The Influence of the Oblique Shock Wave on the Small-Signal Gain Coefficient of CO₂ GDL Employing the Wedge Nozzle*

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When employing a wedge nozzle with the CO₂ gasdynamic laser (GDL), it is necessary to consider generating an oblique shock wave at the nozzle exit. The translational temperature of the gas increases behind the oblique shock wave. Therefore, the population inversion becomes extinct and the small-signal gain is decreased. The laser medium gas is reheated by passing through the oblique shock wave. In this paper, two-dimensional numerical simulation, which considers the influence of the oblique shock wave, is carried out. In the shock wave configuration, good agreement between the schlieren photograph and the result of the calculation is obtained. The experimental value of the small-signal gain coefficient agrees well with the calculated one.

Key Words: Fluid Mechanics, Numerical Analysis, Experimental Study, Two-Dimensional Calculation, CO₂ Gasdynamic Laser, Small-Signal Gain Coefficient, Wedge Nozzle, Oblique Shock Wave

1. Introduction

Various types of nozzles are employed for CO₂ GDL (CO₂ gasdynamic lasers). The authors have studied the application of a wedge nozzle to CO₂ GDL⁽¹⁾⁽²⁾. Compared with the other types of nozzles, the two-dimensional wedge nozzle has two significant advantages. First, the shape of this nozzle has a very simple form. Second, the laser medium gas through this nozzle can be expanded in a comparatively short distance. However, it is considered that an oblique shock wave is generated at the nozzle exit. When an oblique shock wave is generated, both the temperature and the pressure of the laser medium gas increase

behind the shock wave. Consequently, the population inversion of the laser medium gas is decreased. Previous studies have applied the two-dimensional wedge nozzle to CO₂ GDL⁽³⁾⁽⁴⁾; however, there have been very few studies which consider the influence of an oblique shock wave on the small-signal gain coefficient. Therefore, the authors here consider more intensively the influence of an oblique shock wave on the small-signal gain coefficient.

To consider the influence of the oblique shock wave generated at the exit of the wedge nozzle, the numerical simulation of a two-dimensional nonequilibrium flow was carried out in this study. The contour line of the gas density obtained by numerical simulation was similar to that shown in the schlieren photograph. This comparison verifies the propriety of the numerical simulation. Moreover, the experimental values of the small-signal gain coefficient were close to those obtained by numerical simulation.

Nomenclature

C_i : mass fraction of i species gas
 E : total energy per unit volume
 E_{vi} : vibrational energy of mode i per unit volume

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- G : small-signal gain coefficient
 h : Planck's constant
 k : Boltzmann's constant
 N_i : number density of i energy level
 p : pressure
 R_i : gas constant of i species gas
 T : translational temperature
 T_{vi} : vibrational temperature of mode i
 u : velocity in the direction of x
 v : velocity in the direction of y
 $\dot{W}_i$: production rate of vibrational energy of mode i per unit volume⁽⁵⁾
 x : distance in the direction of flow
 y : distance in the direction perpendicular to the flow
 Z : molecular collision frequency
 θ : opening angle of the wedge nozzle
 λ : wavelength of the CO₂ laser
 ν_i : vibrational frequency of i level
 ρ : gas density
 τ : radiative lifetime (=5.38 s)

2. Experimental Apparatus

2.1 Shock tube

The shock tube used in this study is shown in Fig. 1. This experimental apparatus consists of two diaphragm-bursting shock tubes, that is, the main shock tube and the subshock tube. The main shock tube is a uniform cross section shock tube with an inner diameter of 78 mm for both the driver section and the driven section. The subshock tube has a nonuniform cross section with an inner diameter of 78 mm for the driver section and 50 mm for the driven section. This experimental apparatus is designed not only for the mixing GDL but also for the premixed GDL. In the case of the premixed GDL experiment, the main shock tube is used. On the other hand, in the case of the mixing GDL, both the main shock tube and the subshock tube are used.

In this study, the experiment on the premixed GDL was carried out. Therefore only the main shock tube was used. He gas was used as a driver gas, and mixed gas of 10% CO₂-40% N₂-50% He in molar fraction was used as a test gas.

Fig. 1 Schematic diagram of the shock tube used in this study

2.2 Experimental setup for the flow visualization

A schematic diagram of the experimental setup for the flow visualization by means of the schlieren method is shown in Fig. 2. The incident shock Mach number and the stagnation pressure were measured by using two pressure gauges mounted in front of the nozzle. The magnesium arc discharge was used for the light source of the schlieren photograph. A synchronized shot of the spark was obtained by delaying the signal from the pressure gauge set at the driven section. The spark light from the light source was guided to the observation window with a diameter of 100 mm at the test section. After being passed through the test section, the light was reflected by the mirror, passed through the test section again, and then guided to the camera by the beam splitter. The visualization experiment was carried out only for the wedge nozzle having an opening angle of 60°.

2.3 Optical system of the small-signal gain coefficient

A schematic diagram of the measurement for the small-signal gain coefficient is shown in Fig. 3. The CO₂ probe laser used in this study was a discharge-pumped, continuous-wave oscillation laser and the wavelength was selected by using the grating. This probe laser was manufactured in our laboratory. In the process of producing the laser, the reduction of the fluctuation of the discharge voltage and the gas pressure in the laser tube were considered in order to obtain a stabilized laser power. After the probe laser beam was chopped, it was split into two directions by the ZnSe beam splitter. One of the probe laser beams was guided to the ZnSe window set on the test section. The wavelength of the other laser beam was monitored by a monochromator. The probe laser beam passed through the test section was detected by the HgCdTe infrared detector, and the small-signal gain coefficient was measured. The specific detectivity, D^* , of the infrared detector used in this study was $10^{10}(\text{cm} \cdot \text{Hz}^{1/2})$.

Fig. 2 Schematic diagram of the flow visualization experiment

/W). The optical system used for the measurement of the small-signal gain coefficient was set up on the optical flat bench which isolated the vibration of the shock tube by air suspension.

The experimental value of the small-signal gain coefficient was calculated according to the equation given below.

$$G = \frac{1}{L} \ln \left(\frac{I_0 + \Delta I}{I_0} \right) \quad (1)$$

3. Numerical Analysis

3.1 Simulation model

The simulation model in this study was based on the following assumptions.

- (1) Gas is thermally perfect.
- (2) Dissociation, ionization and reaction do not occur in the flow.
- (3) The flow is a two-dimensional nonviscous compressible flow.
- (4) Vibration of the CO₂ and N₂ molecules is that of a harmonic oscillator. Each vibrational mode has its characteristic vibrational temperature. The molecules are assumed to be rigid body rotators.
- (5) The translational and the rotational motion are in equilibrium states.
- (6) In each vibrational mode, the local equilibrium state is assumed because of the rapid energy transfer by molecular collisions.
- (7) In the vibrational energy level system of CO₂-N₂, the two-mode approximation, which assumes local equilibrium between CO₂ [100] and CO₂ [020], CO₂ [001] and N₂ [$\nu=1$], is adopted.

3.2 Equations

In this paper, numerical analysis was carried out

in the region of the nozzle inlet to the duct in which a laser cavity was set. On the basis of the simulation model mentioned in § 3.1, equations of vibrational nonequilibrium flow are given as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (2)$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + p)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = 0 \quad (3)$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2 + p)}{\partial y} = 0 \quad (4)$$

$$\frac{\partial E}{\partial t} + \frac{\partial u(E + p)}{\partial x} + \frac{\partial v(E + p)}{\partial y} = 0 \quad (5)$$

$$\frac{\partial E_{vI}}{\partial t} + \frac{\partial(uE_{vI})}{\partial x} + \frac{\partial(vE_{vI})}{\partial y} = \dot{W}_I \quad (6)$$

$$\frac{\partial E_{vII}}{\partial t} + \frac{\partial(uE_{vII})}{\partial x} + \frac{\partial(vE_{vII})}{\partial y} = \dot{W}_{II} \quad (7)$$

$$p = \rho RT \quad (8)$$

Equation (2) is the equation of continuity. Equations (3) and (4) are equations of momentum in the directions of x and y , respectively. Equation (5) is the equation of energy conservation. Equations (6) and (7) are the rate equations of mode I and mode II, respectively. Equation (8) is the equation of state.

The vibrational energy of mode I and mode II in the two-mode model is shown as follows⁽⁵⁾:

$$E_{vI} = \rho C_v R_c \left\{ (h\nu_1/k) [\exp(h\nu_1/kT_{vI}) - 1]^{-1} + (2h\nu_2/k) [\exp(h\nu_2/kT_{vI}) - 1]^{-1} \right\} \quad (9)$$

$$E_{vII} = \rho C_v R_c \left\{ (h\nu_3/k) [\exp(h\nu_3/kT_{vII}) - 1]^{-1} + \rho C_N R_N (h\nu_N/k) [\exp(h\nu_N/kT_{vII}) - 1]^{-1} \right\} \quad (10)$$

The small signal gain coefficient in P(20) transition of CO₂ laser is approximately expressed as below⁽⁵⁾.

$$G = \frac{\lambda^2}{4\pi\tau Z} (N_{001} - N_{100}) \left(\frac{45.6}{T} \right) e^{-234/T} \quad (11)$$

MacCormack's scheme was employed, and the asymptotic solution was obtained by the time-dependent method. The convergence condition was indicated as follows:

$$\frac{U^{n+1} - U^n}{\Delta t} < \varepsilon,$$

where U : physical quantity, ε : standard of judgement, and Δt : time step. Because of the restraint of fluctuation and divergence in the numerical calculation by the shock wave, an artificial viscosity is used.

The 80×20 mesh system was used for the nozzle region calculation, and the 80×21 mesh system was used for the duct region. The regions of only half of the nozzle part and duct part were calculated because of the advantage of symmetry with respect to the x axis. Boundary conditions were given by the following two conditions: a slip-wall condition on a wall and symmetric condition on a center line. As an upstream boundary, the backward linear extrapolation was applied to the velocity of the x direction, and other physical quantities were fixed. A linear extrapolation was applied to the downstream boundary. The calcu-

Fig. 3 Schematic diagram of the experiment of the small-signal gain coefficient measurement

lation of the nozzle part was carried out by using a coordinate transformation which transforms a physical plane into a rectangular plane.

4. Results and Discussion

4.1 Flow in the nozzle

The contour line of the gas density on the 60° wedge nozzle is shown in Fig. 4. The gas density changes substantially at the nozzle throat and becomes sparse as a result of the rapid expansion at the downstream of the nozzle throat. In Figs. 5 and 6, the temperatures and the small-signal gain coefficient from the nozzle inlet are shown, respectively. From Fig. 5, it can be seen that the state between mode I and

Fig. 4 Contour line of the gas density in the 60° wedge nozzle

Fig. 5 Temperatures in the 60° wedge nozzle

Fig. 6 Small-signal gain coefficient in the 60° wedge nozzle

mode II is already in a nonequilibrium state at the throat. The population inversion is generated at about 20 mm downstream from the nozzle throat.

4.2 Flow in the duct

The distribution of the gas density in the duct, which is set downstream from the 60° wedge nozzle, is shown in Figs. 7(a) and (b). Figure 8 shows the photograph of the flow from the 60° wedge nozzle to the duct obtained by the schlieren method. Figure 9 shows the comparison between the contour line of the gas density and the schlieren photograph. As can be seen from this comparison, good agreement was obtained on the shock wave configuration between the

Fig. 7 Distribution of the gas density in the duct ($\theta=60^\circ$)

Fig. 8 Schlieren photograph ($\theta=60^\circ$)

Fig. 9 Comparison between the contour line of the gas density and the schlieren photograph

schlieren photograph and the numerical result of the present study.

The distribution of the translational temperature and the small-signal gain coefficient in the duct are shown in Figs. 10 (a) and (b) and Figs. 11(a) and (b). The gas passing through the nozzle is continuously expanded in the duct. However, the gas density and the translational temperature of the gas which is passed across the oblique shock wave increase because the gas is compressed and heated by the oblique shock wave generated at the nozzle exit. Consequently, the population inversion is decreased and the small-signal gain coefficient is reduced from about 0.8 m^{-1} to about 0.3 m^{-1} across the oblique shock wave. Figures 12 and 13 show the contour line of the gas density and the small-signal gain coefficient on the duct which is set downstream from the 26° wedge nozzle. In comparison with the case of the 60° wedge nozzle, the angle between the oblique shock wave and the duct wall is larger than that of the 26° wedge nozzle. In the case of the 60° wedge nozzle, the

gas density increases to about 2.3 times in average across the oblique shock wave. However, in the case of the 26° wedge nozzle, the gas density increases to about 1.8 times. The amount of change in the gas density of the 60° wedge nozzle is larger than that of the 26° wedge nozzle. In terms of the pressure and other quantities, similar results were obtained.

In Fig. 14, the result of the two-dimensional calculation in the case of the 60° wedge nozzle is compared with the quasi-one-dimensional calculation and the experimental data on the small-signal gain coefficient measured in the direction of the y axis. Because the quasi-one-dimensional calculation does not consider the oblique shock wave generated at the nozzle exit, the small-signal gain coefficient increases monotonously within the range of 140 mm from the nozzle exit. On the other hand, the small-signal gain coefficient obtained from the result of the two-dimen-

Fig. 10 Distribution of the translational temperature in the duct ($\theta=60^\circ$)

Fig. 11 Distribution of the small-signal gain coefficient in the duct ($\theta=60^\circ$)

Fig. 12 Contour line of the translational temperature in the duct ($\theta=26^\circ$)

Fig. 13 Contour line of the small-signal gain coefficient in the duct ($\theta=26^\circ$)

Fig. 14 Variation of the small-signal gain coefficient downstream from the nozzle exit

Fig. 15 Variation of the small-signal gain coefficient downstream from the nozzle exit ($\theta=26^\circ$ and $\theta=60^\circ$)

Fig. 16 Distribution of the small-signal gain coefficient in the duct calculated by the 80×41 mesh system ($\theta=60^\circ$)

sional calculation in the present study shows the greatest value at the point 40 mm downstream from the nozzle exit and decreases with the distance from the nozzle exit. This tendency arises from the consideration of the influence of the oblique shock wave. Agreement between the result of the two-dimensional calculation and that of the experiments was obtained, as shown in Fig. 14. The values of the small-signal gain coefficient on both the 26° and the 60° wedge nozzle from the two-dimensional calculation taken in the direction of the y axis are shown in Fig. 15. Comparing the two wedge nozzles, we find that the oblique shock wave is weaker and the influence on the small-signal gain coefficient is smaller in the case of the 26° wedge nozzle.

In the 60° wedge nozzle, calculation of the duct region using the 80×41 mesh system was carried out. In this mesh system, the width of the shock wave was narrower than that of the 80×21 mesh system, and the shock configuration agreed with the schlieren photograph. Because the result of the small-signal gain coefficient using the 41 mesh system compared to that using the 21 mesh system is nearly the same, it is thought that the value of the small-signal gain coefficient using the 21 mesh system is reasonable. The distribution of the small-signal gain coefficient is

shown in Fig. 16. The change of the small-signal gain coefficient was expressed clearly.

5. Conclusions

Numerical simulation of the flow in the two-dimensional wedge nozzle and on the flow including the oblique shock wave generated in the duct was carried out. The results from the calculation were compared with those from the flow visualization and the experimental data. The following conclusions were obtained.

(1) The result of the two-dimensional calculation in the case of the 60° wedge nozzle was compared with the schlieren photograph. Good agreement between them was obtained on the shock wave configuration.

(2) From the result of the two-dimensional calculation, the value of the small-signal gain coefficient in the case of the 60° wedge nozzle showed the maximum value at 40 mm from the nozzle exit and decreased monotonously in the downstream. The calculated results of the small-signal gain coefficient agreed with the experimental values, as shown in Fig. 14.

(3) Comparing the calculated result of the 60° wedge nozzle with that of the 26° wedge nozzle, we found that the influence of the oblique shock wave on the small-signal gain coefficient was significant in the case of the 60° wedge nozzle, as shown in Fig. 15.

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