Temperature and/or Density Measurements of Asymmetrical Flow Fields by Means of the Moire-Schlieren Method*

(Method of Transformation of Moire-Data and Its Application to the Temperature Measurement of a Combustion Flame from a Rectangular Burner)

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A mathematical method for transforming moire-schlieren data into internal density (and/or temperature) distributions has been presented. The method of transformation, which is applicable to both symmetrical and asymmetrical flow fields, has been deduced by modifying the Maldonado-Olsen's series. The accuracy of the present method was examined with a mathematical model of a density function of the truncated pyramid type. In order to demonstrate its usefulness for asymmetrical flows, temperature distributions in a combustion flame have been measured for a square nozzle. Experimental moire images of the flame were recorded by a video camera, and the integrated light refraction was converted into an internal temperature distribution. The results showed that the temperature profiles in the flame are distinctive owing to the rectangular nozzle, and that the present method is very effective.

Key Words: Flow Measurement, Flow Visualization, Flame, Thermodynamics, Moire Method, Temperature Measurements, Asymmetrical Flow, CT Method

1. Introduction

Development of flow visualization techniques in recent years has facilitated the efficient measurement of velocity, density and temperature distributions in complicated flow fields such as three-dimensional ones. The optical method with laser light is especially advantageous because it does not disturb the flow fields to be measured⁽¹⁾. The moire-schlieren method is one such technique. With this method, one can easily visualize flow fields and measure their density (and/or temperature) distributions. The method is, in some points, superior to other interferometric methods such as the Machzender interferometer; that is, the measuring system is simple, easy to operate and inexpensive. Thus the method has been widely used to investigate two-dimensional or axisymmetric flow fields such as a combustion flame from a circular burner⁽²⁾, a plasma jet from a circular nozzle⁽³⁾, a high-

density plasma⁽⁴⁾ and a supersonic flow around a circular cone⁽⁵⁾.

The moire method is based upon the fact that the deflection of moire fringes is proportional to the integral of the density gradient in the direction normal to an observed light path. Therefore, to obtain the density profile in the flow, the deflection must be converted into the density by an appropriate transformation. For an axially symmetric flow, such a transformation can be achieved by applying the Abel transformation to the moire data⁽⁶⁾. For an asymmetrical flow, the density must be calculated by a more general transformation method for an integrated equation. The transformation method to convert interferometer fringe shifts to the internal density distribution is well accepted⁽⁷⁾. However, it should be modified when applied to the moire method, because the shift of interference fringes is proportional to the integral of density along the observation path, while that of moire fringes is proportional to the integration of density gradient.

In this paper, a mathematical method to convert moire data into the internal density profiles in asymmetric flows and its application to the temperature

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measurement of a combustion flame from a rectangular burner are presented. The transformation method has been deduced by using an orthogonal polynomial expression previously developed by Maldonado and Olsen for the conversion of integrated intensity fields⁽⁸⁾. The validity of the present method has been examined with a mathematical model of a density function of a truncated pyramid type, and its usefulness has been demonstrated by measuring temperature distributions in a combustion flame from a rectangular burner. Experimental moire images of the flame produced by He-Ne laser light were recorded by a video camera at every angle of its incidence, and then the data were transformed into density profiles, and finally into temperature ones. The results have showed that the temperature profiles in the flame have a fairly complex character, caused by the square nozzle, and that the present transformation method is very useful.

2. Theory

2.1 Moire method

When two grids aligned with a pitch d are put one above the other with a small angle θ , a moire fringe pattern with a spacing of d/θ appears. The grid lines and the moire fringes intersect almost at right angles, and the fine space of the grid is magnified to match that of the moire fringes. Using this characteristic, one can accurately measure the refractive angle of light passing through fluids. Figure 1 illustrates the principle of the moire method. The deflection of moire fringes for a refractive angle ψ is given by

$$w = \psi \Delta / \theta \quad (1)$$

where Δ is the distance between the grids G_1 and G_2 . The angle ψ is the integral of the gradient of refractive index n along an observation light path x' . Therefore, for the rectangular coordinate (x', y') , we have

$$\psi = \int_{-\infty}^{\infty} \frac{\partial n}{\partial y'} dx' \quad (2)$$

Generally, the refractive index n of gas is very close to unity, and can be expressed as $n = 1 + \beta\rho$, where β corresponds to Gladstone-Dale's coefficient. From Eqs. (1) and (2), we obtain

Fig. 1 Principle of the moire method

$$w = \frac{\beta \Delta}{\theta} \int_{-\infty}^{\infty} \frac{\partial \rho}{\partial y'} dx' \quad (3)$$

The inverse transformation of the moire deflection w in Eq. (3) into the density ρ will be shown in the next section. For the density ρ thus obtained, gas temperature T may be calculated by

$$T = P / R_c \rho \quad (4)$$

where P is the gas pressure and R_c is the gas constant.

2.2 Method of inverse transformation

The moire pattern is observed by illuminating a test fluid with paralleled laser light revolving around it. Figure 2 shows two coordinates, the fixed (x, y) and the revolving (x', y') ones. Both the coordinate systems intersect each other at the origin with an angle ξ . From Eq. (3), the relation between the density function $f(x, y)$ and the moire deflection $w(y', \xi)$ is expressed by

$$w(y', \xi) = \int_{-\infty}^{\infty} \frac{\partial f(x, y)}{\partial y'} dx' \quad (5)$$

where $f(x, y)$ is defined by $(\rho - \rho_0)\beta\Delta/\theta$ and ρ_0 is the density at a standard point (out of the test fluid).

The general integrated intensity equation is written as

$$g(y', \xi) = \int_{-\infty}^{\infty} f(x, y) dx' \quad (6)$$

If Eq. (6) is partially differentiated with respect to y' , it assumes the same form as Eq. (5), that is,

$$w(y', \xi) = \frac{\partial g(y', \xi)}{\partial y'} = \int_{-\infty}^{\infty} \frac{\partial f(x, y)}{\partial y'} dx' \quad (7)$$

Here, we apply the following polynomial expression for $f(x, y)$ in Eq. (7), which was previously developed by Maldonado et al.⁽⁷⁾⁽⁸⁾ for the transformation of $g(y', \xi)$ into $f(x, y)$ in Eq. (6).

$$f(x, y) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m \{ C_{m+2k}^m(a) U_{m+2k}^m(ax, ay) + C_{m+2k}^{-m}(a) U_{m+2k}^{-m}(ax, ay) \} \times \exp[-(a^2 x^2 + a^2 y^2)] \quad (8)$$

where the coefficient ε_m equals 1/2 for $m=0$, and 1 for

Fig. 2 Fixed and turning coordinate systems for the test field

$m=1,2,\dots$. The coefficients $C_{m+2k}^{\pm m}$ are unknown factors to be determined from the moire data, and α is an arbitrary scale factor. The functions $U_{m+2k}^{\pm m}$ are defined by

$$U_{m+2k}^{\pm m}(ax, ay) = (-1)^k \alpha [k! (\alpha^2 x^2 + \alpha^2 y^2)^m / \pi(m+k)!]^{1/2} \times \exp(\pm im\phi) L_k^m(\alpha^2 x^2 + \alpha^2 y^2) \quad (9)$$

$$\phi = \tan^{-1}(y/x) - (\pi/2) \quad (10)$$

where ϕ is a phase angle for a polar coordinate system, and L_k^m is the associated Laguerre polynomial series. The function $U_{m+2k}^{\pm m}$ has the nature of a Gaussian transformation:

$$\int_{-\infty}^{\infty} U_{m+2k}^{\pm m}(ax, ay) \exp(-\alpha^2 x'^2) dx' = \frac{\exp(\pm im\xi) H_{m+2k}(\alpha y')}{[k!(m+k)!]^{1/2} 2^{m+2k}} \quad (11)$$

where H_{m+2k} is a Hermite polynomial of an order $m+2k$. By substituting Eq. (8) into Eq. (6) and then using the transformation of Eq. (11), we have

$$g(y', \xi) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m [k!(m+k)! 2^{2(m+2k)}]^{-1/2} \times [C_{m+2k}^m(\alpha) \exp(im\xi) + C_{m+2k}^{\bar{m}}(\alpha) \exp(-im\xi)] \times H_{m+2k}(\alpha y') \exp(-\alpha^2 y'^2) \quad (12)$$

In accordance with Eq. (7), we partially differentiate Eq. (12) with respect to y' . It becomes

$$w(y', \xi) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m [k!(m+k)! 2^{2(m+2k)}]^{-1/2} \times [C_{m+2k}^m(\alpha) \exp(im\xi) + C_{m+2k}^{\bar{m}}(\alpha) \exp(-im\xi)] \times [-H_{m+2k+1}(\alpha y') \alpha \exp(-\alpha^2 y'^2)] \quad (13)$$

In order to determine the above coefficient $C_{m+2k}^{\pm m}$, we use the following orthogonal condition,

$$\int_{-\pi}^{\pi} \exp(\pm im\xi) \exp(\mp in\xi) d\xi \times \int_{-\infty}^{\infty} H_{m+2k+1}(\alpha y') H_{n+2l+1}(\alpha y') \exp(-\alpha^2 y'^2) dy' = \frac{2\pi^{3/2}}{\alpha} [(m+2k+1)!(n+2l+1)! 2^{m+2k+1} 2^{n+2l+1}]^{1/2} \delta \quad (14)$$

where δ is a Kronecker delta function, which is 1 for $m=n$ and $k=l$, and 0 for the others. After multiplying both sides of Eq. (13) by $H_{m+2k+1}(\alpha y')$ we integrate it from $y'=-\infty$ to $+\infty$, and subsequently multiply both sides by $\exp(\mp im\xi)$, and integrate it again from $\xi=-\pi$ to $+\pi$. Consequently, we obtain

$$C_{m+2k}^{\pm m}(\alpha) = \frac{-1}{4\pi^{3/2}} \frac{[k!(m+k)!]^{1/2}}{(m+2k+1)!} \times \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} w(y', \xi) \exp(\mp im\xi) \times H_{m+2k}(\alpha y') dy' d\xi \quad (15)$$

By substituting Eq. (15) and the definition of $U_{m+2k}^{\pm m}$ of Eq. (9) into Eq. (8), we can deduce the inversive transformation in the complex form. By adopting only its real part, we finally obtain the following formula to transform the moire data $w(y', \xi)$ into the density function $f(x, y)$,

$$f(x, y) = \frac{-\alpha}{2\pi^2} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m \frac{(-1)^k k!}{(m+2k+1)!} \times (\alpha^2 x^2 + \alpha^2 y^2)^{m/2} L_k^m(\alpha^2 x^2 + \alpha^2 y^2) \times [B_{m+2k}^m(\alpha) \cos(m\phi) + D_{m+2k}^m(\alpha) \sin(m\phi)] \times \exp[-(\alpha^2 x^2 + \alpha^2 y^2)] \quad (16)$$

where

$$\left. \begin{aligned} B_{m+2k}^m(\alpha) &= \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} w(y', \xi) \cos(m\xi) H_{m+2k+1}(\alpha y') dy' d\xi \\ D_{m+2k}^m(\alpha) &= \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} w(y', \xi) \sin(m\xi) H_{m+2k+1}(\alpha y') dy' d\xi \end{aligned} \right\} \quad (17)$$

Here, we note that if the density function has an axisymmetric form, we take only the terms with $m=0$ in Eq. (16) into account, and that if the density function is symmetric with respect to the x axis, only the terms including $\cos(m\phi)$ must be considered.

2.3 Example calculation with a mathematical model

In order to examine the validity and the accuracy of the present theory, a mathematical model of the density function was tested. A computer program for Eq. (16) was developed by modifying the program previously developed by Olsen et al.⁽⁹⁾ for Eq. (6).

In Fig. 2, we consider two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, shifted by $\pm \Delta y$ from a point P in the direction normal to the line A-A'. Then the moire deflection for the density function f is calculated from

$$\left. \begin{aligned} w(y', \xi) &= \int_{-\infty}^{\infty} \frac{\Delta f}{\Delta y'} dx' \\ \frac{\Delta f}{\Delta y'} &= \frac{f(x_1, y_1) - f(x_2, y_2)}{2\Delta y'} \end{aligned} \right\} \quad (18)$$

In the present paper, the following mathematical model of a truncated pyramid type was adopted as an example,

$$f = 10 \left\{ 1 - \frac{|x| + |y|}{0.8} \right\} \quad (19)$$

with

$$\begin{aligned} f &= 0 \quad \text{for } |x| + |y| \geq 0.8 \\ f &= 5 \quad \text{for } |x| + |y| \leq 0.4 \end{aligned}$$

Figure 3 represents the moire deflection data $w(y', \xi)$, thus calculated from Eqs. (18) and (19) with $\Delta y' = 0.0125$ and $\Delta x' = 0.02$ for the observation angles, $\xi = 0^\circ, 15^\circ, 30^\circ$, and 45° . Note that the diagrams at these angles are quite different from one another. All of them are symmetrical with respect to the origin, since, in this case, the axis of the assumed density function coincides with that of the turning coordinate.

Figure 4 shows three-dimensional diagrams of the density function assumed as a model. Figure 4(a) is the original form given by Eq. (19). Figures 4(b), 4(c) and 4(d) are the reconstructed forms. These are calculated from the moire data (Fig. 3) and Eq. (16) with the upper limits of the summation of $m_{\max} =$

30 and $k_{\max}=50$, $m_{\max}=10$ and $k_{\max}=200$, and $m_{\max}=30$ and $k_{\max}=200$, respectively. The scale factor α was taken as 2 in consideration of the convergency of the polynomial⁽⁹⁾. These figures indicate that the truncation of $m_{\max}$ affects the reconstruction of the region far from the origin, and that the truncation of $k_{\max}$ affects the region near the origin. The values of $m_{\max}=30$ and $k_{\max}=200$ enable us to faithfully reconstruct the original form except near the edges. Plane and side views of Fig. 4(d) are shown in Fig. 5, where the cross section B-B' is the position of a half-height of the assumed pyramid. It is clear that the reconstructed form (shown by dots) agrees well with the original one (solid lines).

Fig. 3 Moire deflection for mathematical model of truncated pyramid type

Fig. 4 Three-dimensional figures of original and reconstructed density functions for the truncated pyramid model

3. Experiments

An experimental application of the present method was performed to obtain temperature measurements of a combustion flame from an asymmetrical burner. Figure 6 shows the schematic of our experimental apparatus. The burner has a square nozzle (the length of a side is 8.5 mm and that of the burner is 160 mm) and a baffle plate 90 mm in diameter at the nozzle exit. The fuel gas is a natural gas whose main components are methane 86.5 %, ethane 5.4 %, and butane 4.0 % in volume. The parallel light of a He-Ne laser collimated by a concave mirror enters the ronchi rulings where the moire fringe images occur. The fringe images, which were distorted by the lowering of refractive index due to the lowering density, were projected onto a screen of the

Fig. 5 Top and side plane views of original and reconstructed density functions for the truncated pyramid model

Fig. 6 Schematic of experimental apparatus

schlieren optical system and recorded by a video camera. The blur of the images was reduced by passing one of the interference light spots of higher order through a slit set at the focal point of an image lens⁽⁴⁾. The image data of the video pictures were treated by using the PIAS LA-500 image processing system. The burner was set on a turntable revolving about its axis. By doing so, we could observe the flame from an arbitrary direction corresponding to the angle ξ in Figs. 2 and 6. The dimensions of the optical system were $\Delta=71$ mm, $d=0.0726$ mm, $\theta=1.6^\circ$, the focal distance of the image lens 480 mm, and the distance between the burner and the screen 1500 mm. The experimental conditions were the flow rate of fuel gas of $72 \text{ cm}^3/\text{s}$ and the pressure of gas reservoir of 1.6 kPa.

4. Results and Discussion

Figure 7 shows two typical photographs of the combustion flames from the square burner and their moire images. These are observed diagonally ($\xi=0^\circ$) and from the side ($\xi=45^\circ$). The photographs are significantly different from the moire images concerning the flame width and shape. This is because the emission intensity from the flame is closely connected with the accompanying chemical reactions and does not always reflect the temperature as the moire images do. The difference between these two moire images is mainly noticeable inside the flame core, but not outside. It indicates that the effect of the rectangular burner on the diffusive combustion flame may appear inside rather than outside. Figure 8 shows the video picture data after image processing, which includes the following procedures; the subtraction of

background lights from the images, the conversion to binary pictures, and the slenderization of the moire fringes. In the present work, the experimental data were taken at $\xi=0^\circ$ to 45° with a step of 5° , and were transformed to the density function by using Eq. (16) with the parameters of $k_{\max}=300$, $m_{\max}=30$ and $\alpha=2$.

The Gladstone-Dale's coefficient of the refractive index of the flame is necessary for the temperature measurements. Generally, its dependency on temperature is difficult to determine accurately owing to the complexity of the components of the combustion gas. Therefore, we supposed that the coefficient β was fixed throughout the flame, and the effective value β was determined so that the temperature from the moire method would agree with that detected by a thermocouple at a reference point in the flame. Consequently, the temperature T at an arbitrary point, where the density function is given by f , can be calculated by Eq. (20) under the condition of constant pressure as in the present case.

$$T = \frac{f_m T_m T_0}{f_m T_m - f(T_m - T_0)} \quad (20)$$

where the suffix m means the reference point in the flame and o the condition outside of the flame.

Figure 9 shows the temperature profiles measured by the present method. Figures 9(a) and (b) are the profiles at a height of $z=15$ mm and $z=5$ mm from the nozzle, respectively, and Figs. 9(c) and (f) indicate the shape of the nozzle exit and the area tested (35 mm in diameter). Figures 9(d) and (e) are the temperature profiles viewed from above.

Fig. 7 Photographs of combustion flames and their moire images

Fig. 8 Moire data treated by image processing system

Fig. 9 Three-dimensional temperature distributions of combustion flame, ground planes, and shape of square nozzle

These correspond to Figs. 9(a) and (b), respectively. In the figures, the temperature region over 500 K is indicated by the slender lines, and the region over 900 K by the thick lines. From these figures, it is clear that the temperature is distributed like a distorted ring, and shows a distinct feature effected by the nozzle shape; that is, the profile inside the flame is close to a square, reflecting the nozzle form, but the outside seems to be almost circular. The high temperature region in the flame gradually spreads upward.

In Fig. 10, the temperature profiles measured along the diagonal line of $\xi=45^\circ$ (shown by the solid lines) are compared with those measured with a C-A thermocouple of 0.1 mm in diameter (small circles). The outer points obtained by the thermocouple were measured by inserting it from the outside of the flame, while the inner point was measured from the inside of the nozzle to avoid the influence of thermal conduction. As mentioned before, the temperature profile from the moire data was adjusted by changing β so as to coincide with that found by the thermocouple at the maximum temperature point for $z=15$ mm. The agreement between the results by both methods is good on the whole. The difference between them is about 10% or less, depending on the position. The cause of this may be the nonuniform composition of

Fig. 10 Comparison of flame temperatures measured by moire method (solid lines) and thermocouple (small circles)

the combustion gas throughout the flame, a slight fluctuation of the flame, and the disturbance of the combustion flow caused by the insertion of the thermocouple.

It is worth pointing out that the present method can be effectively applied to a fast transient flow, a more complex flow, a higher temperature flow such as a plasma jet, and so on.

5. Conclusions

In order to measure the density (and/or temperature) profile in an asymmetric flow field, the moire method is presented, including the mathematical transformation. The transformation inverting the moire data to the density profile was derived by modifying the Maldonado-Olsen series. The accuracy of the transformation was examined by use of a density function of the truncated pyramid type. The validity of the method was demonstrated by applying it to the temperature measurement of a combustion flame from a rectangular burner. It was clearly shown that the effect of the nozzle shape on the flame temperature tends to appear mainly inside of the flame rather than outside, and that the region of high temperature spreads upwards. This method can be applied to a variety of complicated flow fields with density variation.

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