An Experiment on the Pulsatile Flow of Bingham Plastic Fluids in a High Reynolds Number Region* (Influence of the Pulsatile Amplitude)

Masahide NAKAMURA** and Tadashi SAWADA**

To evaluate the influence of pulsatile amplitude, experiments on pulsatile flow of Bingham plastic fluids are performed under the conditions that the time-averaged Reynolds number of a cycle is greater than the critical Reynolds number in the steady state. On the other hand, calculations based on the quasi-steady approximation are performed. These experimental and calculational results show that the characteristic parameters of the pulsatile flow ($\Phi_{t,1}$ and $\Phi_{z,1}$) obtained from the experiments become greater than the ones obtained from the quasi-steady approximation in a large-amplitude region. Next, the causes of this difference are investigated, and it is suggested that the acceleration parameter plays an important role in explaining the cause of this difference. Finally, the influence of the Hedström number is evaluated, and it is suggested that the change of the turbulent viscosity coefficient due to the acceleration becomes notable with the increase in the Hedström number.

Key Words: Non-Newtonian Fluid, Unsteady Flow, Pulsatile Amplitude, Acceleration Parameter, Plastic Fluid

1. Introduction

As the pulsatile flow of Bingham plastic fluids in a high Reynolds number region is closely related to slurry transport or blood flow in an aorta, this pulsatile flow is expected to be very important from practical viewpoints. However, little information is available with respect to this flow, and it is necessary to perform a fundamental study. Hence, the authors began a study on the pulsatile flow of Bingham plastic fluids in a high Reynolds number region. The effects of the angular frequency on the nature of the small-amplitude pulsatile flow of Bingham plastic fluids were reported in earlier papers⁽¹⁾⁽²⁾, but large-amplitude pulsatile flow of Bingham plastic fluids is encountered in everyday life (for example, in blood flow in an aorta), and it is easily expected that the pulsatile amplitude has a direct influence on the nature of the pulsatile flow.

In this paper, an experiment on the pulsatile flow

of Bingham plastic fluids is performed, and the effect of the pulsatile amplitude is investigated. Since the main target of this study is the high Reynolds number flow, an experiment is performed under the condition that the time-averaged Reynolds number is greater than the critical Reynolds number in the steady-state. On the other hand, the calculation based on the quasi-steady approximation is performed, and the results are compared with the experimental data. On the basis of these comparison results, the validity of the quasi-steady approximation is considered. Moreover, the characteristics of the experimental results, the effect of the Hedström number (non-Newtonian effect) on the experimental results, and the effect of the pulsatile amplitude on the time-averaged flow rate⁽³⁾⁽⁴⁾ in the large-amplitude and high Reynolds number regions are considered.

2. Nomenclature

a : relative plug radius
 d : diameter of the capillary tube (=2.0 mm)
 D : diameter of the test section (=50.0 mm)
 He : Hedström number, $He = \rho p_y D^2 / \mu_b^2$
 I : change of the mean axial velocity due to the

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**Department of Mechanical Engineering, Akita University, 1-1 Tegatagakuen-cho, Akita, 010, Japan

- pulsation, see Eq. (9)
- K : acceleration parameter, see Eq. (8)
- p : pressure
- p_y : yielding stress
- Q : flow rate in the capillary tube
- R_e : Reynolds number, $R_e = \rho \bar{u}_s D / \mu_B$
- R_{eB} : generalized Reynolds number, see Eq. (6)
- $\bar{u}$: mean axial velocity
- z : axial direction
- Δu^* : non-dimensional amplitude of pulsation, see Eq. (1)
- $\Delta \bar{u}_p$: amplitude of the mean axial velocity
- $\Delta \tau_{wp}$: amplitude of the wall shear stress
- ε : constant, see Eq. (3)
- λ : frictional coefficient, see Eq. (5)
- μ_B : plastic viscosity
- ν_B : $\nu_B = \mu_B / \rho$
- ρ : density
- τ_w : wall shear stress
- $\Phi_{t,1}, \Phi_{z,1}$: characteristic parameters, see Eq. (1)
- ω : angular frequency
- ω^* : non-dimensional angular frequency,
 $\omega^* = D/2 \cdot \sqrt{\omega/\nu_B}$
- $\langle \rangle$: time average in a cycle
- subscripts**
- p : pulsation
- s : steady-state

3. Experimental Methods and Approximated Analysis

3.1 Experimental apparatus and methods

As the experimental apparatus and methods used to obtain the nature of the pulsatile flow and the rheological parameters of test fluids are the same as the ones used in the earlier paper⁽¹⁾, only brief explanations are presented here. The rheological parameters of test fluids are determined by the modified capillary viscometer⁽¹⁾, and kaolin clay slurries are used in this study. Figure 1 shows an example of the flow curves of kaolin clay slurry. This figure shows that the fluid can be considered as a Bingham plastic fluid.

Next, the experimental apparatus for the pulsatile flow is explained. The pulsation is generated by the piston which is driven by the scotch-yoke mechanism. So, the motion of the piston can be approximated very well by the sinusoidal curve, and the pulsatile amplitude can be changed easily by the piston stroke. The pressure gradient and the mean axial velocity are measured by the variable inductance-type pressure transducer and the electromagnetic flowmeter, respectively. As the calibration of these experimental apparatus is performed in the range of frequency less than 1.4 Hz⁽¹⁾, this study is also performed within this fre-

quency limit. It is easily expected that the generation of deposit in a pipe makes the nature of the pulsatile flow complex. The present experiments are performed under the conditions that the generation of a deposit in a pipe is insignificant, and the effect of a deposit is beyond the scope of this study. As the solid particles contained in the kaolin clay slurry are difficult to precipitate to generate a deposit, this fluid is suitable for the present study.

Experiments on the pulsatile flow of Newtonian fluid in a high Reynolds number region have already been performed by Ohmi et al.⁽⁵⁾ and Ohmi and Iguchi⁽⁶⁾, and they have shown that the non-dimensional amplitude Δu^* does not have a strong effect on the characteristic parameters $\Phi_{t,1}$ and $\Phi_{z,1}$, where Δu^* , $\Phi_{t,1}$ and $\Phi_{z,1}$ are defined as:

$$\Delta u^* = \frac{\Delta \bar{u}_p}{\bar{u}_s}, \quad \Phi_{t,1} = \frac{\rho \omega \Delta \bar{u}_p}{|\varepsilon (\partial p / \partial z)_s|},$$

$$\Phi_{z,1} = \frac{4 \Delta \tau_{wp} / D}{|\varepsilon (\partial p / \partial z)_s|}. \quad (1)$$

For the verification of the experimental apparatus, a preliminary experiment is performed using a Newtonian fluid (water). Results are given in Fig. 2, which shows that the values of $\Phi_{t,1}$ and $\Phi_{z,1}$ are almost constant and lie near the results of Ohmi and Iguchi⁽⁶⁾. These results confirm the validity of this experimental apparatus.

3.2 Quasi-steady approximation

The basic equation is obtained from the integration of Hencky's equation⁽⁷⁾ over the cross section of a pipe.

$$\rho \frac{d\bar{u}_p}{dt} = - \left(\frac{\partial p}{\partial z} \right)_p - \frac{4}{D} \tau_{wp} \quad (2)$$

Pressure gradient is written as:

$$(\partial p / \partial z)_p = (\partial p / \partial z)_s (1 + \varepsilon \sin \omega t). \quad (3)$$

This equation can be understood by considering that the value of $\langle (\partial p / \partial z)_p \rangle$ is almost the same as $(\partial p / \partial z)_s$ in the present study.

It is easily expected that the flow pattern is

Fig. 1 Flow curve of kaolin clay slurry

strongly affected by the ratio of these three terms in Eq. (2) $[\rho d\bar{u}_p/dt, -(\partial p/\partial z)_p, 4\Delta\tau_{wp}/D]$, and the non-dimensional parameters which denote the ratio of these three terms should be introduced. For these reasons, the non-dimensional parameters $\Phi_{t,1}$, $\Phi_{z,1}$ are introduced, and $\Phi_{t,1}$ denotes the ratio of the inertial term $(\rho d\bar{u}_p/dt)$ to the pressure gradient term $-(\partial p/\partial z)_p$ while $\Phi_{z,1}$ denotes the ratio of the viscous term $(4\Delta\tau_{wp}/D)$ to the pressure gradient term⁽⁵⁾.

The calculation method of τ_{wp} has not yet been established, but the wall shear stress in the steady-state τ_{ws} can be obtained by the following equations⁽⁷⁾.

$$\tau_{ws} = 1/8 \cdot \lambda_s \cdot \rho \bar{u}_s^2 (1 - a_s) \quad (4)$$

$$\lambda_s = \begin{cases} 0.3164 R_{eBS}^{-1/4} & (\text{turbulent flow}) \\ 64/R_{eBS} & (\text{laminar flow}) \end{cases} \quad (5)$$

$$R_{eBS} = \frac{\rho \bar{u}_s D}{\mu_B} \cdot \frac{a_s^4 - 4a_s + 3}{3} \cdot (1 - a_s) \quad (6)$$

$$\frac{a_s^4 - 4a_s + 3}{12a_s} = \frac{2\bar{u}_s \mu_B}{D \rho \bar{u}_p} \quad (7)$$

The validity of these equations becomes poor under the unsteady conditions, but the analysis is developed on the basis of the assumption that these equations are effective under the unsteady conditions (quasi-steady approximation). One of the purposes of this study is to evaluate the validity of this approximation. These equations can be solved easily by using the Runge-Kutta-Gill method.

The calculated results are summarized as follows:

- (1) Non-dimensional amplitude Δu^* does not have a strong effect on $\Phi_{t,1}$ and $\Phi_{z,1}$, and the values of $\Phi_{t,1}$ and $\Phi_{z,1}$ are almost constant (see Figs. 4 and 5).
- (2) The increase in the non-dimensional amplitude Δu^* causes a decrease in the time-averaged

Fig. 2 Relationship among $\Phi_{t,1}$, $\Phi_{z,1}$ and Δu^* (Newtonian fluid)

mean axial velocity $\langle \bar{u}_p \rangle$ compared with the mean axial velocity without the pulsation $\bar{u}_s$ (see Fig. 7). These results are similar to the ones for the Newtonian fluid⁽⁵⁾⁽⁶⁾, but it should be noted that these results are obtained from the quasi-steady approximation. These results should be compared with the experimental results and the validity of this approximation is examined in the following section.

4. Results and Discussion

An example of the measured results is given in Fig. 3, which shows the change in the mean axial velocity $\bar{u}_p$ and pressure gradient $(\partial p/\partial z)_p$. The pressure gradient and the mean axial velocity are measured by the variable inductance-type pressure transducer and the electromagnetic flowmeter, respectively. This figure shows that the change in these values can be approximated very well by the sinusoidal curve, and high-frequency fluctuation is not found in the signal of $\bar{u}_p$.

Figure 4 shows the results concerning the small amplitude and low-frequency pulsatile flow, and Fig. 5 shows those for the large amplitude and high-frequency pulsatile flow. Comparison of these figures yields the following findings.

- (1) Nondimensional amplitude Δu^* has a strong effect on $\Phi_{t,1}$ and $\Phi_{z,1}$, and the increase in Δu^* increases the experimental results of $\Phi_{t,1}$ and $\Phi_{z,1}$.
- (2) Quasi-steady approximation is effective in the low Δu^* region, but the experimental results of $\Phi_{t,1}$ and $\Phi_{z,1}$ become greater than the calculated ones based on the quasi-steady approximation in the high Δu^* region.

The quasi-steady approximation is effective under the condition that the unsteadiness effect is weak, as it is in the low Δu^* region. Figures 4 and 5 are consistent with this consideration. It is well known that the turbulent viscosity coefficient decreases in the acceleration phase and increases in the deceleration

Fig. 3 Waveform of the mean axial velocity and the pressure gradient ($H_e = 1.9 \times 10^5$, $R_e = 2.7 \times 10^4$, $\omega^* = 5.5 \times 10^2$, $\Delta u^* = 0.52$)

phase. Hence, the decreased turbulent viscosity coefficient promotes the acceleration of the flow in the acceleration phase and the increased turbulent viscosity coefficient promotes the deceleration of the flow in the deceleration phase. Therefore, it is expected that the change in the turbulent viscosity coefficient due to the acceleration of the flow increases the amplitude of

axial velocity and wall shear stress. This explanation is given in the previous paper⁽²⁾ by the authors, and it is clear that this effect becomes marked with the increase in the acceleration of the flow. The increase in $\Phi_{t,1}$ and $\Phi_{z,1}$ is equivalent to the increase in the pulsatile amplitude of the mean axial velocity and wall shear stress, and it is clear that the increase in

Fig. 4 Relationship among $\Phi_{t,1}$, $\Phi_{z,1}$ and Δu^* (Bingham plastic fluid, low frequency and small amplitude)

Fig. 5 Relationship among $\Phi_{t,1}$, $\Phi_{z,1}$ and Δu^* (Bingham plastic fluid, high frequency and large amplitude)

the pulsatile amplitude increases the acceleration of the flow. On the basis of these considerations, the experimental results mentioned above can be explained qualitatively.

Next, the effects of the non-dimensional angular frequency ω^* and Reynolds number Re are discussed. Experimental results are shown in Fig. 6, and the calculated results based on the quasisteady approximation are given in Table 1. Calculated results are not included in Fig. 6 for the sake of readability. The following findings are obtained from Table 1 and Fig. 6.

(1) The effects of the large ω^* on $\Phi_{t,1}$ and $\Phi_{z,1}$ qualitatively resemble the effects of the small Re on $\Phi_{t,1}$ and $\Phi_{z,1}$.

(2) The conditions that increase the difference between the experimental results and the calculated results based on the quasi-steady approximation are: i) increase in ω^* , ii) decrease in Re , iii) increase in Δu^* .

In case of the Newtonian fluid, Ohmi and Iguchi⁽⁶⁾ indicate that the effects of the large ω^* on $\Phi_{t,1}$ and $\Phi_{z,1}$ resemble the effects of the small Re on $\Phi_{t,1}$ and $\Phi_{z,1}$. Finding (1) means that this indication is also qualitatively effective in the case of the Bingham plastic fluid. The effects of ω^* on $\Phi_{t,1}$ and $\Phi_{z,1}$ are investigated in the earlier paper⁽¹⁾⁽²⁾ and the following parameter (acceleration parameter) is used to explain the difference between the experimental results and the calculated ones based on the quasi-steady approximation⁽²⁾:

$$K = \frac{\nu_B}{\bar{u}_p^3} \cdot \frac{d\bar{u}_p}{dt} \quad (8)$$

This equation implies that the value of K increases with i) increase in Δu^* , ii) increase in ω^* , iii) decrease in Re . These conditions agree with the conditions that increase the difference between the experimental results and the calculated results based on the quasi-steady approximation mentioned above. Thus, the validity of this parameter is also shown qualitatively from the present results. The importance of the acceleration parameter or the amplitude has already been suggested by Iguchi and Ohmi⁽⁸⁾, but these parameters do not play an important role within the present experimental conditions in the case of the Newtonian fluid⁽⁵⁾⁽⁸⁾. The present study shows that the non-Newtonian property changes the role of these parameters significantly.

Table 1 Calculated results of $\Phi_{t,1}$ and $\Phi_{z,1}$ based on the quasi-steady approximation

	1	2	3
He	5.1E5	5.1E5	5.1E5
Re	2.7E4	2.7E4	3.9E4
ω^*	8.9E2	5.7E2	8.9E2
Δu^*	0.10-0.70	0.10-0.70	0.10-0.70
$\Phi_{t,1}$	1.00-1.07	1.00-1.05	0.97-1.00
$\Phi_{z,1}$	0.10-0.12	0.15-0.17	0.13-0.15

$$(5.1E5 = 5.1 \times 10^5)$$

Fig. 6 Influence of ω^* and Re on $\Phi_{t,1}$ and $\Phi_{z,1}$ ($He=5.1 \times 10^5$)

The change in the time-averaged flow rate due to the pulsation is discussed. The following parameter is defined⁽³⁾⁽⁴⁾.

$$I = 100(\langle \bar{u}_p \rangle - \bar{u}_s) / \bar{u}_s \quad (9)$$

One example of this value is shown in Fig. 7. The experimental value of I is indicated as almost constant and equal to zero, whereas the calculated results based on the quasi-steady approximation decrease with the increase in Δu^* . This figure means that the quasi-steady approximation is also effective in the low Δu^* region. Within the limits of the present studies, a large part of the experimental results and the calculated results shows the same tendency as this figure. The physical meaning of this result is not clear, and the explanation is left as a subject for future study.

The effect of the Hedström number expressing the non-Newtonian flow characteristics is given by Figs. 6 and 8. The nondimensional parameters of Fig. 8 are almost the same as the ones of Fig. 6 except the Hedström number. Figures 6 and 8 show that the change in $\Phi_{t,1}$ and $\Phi_{z,1}$ becomes great with the increase in Hedström number. Thus, it is suggested that the change in the turbulent viscosity coefficient due to the acceleration is affected by the non-Newtonian effect and becomes marked with the increase in the non-Newtonian effect. This result is consistent with those of earlier studies⁽¹⁾⁽²⁾.

Lastly, the pulsatile blood flow in an aorta is discussed qualitatively as a concrete application of this study. The measurements of blood flow in an aorta are performed by many researchers⁽⁹⁾, and it is

shown that the intensity of the turbulent fluctuation in the acceleration phase is weaker than that in the deceleration phase. As mentioned above, these characteristics of the blood flow are the causes of the increase in $\Phi_{t,1}$ and $\Phi_{z,1}$, and the increase in $\Phi_{t,1}$ and $\Phi_{z,1}$ is equivalent to the increase in the amplitude of the mean axial velocity and the increase in the amplitude of the wall shear stress. Therefore, it is suggested that these parameters of blood flow in an aorta lie in the conditions that increase the amplitude of the mean axial velocity and wall shear stress. The role of the blood flow parameters in an aorta has not yet been clarified, but there is a theory that the pulsation plays an important role for the maintenance of life⁽¹⁰⁾. If this theory can be recognized, the results obtained above

Fig. 7 Change of the time-averaged flow rate due to the pulsation

Fig. 8 Influence of ω^* and Re on $\Phi_{t,1}$ and $\Phi_{z,1}$ ($He = 1.5 \times 10^5$)

seem to be favorable the maintenance of life. Of course, the present suggestion is only a hypothesis that explains the role of the blood flow parameters in an aorta, and more detailed studies should be performed in the future.

5. Conclusions

The experimental results and the calculated ones based on the quasi-steady approximation are summarized as follows:

(1) The increase in the pulsatile amplitude increases the value of $\Phi_{t,1}$ and $\Phi_{z,1}$, and this result cannot be explained by the quasi-steady approximation. The calculated $\Phi_{t,1}$ and $\Phi_{z,1}$ based on the quasi-steady approximation lie near the experimental results when the value of Δu^* is small.

(2) The conditions that increase the difference between the experimental results and the calculated results based on the quasi-steady approximation are i) increase in ω^* and Δu^* , ii) decrease in Re . These results show that the acceleration parameter K plays an important role in explaining the cause of this difference.

(3) The increase in the Hedström number makes the change in $\Phi_{t,1}$ and $\Phi_{z,1}$ with Δu^* marked. It is thus suggested that the change in the turbulent viscosity coefficient due to the acceleration also becomes marked with the increase in the non-Newtonian effect.

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