Study of Heat Transfer Phenomena in Quenching of Steel*

(Effects of Boiling Heat Transfer on Cooling Curves and Water Temperature on Hardness of Steel)

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Unsteady heat conduction and transient boiling heat transfer problems during quenching are studied experimentally and numerically. The relation between the hardness of S 45 C steel and the temperature of cooling water is also examined experimentally. The cooling curves of a gold cylinder under various subcooled conditions of water are measured and the transient boiling heat transfer is calculated using those results. The experimental cooling curves for different diameters of steel were measured using the CA sheathed thermocouple. The cooling curves during quenching were calculated using an implicit finite difference method. The characteristics of transient boiling heat transfer of a gold specimen to liquid subcooling from 0 to 80 K became clear. A model including the effect of curvature on the measured boiling curve was constructed. The calculated cooling curves using the boiling curve model as the boundary condition agreed well with the experimental results of steel. High cross-sectional and surface hardness are obtained below 40°C.

Key Words : Cooling Curve, Boiling Curve, Thermal Conduction, Latent Heat of Phase Transformation, Hardness, Temperature of Water, Quenching

1. Introduction

The purpose of our study is to estimate numerically the cooling curve during the quenching of steel. First we discuss the phase transformation of JIS S45C carbon steel, using the measured cooling curves⁽¹⁾. Next we deal numerically with the heat conduction accompanying the phase transformation and compare the calculated cooling curves with the measured results⁽²⁾. On this occasion, the cooling curves during quenching are analyzed numerically using the referenced boiling curve [3] as the boundary condition. Because the studies of transient boiling heat transfer and associated data are rather elementary^{(3)~(6)}, these discussions are as yet inconclusive.

The purpose of quenching steel is to harden it and

improve its other mechanical properties. The relation between the hardness and the other mechanical properties has already been written about by many researches^{(7)~(9)}. Therefore, knowing the hardness of quenched steel enables one to estimate the other mechanical properties. It is well known empirically that in water quenching, the hardness decreases as the water temperature rises. No detailed reports on the relation between the hardness and the water temperature have been published to date.

In this study, we first measure the transient boiling curve in subcooled water. The specimen is a gold (Au) cylinder (4 mm in diameter, 40 mm long, ϕ 4-40 mm abridged notation) of clean surface and large thermal diffusivity. The gold specimen is immersed vertically into water in various conditions of subcooling. The subcooled pool boiling heat transfer of transient states is then measured experimentally and compared with three reference^{(3)~(5)}. Next, the heat conduction with the measured boiling curve of the gold specimen as the boundary condition is numerically analyzed for S 45 C steel. The relations between the calculated and the measured cooling curves for various diameters of cylinder are compared. Then we

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discuss the boiling curve as the boundary condition of numerical analysis. The effect of the temperature of the cooling water on the hardness of S 45 C steel is also examined experimentally.

Nomenclature

- a : thermal diffusivity, $=\lambda/(c \cdot \rho)$ m²/s
 A : area, m²
 c : specific heat, J/(kg·K)
 D : diameter, mm
 h : heat transfer coefficient, W/(m²·K)
 q : heat flux, W/m²
 Q : heat flow rate, W
 t : time, s
 Δt : time step size, s
 T : temperature, °C
 T_A : measured temperature of gold specimen, °C
 T_b : saturated temperature of liquid, °C
 T_s : surface temperature of specimen, °C
 T_w : liquid temperature, °C
 ΔT_{sat} : degree of superheat, K
 ΔT_{sub} : subcooled temperature of liquid, K
 V_1 : A_1 cooling rate, °C/s
 V : volume, m³ or cooling rate, °C/s
 ρ : density, kg/m³

2. Experimental Apparatus and Specimens

2.1 Experimental apparatus

The apparatus shown in Fig.1 consists of an electric furnace (2 kW), a pipeline through which an ambient gas flows into the furnace, a water bath (0.036 m³), a cylinder which maneuvers the specimen, and devices for measuring the temperature of specimen. The specimen is cylindrical and its diameters are relatively small, from 4 to 30 mm. The specimen is

Fig. 1 Experimental apparatus

vertically immersed into the water bath by a cylinder (stroke 600 mm, velocity 0.2 m/s). Heat-resistant glass is fitted to the parallel sides of the water bath, to allow observation of the boiling state of the specimen during quenching. The cooling water in the bath is boiled once for deaeration. This water is kept at a specified temperature. The electric furnace can be moved horizontally by hand and it is held at constant temperature of 850°C by means of a PID controller. The hot junctions of the thermocouple are soldered onto the specimen with silver solder. The thermocouple is calibrated using the solidifying points of particular pure metals.

2.2 Specimen for boiling heat transfer

In quenching, the heated specimen is cooled rapidly by the water. Heat transfer in this case is film, transition and nucleate boiling and convection. Boiling heat transfer is especially influenced by any small projection or cavity in the specimen surface, but then measurement of the surface temperature of the specimen is very difficult for us. Boiling heat transfer is significantly affected by the surface condition of the material, namely, the purity, oxidation and the thickness and roughness of the specimen. In this study, a material of clean surface and large thermal diffusivity (Au, $\phi 4-40$ mm) is used. The transient temperatures (T_A) of the cylindrical core of the specimen are measured. Heat fluxes are calculated by Eq.(1).

Fig. 2 Dimensions of cylindrical specimen

$$q = \frac{V}{A} \rho c (T_A) \frac{dT_A}{dt} \quad (1)$$

As shown in Fig. 2(a), the transient temperatures of a specimen are measured by means of a CA sheathed thermocouple (sheathed diameter 1.0 mm, thermocouple wire 0.2 mm, time lag 47 ms). They are recorded by personal computer through an AD converter (14 bit, 20 ms). The conditions of cooling water cover the subcooling range from 0 to 80 K.

2.3 Specimen for cooling curve

The specimen is a common carbon steel JIS S45C (C=0.44%) with cylindrical diameters of 6 to 30 mm. This is quenched vertically into the water at rest. The dimensions and the locations for the measurement of the temperature of the specimen are shown in Fig. 2(b). Each specimen has from one to five holes corresponding to its diameter. The holes are drilled along the path of the same temperature range from the upper end to the center of the specimen. A CA sheathed thermocouple (1.6 mm sheathed diameter, 0.32 mm wires, 110 ms time lag) is inserted into the holes so as to make errors of the measured temperature as small as possible. The temperatures are recorded by personal computer through an AD converter (12 bit, 8 ms). The initial temperature of the specimen is 850°C and that of the cooling water is 20°C.

2.4 Specimen for hardness

The specimen is S45C steel 6 mm in diameter and 80 mm in length ($\phi 6-80$ mm) as shown in Fig. 2(c). For hardness, the specimen is cut by an abrasive cutting machine which keeps the specimen temperature sufficiently low. The surface hardness is measured by a Rockwell hardness tester and the cross-sectional hardness by a micro-Vickers hardness tester.

3. Subcooled Transient Pool Boiling Heat Transfer

Though saturated boiling heat transfer is not used for the quenching of steel, we examine the relation between our data and the literature on saturated boiling. The results are shown in Fig. 3. The result obtained by Bergles and Thompson⁽⁶⁾ is indicated by a broken line. The result for the nucleate boiling region of the steady-state calculated by Inada et al.⁽¹⁰⁾ is indicated by a dot-dash line. The result of the transient nucleate boiling of horizontal cylinder calculated by Nishio and Uemura⁽¹¹⁾ is indicated by the double dot-dash line. The results of the film boiling of a horizontal cylinder obtained by Bromley⁽¹¹⁾ and Sakurai et al.⁽¹²⁾ are also indicated in Fig. 3.

The film boiling results obtained by our study (Au) lie in the region between Bromley's and Sakurai et al.'s results. This result is considered to be proper. Comparing the result of the boiling curve obtained by

Bergles with our result for gold, we find that the shape of the boiling curve completely shifted toward a low value of the degree of superheat. Moreover, the minimum heat flux of our result took a low value and the critical heat flux conversely took a high value. In the nucleate boiling region, the result obtained was of the same order as that of Inada at the lower degree of superheat, but higher degrees of superheat increased the difference between both results. The slope of the nucleate boiling region was the same as obtained by Nishio, but its value was of a higher order. Even for the transient pool boiling of the saturated state, the boiling curves for each study differ. The saturated boiling curves cannot be determined quantitatively due to several factors.

The boiling curves of the high subcooled states of water described in three referenced works are shown in Fig. 4. The result of liquid subcooling of gold at 70 K is also shown in Fig. 4. The shapes of the gold and Nishio and Uemura's boiling curves are similar and the critical heat flux temperatures are of the same order, but every heat flux value of gold is large. The difference between heat fluxes increased at lower degrees of superheat in the nucleate boiling region.

The experimental cooling curves of the gold specimen are shown for various liquid subcooling rates in Fig. 5. When the liquid subcooling (water temperature) decreases (increases), the quench point of the cooling curve is delayed and its temperature is reduced.

The boiling curves of gold specimen as affected by various subcoolings are shown in Fig. 6. Critical

Fig. 3 Saturated transient pool boiling

heat flux increases in proportion to an increase of subcooling and the temperature change in accordance with the degree of superheat. For liquid subcooling at 40 and 50 K, the difference between critical heat fluxes is very large. Up to the 40 K subcooling, the heat transfers of film boiling are very stable and continuous. At subcooling above 50 K, the situation is reversed. The heat flux of the film boiling with a larger degree of subcooling increases because the energy required to form vapor increases. The formation of vapor film at the temperature of 850°C is insufficient to promote vapor formation. Therefore the boiling state shifts at once to transition and nucleate boiling heat transfers. In the lower region of the degree of superheat below 50 K, nucleate boiling is influenced considerably by the subcooling.

Fig. 4 Subcooled transient pool boiling

Fig. 5 The experimental cooling curves of gold

4. Numerical Analysis of Transient Temperature with the Measured and Reference Boiling Curves as the Boundary Condition

To calculate the transient temperature during quenching numerically, it is necessary to consider the properties (ρ, c, λ) of steel, the latent heat, the temperature of phase transformations and heat flux (or surface temperatures, or heat transfer coefficient) as the boundary conditions. The numerical analysis considers one-dimensional unsteady heat conduction in the radial direction of the cylinder. An implicit finite difference method is employed to perform the numerical calculations. The numerical analysis, the temperature and the latent heat of phase transformations are the same as those described by the previous way of thinking.

The heat fluxes forming the boundary condition of analysis are shown in Fig. 7. The transient temperatures are calculated for the center of various diameters from 6 to 30 mm. The center cooling curves depicted in Fig. 8 are based on the results of the 80 K subcooling shown in Fig. 7. The experimental cooling curves are represented by the broken line and the calculated results by the solid line. All the calculated cooling curves increase rapidly compared with the experimental results. This difference between the experimental and the calculated results increases with increasing curvature.

In the previous study, a boiling curve model (double dot-dash line in Fig. 7) was constructed. The

Fig. 6 Subcooled transient pool boiling

calculated and the experimental results only coincide when the specimen diameter is 30 mm. The center cooling curves using this model are shown in Fig. 9. The two curves for a diameter of 30 mm agree exactly, but all the calculated results decrease with decreasing diameter.

The center cooling curves when the boundary condition is the boiling curve obtained by Nishio and Uemura in Fig. 7 ($\Delta T_{\text{sub}} = 68.7$ K, a dotted line) are shown in Fig. 10. The critical heat flux obtained by them is small compared to the result of the 80 K subcooling and is large compared to the model. Therefore, below 15 mm in diameter, the calculated cooling curves decrease compared with the experimental results; above 15 mm, the situation is reversed. On the basis of the comparison of the calculated and measured cooling curves when the boundary condition for

various boiling curves is applied, the effect of curvature is assumed to be very large.

5. Numerical Analysis of Transient Temperature with Boiling Curve Considering Curvature as the Boundary Condition

In section 4, subcooled transient pool boiling curves for the present (Au), model and referenced cases were examined as the boundary condition. Only partial agreement between calculated and experimental cooling curves was attained. In particular, the effect of curvature on the saturated film boiling heat transfer of a horizontal cylinder was investigated. These referenced results are shown in Fig. 11. The result of Bromley's equation is shown by the solid line. The large sizes of cylinder agree well with most referenced results, but those differences increase with smaller size. In Bromley's equation, if the liquid is water, then the properties are constant. The heat transfer coefficient is inversely proportional to a quar-

Fig. 7 Subcooled transient pool boiling

Fig. 8 Cooling curves at the center of various diameters (the case with the boiling curve of $\Delta T_{\text{sub}} = 80$ K as the boundary condition)

Fig. 9 Cooling curves at the center of various diameters (the case with the boiling curve of model in Fig. 7 as the boundary condition)

Fig. 10 Cooling curves at the center of various diameters (the case with the referenced boiling curve (3) as the boundary condition)

ter of the power of the diameter. The ratios of the heat transfer coefficient obtained by Bromley's equation are $\phi 4/\phi 15=1.39$, and $\phi 4/\phi 30=1.67$. The effect of curvature is fairly large. The difference between Bromley's and the referenced results increases for smaller curvatures.

The results of exponential expression analysis using reference values are shown by a dotted line in Fig. 11. The results of this expression agree approximately with those values. Although the heat transfer coefficient value of saturated film boiling is very low, the effect of curvature is unexpectedly large. In respective boiling regions of larger subcooling, therefore, the effect of curvature is also large.

The effect of curvature on the boiling region, except for saturated film boiling, is investigated with regard to the burn-out heat transfer of a horizontal cylinder⁽¹³⁾. For a curvature of more than 4 mm in diameter, the amount of scatter in these results is very large. Thus the effect of curvature cannot be detected and referred to. Few works dealing with water with large subcooling are available; such studies relating to curvature are particularly scarce.

The model shows the following according to the result of the boiling curve for the 80 K subcooling. The heat fluxes for the temperatures of critical and minimum heat fluxes, 100, 200 and 1 000°C, are set close to the measured results. Then each is approximated by a linear function. The model based on the 80 K subcooling is built on a basic concept. The expressions including the effect of curvature are compared to the referenced result of curvature in the saturated film boiling. The boiling curves for various diameters are

Fig. 11 Film boiling heat transfer of saturated state for the horizontal cylinder in water

shown in Fig. 12. The boiling curve measured with the gold specimen is for a 4 mm diameter; boiling curves for various diameters are determined from this basic cooling curve. Film boiling, minimum, critical and nucleate boiling heat fluxes are each given in Eqs. (2) to (5), respectively.

$$q_{\text{film}} = 1.17 \cdot 10^6 \exp\left(1.11 \frac{D_4}{D}\right) (T_{\text{fm}} = 1000^\circ\text{C}) \quad (2)$$

$$q_{\text{min}} = 9.40 \cdot 10^5 \exp\left(0.468 \frac{D_4}{D}\right) (T_{\text{min}} = 758 + 32.3 \times \log\left(\frac{D_4}{D}\right)^\circ\text{C}) \quad (3)$$

$$q_{\text{max}} = 7.18 \cdot 10^6 \exp\left(0.637 \frac{D_4}{D}\right) (T_{\text{max}} = 400^\circ\text{C}) \quad (4)$$

$$q_{\text{sbo}} = 1.18 \cdot 10^6 \exp\left(0.943 \frac{D_4}{D}\right) (T_{\text{sbo}} = 200^\circ\text{C}) \quad (5)$$

where

$$D_4 = 4 \text{ mm},$$

Fig. 12 Model of subcooled transient boiling curve in water

Fig. 13 Cooling curves at the center of various diameters (the case with the boiling curve in Fig. 12 as the boundary condition)

The comparative results of the calculated and measured center cooling curves of various diameters as the boundary condition for these boiling curves are shown in Fig. 13. Both cooling curves and these temperature regions generated phase transformations agreed sufficiently well. It is concluded that we cannot accurately estimate the cooling curve when constant heat flux regarding cylinders of various diameters is used as the boundary condition. From the above discussion, it is assumed that the boiling curves of large liquid subcooling involving various specimen diameters are distributed as shown in Fig. 12. The effect of curvature on the boiling curve for various specimen diameters of large subcooling is a subject for future study.

6. Results and Discussion of Hardness

6.1 Hardnesses obtained by quenching at various water temperatures

The relations between the temperature of cooling water and the hardnesses of the surface or the cross section are shown in Figs. 14 and 15. The circle is the averaged value of the hardness and the vertical lines indicate dispersion. The cross-sectional hardnesses are almost uniformly high values. The amount of scatter is also small. The specimen dimension is cru-

Fig. 14 The relation between the surface hardnesses and the temperature of cooling water

Fig. 15 The relation between the cross-sectional hardnesses and the temperature of cooling water

cial to the required hardness being achieved. It is evident that both surface and cross-sectional hardnesses are very large around 40°C. Both high-value hardnesses are obtained with the water temperature below 40°C. At water temperatures above 40°C, the hardnesses assume extremely low values.

6.2 Cooling curves for the various water temperatures.

Figure 16 shows the measured cooling curves in the core of a 6 mm S45C steel cylinder. We take the ordinate as temperature and the abscissa as a logarithm of time. Continuous cooling transformation diagrams (CCT) of S 45 C steel are also plotted in Fig. 16. The effect of phase transformation is not clearly evident in the cooling curves for water temperatures below 40°C. In the cooling curves for water temperatures over 40°C, Ar' transformation is marked at around 660°C. At 40°C and 45°C, these cooling curves differ according to the presence or absence of heat generated by the phase transformation. The measured cooling curves correspond with the CCT diagram.

6.3 Discussion of hardness

Figure 17 shows the cooling rate obtained from previous cooling curves. To better understand the effects of latent heat, we take the ordinate as temperature and the abscissa as a logarithm of the cooling rate. With the water temperature above 45°C, the cooling rate is slower than when the temperature is around 660°C. From this result, Ar' transformation develops fully. At a water temperature of 40°C, no effects of Ar' transformation are detected in the cooling curve. The cooling rate is slower at around 630°C, therefore slight Ar' transformation occurs. With the least amount of Ar' transformation, the

Fig. 16 The measured cooling curves of various water temperatures

Fig. 17 The cooling rate curves of various water temperatures

hardness obtained with a water temperature of 40°C is a low value compared with the results below 35°C.

The relation between A_1 cooling rate ($V_1 = dT/ds$, 727°C) and the water temperature is shown in Fig. 18. With the water temperature above 40°C, the A_1 cooling rate decreases rapidly. The A_1 cooling rate changes markedly in the narrow range of 40°C to 50°C. At water temperatures above 50°C, the variation in the cooling rate is very small.

The relation between cross-sectional hardnesses and A_1 cooling rates is shown in Fig. 19. The cross-sectional hardnesses for the 30 mm cylinder obtained by the previous study are also shown in Fig. 20. With the exception of the results at water temperatures of 50°C and 55°C, these hardnesses agree well with previous results. In the narrow temperature range of 40°C to 45°C, the difference in the A_1 cooling rate is about 70°C/s and the cross-sectional hardness is 260 HmV. The high hardnesses are obtained with water at rest below 40°C. It is obvious that in the water quenching we are usually stirring up the water or moving the quenched parts.

7. Conclusions

In order to determine the effect of transient boiling heat transfer of large subcooling on the water quenching, we measured the transient temperatures with a gold specimen (Au, $\phi 4$ -40 mm) and then calculated the boiling curves by using these temperatures. Next, specimens of S45C steel of various diameters ($\phi 6$ -30 mm) were quickly immersed in water at 20°C, and the cooling curves were measured. The measured cooling curves of a central portion of the cylinder were compared with the calculated results of unsteady

Fig. 18 A_1 cooling rate with water temperature

Fig. 19 The hardnesses with A_1 cooling rate

heat conduction when the boundary condition is the subcooled boiling curve of the gold specimen or of a referenced work. The relation between the hardness of S45C steel and the temperature of the cooling water was also examined experimentally. The results obtained in this study were:

(1) The transient pool boiling heat transfer of a gold specimen to liquid subcooling from 0 to 80 K was measured and clarified. Hitherto insufficient data of boiling curves were improved.

(2) Heat conduction analysis using the boiling curve obtained by the gold specimen or through references as the boundary condition was performed. Partial agreement between calculated and experimental cooling curves was reached. From these discussions, the effect of curvature on boiling heat transfer is deemed to be very large.

(3) In particular, the effect of curvature on the saturated film boiling heat transfer of a horizontal cylinder was investigated. A model of a boiling curve,

considering the curvature in the results obtained with the gold specimen, is made. The calculated cooling curves of the central portion of the cylinder during quenching are compared with the measured results and are found to agree well.

(4) High cross-sectional and surface hardnesses are obtained with water temperatures below 40°C.

(5) The reason for the low hardness obtained at water temperatures above 40°C is that Ar' transformation of steel occurs at around 660°C.

(6) The behavior of cross-sectional hardness is well explained by the A₁ cooling rate.

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