Flow-Induced Vibration of Long-Span Gates*

(Verification of Added Mass and Fluid Damping)

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Long-span gates vibrating in the streamwise direction push and draw water in the reservoir ; thus the gate is accompanied by the added mass of water. Moreover, waves propagating in the upstream direction are formed on the free reservoir surface, resulting in a fluid damping effect on the vibrating gates. The added mass and the fluid damping coefficient were derived theoretically in a dimensionless form in a previous study. To verify the theoretical results of the added mass and the fluid damping coefficient, the experimental results of a model test are presented in this study. A planar vertical gate undergoes free damped vibrations in air and still water. The added mass and the fluid damping coefficient are calculated from the measured frequencies and damping ratios of the damped vibrations. Consequently, it is shown that there is good agreement between the theoretical and experimental results.

Key Words : Fluid Mechanics, Dynamics of Machinery, Flow-Induced Vibration, Gate, Long-Span Gate, Added Mass, Fluid Damping

1. Introduction

Figure 1 shows a long-span gate which is free to vibrate transversely in the streamwise direction. The discharge flow under the gate is jetted into the air. When the riverbed just below the gate is sloped, the streamwise vibration of the gate causes a variation in discharge flow rate. Therefore, there are two factors which induce a variation in hydraulic pressure. One is the push-and-draw effect of the streamwise-vibrating gate on the water, and the other is the variation in discharge flow rate. The hydraulic pressure induced by the push-and-draw effect results in an added mass of water and a fluid damping effect on the vibrating gate. Under certain conditions, however, the hydraulic pressure induced by the variation in the discharge flow rate from a small gate opening results in an energy source which may cause vibrations, thereby leading to a severe flow-induced streamwise vibration^{(1)~(6)}.

As a first step in studying such a flow-induced streamwise vibration of long-span gates under a small gate opening, the fluctuating hydraulic pressures for an arbitrary gate movement were theoretically analyzed⁽⁷⁾ by using the potential theory developed by Rayleigh⁽⁸⁾⁽⁹⁾ and Lamb⁽¹⁰⁾⁽¹¹⁾, which treats the dissipative wave radiation problems as an initial-value and boundary-value problem. The same theory was applied to a flow-induced vibration problem^{(12)~(18)} of Tainter gates⁽¹⁹⁾⁽²⁰⁾ and it was shown that theoretical results in good agreement with experimental ones can be obtained. Secondly, the hydraulic pressure induced by the push-and-draw effect of the vibrating gate on the water was calculated in order to derive the added mass of the water and the fluid damping coefficient in a dimensionless form⁽²¹⁾. These are quite significant for discussion of the vibration frequency in water and the level of fluid damping.

In this study, the theoretical results of the added mass and the fluid damping coefficient induced by the push-and-draw movement of the gate are verified by performing model test. It is supposed that the gate opening is comparatively small, and hence the mean discharge flow rate is so small that the reservoir flow

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field can be regarded as still water. Therefore, a two-dimensional vertical model gate which is suspended elastically in the streamwise direction is submerged in still water in an 8 m-long-by-300 mm wide channel with a horizontal floor. The gate opening between the bottom of the gate and the horizontal channel floor is adjusted to be as small as possible. Then, the model gate is accelerated to produce free damped vibrations for various frequencies and water levels. The measured free damped vibrations are then analyzed by a computer to reveal the vibration frequencies and damping ratios. The added mass and the fluid damping coefficient for the model gate can be calculated from the vibration frequencies and the damping ratios in air and water. The water motions at both sides of the gate induce the same amount of added mass and fluid damping, respectively. Thus, one-half of the calculated results is the added mass and fluid damping coefficient for the long-span gate in contact with water on one side. Finally, the experimental results for the added mass and the fluid damping coefficient are then compared with the theoretical ones.

Nomenclature

- C : mechanical damping coefficient
 d_0 : water depth to the bottom of the gate
 d_r : water depth of the reservoir
 F : Froude number
 F_a : vibration frequency of the gate in air (Hz)
 F_w : vibration frequency of the gate in water (Hz)
 g : acceleration of gravity
 I : spring constant
 K : moment of inertia of the gate
 l : length of the stay
 L : length of the gate
 X : displacement of streamwise gate vibration
 β : water depth ratio of the bottom of the gate to the reservoir
 $\Delta c, \Delta C, \Delta C_i$: fluid damping coefficient
 $\Delta m, \Delta M, \Delta M_i$: added mass of water
 ζ_a : damping ratio of the gate in air
 ζ_w : damping ratio of the gate in water
 Θ : rotation angle of the stay
 ρ : specific mass of water
 ω_a : damped vibration circular frequency of the gate in air
 ω_{a0} : undamped vibration circular frequency of the gate in air
 ω_w : damped vibration circular frequency of the gate in water
 ω_{w0} : undamped vibration circular frequency of the gate in water

2. Dimensionless Added Mass and Fluid Damping Coefficient

The added mass ΔM and the fluid damping coefficient ΔC per unit length of the gate are represented by Δm and Δc in the following dimensionless forms⁽²¹⁾:

$$\begin{aligned}\Delta m &\equiv \Delta M / (\rho d_0^2) \\ \Delta c &\equiv \Delta C / (\rho d_0 \sqrt{g d_0}),\end{aligned}\quad (1)$$

where ρ represents the specific mass of water and d_0 the water depth to the bottom of the gate (see Fig. 1). The representative mass is ρd_0^2 , which represents the mass of water captured in the rectangular prism shown in Fig. 2. The basic plane of the rectangular prism is a unit-length gate surface submerged by d_0 in the water, which is drawn by the oblique lines in Fig. 2. The rectangular prism has the length d_0 toward the upstream side. The representative damping coefficient is $\rho d_0 \sqrt{g d_0}$, which represents the ratio of the gravity

Fig. 1 Schematic view of long-span gate

Fig. 2 Representative added mass for long-span gate

force $\rho g d_0^2$ acting on the representative mass to the wave propagation speed $\sqrt{g d_0}$.

The dimensionless added mass Δm and the dimensionless fluid damping coefficient Δc are functions of the water depth ratio β and the Froude number $F^{(21)}$ which are defined by

$$\beta \equiv d_0/d_r \quad (2)$$

$$F \equiv \sqrt{d_0/g} \cdot \omega_w \quad (3)$$

β represents the ratio of the water depth d_0 to the water depth d_r of the reservoir. F is the Froude number. The square of the Froude number represents the ratio of the inertia force $\rho d_0 \omega_w^2$ to the gravity force ρg acting on a unit-volume fluid⁽³⁾, where ω_w represents the vibration circular frequency of the gate in water. The calculated added mass Δm and fluid damping coefficient Δc for $\beta=1.0$ are shown in Fig. 3. The added mass Δm is almost zero when the Froude number F is smaller than about 1.0, but Δm increases rapidly as F increases, and reaches the constant value of 0.54 when F is about 11.0. On the contrary, the fluid damping coefficient Δc takes a large value when F is smaller, but Δc decreases rapidly as F increases, and reaches zero when F is about 6.0.

3. Gate Model

A gate model is shown in Fig. 4. A vibrating table is suspended from the frame (base) by two stays with the length l of 37.5 cm, in the horizontal. The vibrating table, the two stays and the frame form a parallel-motion mechanism. Thus, the vertical gate attached to the vibrating table undergoes a parallel motion given by $l\theta$, where θ represents the rotation of the stays. When the rotation θ is small, the parallel motion of the vertical gate can be considered to be a small linear motion in the horizontal X -direction. The model gate with the length L of 298 mm is inserted in a water channel with the width of 300 mm. The gate opening between the gate bottom and the horizontal reservoir bed is adjusted to be 0.5 mm which is

Fig. 3 Added mass Δm and fluid damping coefficient Δc in dimensionless form

far smaller than the gate width of 20 mm. Thus, the water depth ratio β defined by Eq. (2) is 1.0 for any water level in the reservoir. The vibrating table is pulled, at both sides, by a coiled spring through wires and arms. The vibration frequency of the model gate can be adjusted by sliding the setting position of the coiled spring along the arms.

The moment of inertia I of the model gate can be calculated from the vibration frequency of the model gate in air. The moment of inertia I was 4.586 kg·m². The reciprocating mass M of the model gate can be calculated by I/l^2 . The reciprocating mass M was 32.5 kg. When the water depth d_0 to the bottom of the gate is 400 mm for instance, the added mass of water becomes as heavy as about 35 kg, and hence the reciprocating mass M was adjusted to be a comparatively large value. When the reciprocating mass of the model gate is too small, the gate free vibration in water damps too quickly to calculate the vibration frequency and the damping ratio with high accuracy.

4. Calculation of Added Mass and Fluid Damping Coefficient

The model gate in the reservoir undergoes a small-amplitude streamwise vibration represented by $X(=l\theta)$. When the damping coefficient which represents all mechanical damping effects in the vibration system is denoted by C and the spring constant by K , the gate motion in air is subject to the following equation of motion:

$$M\ddot{x} + C\dot{x} + KX = 0. \quad (4)$$

When the model gate is submerged in still water to d_0 and pushed to produce a damped free vibration, the water motion at both sides of the model gate induces an added mass ΔM_i and a fluid damping coefficient ΔC_i . Then, the equation of motion of the submerged gate is given by

$$(M + \Delta M_i)\ddot{x} + (C + \Delta C_i)\dot{x} + KX = 0. \quad (5)$$

When the damped vibration frequency ω_a and the damping ratio ζ_a in air are measured, the undamped

Fig. 4 Model gate submerged in still water

vibration frequency ω_{a0} can be calculated by

$$\omega_{a0} = \omega_a / \sqrt{1 - \zeta_a^2} \quad (6)$$

ω_{a0} and ζ_a are, of course, defined by the following expressions:

$$\omega_{a0} = \sqrt{K/M}, \quad \zeta_a = C/(2\omega_{a0}M) \quad (7)$$

When the damped vibration frequency ω_w and the damping ratio ζ_w in water are measured, the undamped vibration frequency ω_{w0} ,

$$\omega_{w0} = \sqrt{\frac{K}{M + \Delta M_t}} \quad (8)$$

can be calculated by

$$\omega_{w0} = \omega_w / \sqrt{1 - \zeta_w^2} \quad (9)$$

where ζ_w is defined by

$$\zeta_w = \frac{1}{2\omega_{w0}} \cdot \frac{C + \Delta C_t}{M + \Delta M_t} \quad (10)$$

Arranging Eqs. (8) and (10) by using Eqs. (6), (7) and (9), the added mass ΔM_t and the fluid damping coefficient ΔC_t are given by the following expressions:

$$\Delta M_t = \left\{ \frac{1 - \zeta_w^2}{1 - \zeta_a^2} \left(\frac{\omega_a}{\omega_w} \right) - 1 \right\} M \quad (11)$$

$$\Delta C_t = \sqrt{\frac{2\omega_a}{1 - \zeta_a^2}} \left(\sqrt{\frac{1 - \zeta_w^2}{1 - \zeta_a^2}} \cdot \frac{\omega_a}{\omega_w} \zeta_w - \zeta_a \right) M \quad (12)$$

When the damping ratio ζ_a of the vibration system in air is very small, ΔM_t and ΔC_t can be calculated under $\zeta_a^2 \ll 1$.

5. Experimental Results

The model gate was pushed to produce a damped free streamwise vibration first in air, and then in water. The measured analogue data of the free damped vibration were input to a computer through an A-D converter to calculate the damped vibration frequency and the damping ratio with high accuracy. The calculated results of the vibration frequency $F_a (= \omega_a/2\pi [\text{Hz}])$ and the damping ratio ζ_a in air are shown in Fig. 5. The vibration frequency F_a was adjusted from 1.49 to 3.16 Hz. When the vibration frequency F_a is too small, standing waves appeared in the 8 m-long reservoir and thus, accurate experimen-

Fig. 5 Damping ratio of the model gate in air

tal data could not be obtained. When the vibration frequency F_a is too high, the frame suspending the vibration system started to vibrate because of its lower rigidity, and thus experimental data with high accuracy could not be obtained. The damping ratio ζ_a shows comparatively small values of 0.0027 to 0.005, since ball bearings with low friction were used at the turning pairs of model gate.

Some examples of the measured free damped vibrations in air and water are shown in Fig. 6. The first analogue wave is a damped vibration in air. The vibration frequency F_a is 2.41 Hz and the damping ratio ζ_a is 0.0027. The other three analogue waves are the damped vibrations in water. The gate submergence d_0 increases to 20 cm, 30 cm and further to 40 cm. As the gate submergence d_0 increases, the vibration frequency $F_w (= \omega_w/2\pi)$ decreases to 2.22, 2.01 and, then to 1.76 Hz, and to the contrary, the damping ratio ζ_w increases to 0.046, 0.073 and then to 0.085. These results show that as the gate submergence increases, both the added mass and the fluid damping increase. Similar experiments were performed for various vibration frequencies, resulting in the vibration frequency ratio $F_w/F_a (= \omega_w/\omega_a)$ and the damping ratio ζ_w , which are shown in Figs. 7(a) and (b), respectively. The calculated results for $d_0 = 20, 30$ and 40 cm are marked by a circle, a square and a triangle, respectively. Both F_w/F_a and ζ_w show a tendency to

Fig. 6 Measured damped free streamwise vibration

decrease as F_a increases. This tendency demonstrates that as the vibration frequency F_a in air increases, the added mass increases and the fluid damping effect decreases. Substituting the experimental data shown in Figs. 7(a) and (b) into Eqs. (11) and (12), the added mass ΔM_t and the fluid damping coefficient ΔC_t can be obtained, as shown in Figs. 8(a) and (b). The abscissa is the vibration frequency F_w in water, which varies from 1.42 to 2.82 Hz. As the vibration frequency F_w increases, the added mass ΔM_t increases and reaches about 35 kg when the gate submergence d_0 is 40 cm. The fluid damping coefficient ΔC_t shows a tendency to decrease as F_w increases.

In order to compare the experimental results shown in Fig. 8 with the theoretical ones shown in Fig. 3, ΔM_t and ΔC_t must be reduced to the same dimensionless forms as Eq. (1):

$$\Delta m = \Delta M_t / (2L\rho d_0^2), \quad \Delta c = \Delta C_t / (2L\rho d_0 \sqrt{gd_0}). \quad (13)$$

In this model test, the added mass ΔM_t and the fluid damping coefficient ΔC_t , which act on both sides of the model gate with the length L were obtained. They

should be reduced to the added mass Δm and the fluid damping coefficient Δc per unit length for one side of the gate, and thus ΔM_t and ΔC_t are divided by $2L$. Moreover, the abscissa must be reduced to the Froude number F defined by Eq. (3):

$$F \equiv 2\pi\sqrt{d_0/g} \cdot F_w \quad (14)$$

The experimental values calculated from Eqs. (13) and (14) in the dimensionless forms are shown in Figs. 9(a) and (b) in which the theoretical results are shown by the solid curves.

The Froude number range covered in this model test is from 1.27 to 2.81. This range is comparatively narrow, but the experimental data for different values of gate submergence d_0 overlap well, and furthermore, they distribute along the theoretical curves. When the Froude number is larger, the experimental data of the fluid damping coefficient show a tendency to be a little larger than the theoretical curve. It is considered that this tendency was caused by low rigidity of the frame suspending the vibration system. When the Froude number was larger, that is, the vibration frequency was higher, frame vibrations were also induced by the model gate vibrations, which had a worse effect on the

(a) Vibration frequency ratio F_w/F_a

(b) Damping ratio ζ_a

Fig. 7 Vibration frequency ratio and damping ratio of damped free streamwise vibration in water

(a) Added mass ΔM_t

(b) Fluid damping coefficient ΔC_t

Fig. 8 Added mass and fluid damping coefficient

(a) Added mass Δm (b) Fluid damping coefficient Δc

Fig. 9 Comparison of experimental and theoretical results for added mass and fluid damping coefficient in dimensionless form

fluid damping coefficient. The depth of the water channel used for this model test was limited to 40 cm, and hence the vibration frequency had to be increased for large Froude numbers. It may be expected that a more rigid frame and deeper water channel will lead to more accurate experimental results of the fluid damping coefficient which coincide better with the whole theoretical curve.

6. Conclusions

In order to verify the theoretical results of the added mass of water and the fluid damping coefficient for long-span gates, a model test for the two-dimensional 298 mm-long gate suspended in a water channel was carried out. Consequently, the experimental data of the added mass and the fluid damping coefficient, which coincide well with the theoretical results, were obtained in the range of the Froude number from 1.27 to 2.81.

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