream boundary condition.

Fig. 10 Relation of phase velocity C_p and group velocity C_g to wave number M

5.2 Bubble growing process

The flow field is calculated from the above theoretical model, and its result is compared with the experimental result. The experimental result is shown in Fig. 11 when the vane angle is rapidly changed from 28° to 30° over 0.8s with $Re=1310$. Figure 11 (a) represents the azimuthal velocity at $r/a=0.3$ and Fig. 11 (b) is the axial velocity at the pipe axis for several axial positions. As seen in Fig. 5, a bubble in the steady flow field does not occur thus far, but there is a minimum axial velocity along the pipe axis which corresponds to a stationary wave of small amplitude. Figure 11 shows the dispersion and the propagation of the wave packet that takes place in the actual swirling flow. According to the step change of the vane angle, the velocity at $Z=3.3$ began to change at $t=14$ s, and became almost steady at $t=25$ s. Downstream, however, the beginning of the velocity change is later, and the wave form is more complex. Figure 12 represents the calculated transitions of the azimuthal and axial velocities of the disturbance for several axial positions for the step change of the azimuthal velocity at the upstream boundary ($R_o=0.5 < 2/\sigma_1$). From comparison of Fig. 11 with Fig. 12, it can be said that the dispersion and the propagation of the wave disturbance are in qualitative agreement.

Next, we consider the traveling of the bubble, as shown in the section 3 (Fig. 8). Figure 13 shows the theoretical result: the axial velocity along the pipe axis at various times when the step change is applied at the upstream boundary ($R_o=0.5$). The progressive waves, whose wavelengths are shorter than that of stationary wave, propagate faster than the stationary wave propagates. Because the amplitudes of these waves are large, a reversal flow occurs near the axis relatively early ($T=6$). This reversal flow travels downstream according to the propagation of the wave, and finally, the reversal flow region is fixed so as to correspond to the stationary wave. Thus, the bubble travels downstream and settles at the stationary position. Figure 14 (a) shows the streaklines on the R - Z plane in the same condition as in Fig. 13. As mentioned above, the bubble moves downstream. Figure 14 (b), on the other hand, shows the streaklines on condition that the upstream azimuthal velocity is changed slowly ($T_c=50$) up to the same velocity as Fig. 14 (a). In this case, the amplitudes of the wave disturbances except the stationary wave are relatively small so that the effects of the propagation and dispersion are not observed clearly, therefore a bubble takes place and grows at a certain axial position corresponding to the stationary wave.

Thus the downstream traveling of the bubble is qualitatively explained. However, the upstream

Fig. 11 Measured velocity components for several axial positions

Fig. 12 Calculated behavior of velocity components ($R_o=0.5$)

traveling cannot be explained from the theoretical model assuming the primary flow to be constant, i. e., $R_o = \text{constant}$. We guess that after the change in the upstream boundary condition, the Rossby number becomes small. From both numerical results on the N-S equation and the experimental results, the axial position of the bubble reaches more upstream as the upstream azimuthal velocity increases.

6. Conclusions

In this study, the birth and growing process of an axisymmetric vortex breakdown is investigated. By the method of an intermittent injection of dye, the growing process of a bubble is clearly visualized. The dispersion and propagation of the internal wave due to the change in the upstream boundary condition are measured by LDV. The theoretical results based on

Fig. 13 Axial velocity distribution along the axis for various times

Fig. 14 Comparison of growth of streak surface corresponding to the upstream boundary condition

the inviscid linear theory are compared with the experimental results and the results based on the N-S equation. Then the theoretical model explains the growing process of a bubble qualitatively. The travel of a bubble in relation to the upstream condition is discussed.

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