A Numerical Analysis of the Flow Passing through a Cascade with Tip Clearance*
(Subsonic Flow through a Linear Cascade Composed of Flat Plates)

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The three-dimensional flow field through a linear cascade with tip clearance was numerically studied by solving Navier-Stokes equations. A fundamental cascade model composed of flat plates, for which corresponding experimental data were available, was adopted to develop the appropriate solution method for the flow through the tip clearance. The computed normal force distributions on blades and velocity vectors in the downstream flow field showed good agreement with the experimental data. The detailed flow phenomena around the blade tip, such as the formation of separation bubbles on the tip surface, were clearly described. The spanwise distributions of the normal force on the blade were investigated for various tip clearances, and it was found that the normal force on the extremity of the blade did not diminish in the case of small clearances of less than 0.6% of the blade span because of the blockage of leakage flow due to the effect of viscosity.

**Key Words**: Fluid Mechanics, Numerical Analysis, Navier-Stokes Equations, Cascade Flow, Tip Clearance, Secondary Flow, Tip Leakage Flow

1. Introduction

In the cascades of turbomachines, there are tip clearances between blade tips and casing walls. The tip leakage flow and the tip vortex owing to the tip clearance have considerable influence on the three-dimensional flow fields around cascades. Moreover, the flow through tip clearance has a fundamental relation with the loss generation. Therefore, the understanding of the detailed flow field near the blade tip including the effect of tip clearance is one of the most important matters for the present turbomachinery.

Since the flow field in the tip region is governed by the viscous property of the fluid, the analytical studies based on the potential theory is not effective for the practical reasons mentioned above.

In the present study, the three-dimensional viscous flow through cascades with tip clearance was numerically investigated by solving the Navier-Stokes equations. The assumption of the thin-layer approximation was adopted to reduce the computational load. The LU-ADI factorization method was used as the numerical algorithm. The computational grid was generated by a zonal method with an overlapping grid technique. This method enabled us to represent the shape of blade tip correctly and to investigate the tip leakage flow more thoroughly than the previous methods. The computational method for the cases without tip clearance was presented before and it was improved in this study for the flow through the tip clearance.

A fundamental linear cascade composed of flat plates, on which corresponding experimental data were available, was adopted for the calculation in order to establish the appropriate solution method. The calculated results were compared with the experimental data and the effectiveness of the developed method was confirmed. By the developed method, the
details of the flow field around the blade tip were clearly revealed and interesting phenomena, such as the formation of separation bubbles on the tip surface, were found. The spanwise distribution of the normal force on the blade was also investigated from the computed results with special emphasis on the force near the tip.

Nomenclature

$C_n$: normal force Coefficient
$F$: Jacobian
$P_r$: Prandtl number
$a$: speed of sound
$c$: local coordinate on the blade chord
$c_o$: chord length
$e$: energy
$h_o$: span length between hub and tip walls
$j,k,l$: numbers of grid points
$\rho$: static pressure
$t$: time
$u,v,w$: velocity components in $x$, $y$ and $z$ directions, respectively
$x,y,z$: Cartesian coordinates
$\alpha$: angle of attack
$\beta$: inlet flow angle
$\gamma$: ratio of specific heat
$\delta$: tip clearance
$\xi, \eta, \zeta$: general curvilinear coordinates
$\mu$: viscosity coefficient
$\rho$: density

2. Computational Method

2.1 Cascade model

Figure 1 shows a linear stationary cascade composed of flat plates adopted for the computation. The aerodynamic characteristics of this cascade were studied experimentally by one of the authors. Each blade has a chord length of 60 mm and a thickness of 6 mm. Their leading and trailing edges are shaped into semicircles in the cross section. The blade span is 180 mm including the tip clearance. The clearance is changed from 0 mm to 5 mm.

2.2 Governing equations and numerical algorithm

The equations of continuity, momentum and energy are written on the cartesian coordinate system and transformed to the general coordinate system $(\xi, \eta, \zeta)$. These equations are expressed in the conservation-law form as follows:

$$
\frac{\partial \hat{Q}}{\partial t} + \frac{\partial \hat{E}}{\partial \xi} + \frac{\partial \hat{F}}{\partial \eta} + \frac{\partial \hat{G}}{\partial \zeta} = \frac{1}{Re} \left( \frac{\partial \hat{S}_1}{\partial \eta} + \frac{\partial \hat{S}_2}{\partial \zeta} \right),
$$

where

$$
\hat{Q} = \begin{bmatrix}
\rho \\
\rho u \\
\rho v \\
\rho w \\
e \\
\end{bmatrix}, \quad \hat{E} = \begin{bmatrix}
\rho U \\
\rho uU + \xi_k P \\
\rho vU + \xi_k P \\
\rho wU + \xi_k P \\
U(e + p) \\
\end{bmatrix}
\hat{F} = \begin{bmatrix}
\rho V \\
\rho uV + \eta_k P \\
\rho vV + \eta_k P \\
\rho wV + \eta_k P \\
V(e + p) \\
\end{bmatrix}, \quad \hat{G} = \begin{bmatrix}
\rho W \\
\rho uW + \zeta_k P \\
\rho vW + \zeta_k P \\
\rho wW + \zeta_k P \\
W(e + p) \\
\end{bmatrix}
$$

Fig. 1 Computational grid for linear cascade
layer approximation is used. Equation (1) is solved by the LU-ADI factorization scheme, the details of which are given in Ref. (1). In the present study, flow fields are assumed to be laminar.

2.3 Computational grid and boundary conditions
In Fig. 1, the schematic diagram of the computational grid on the blade surface and hub wall is shown. Two-dimensional C-grids around a blade are stacked in the spanwise direction from the hub side wall to the tip side wall in order to generate the three-dimensional grid. The region between the tip surface of the blade and the tip wall is filled with H-grid as shown in Fig. 2. By this method, the flat tip surface and its sharp edge can be accurately represented. C- and H-grids are overlapped at their interface for the smooth connection of computed values. Overlapping grids are also used on the periodic boundary. Communication on the overlapped boundaries is achieved by means of the linear interpolation along the grid lines.

The number of C-grid points is $181 \times 65 \times 31$ in the $\xi, \eta$ and $\zeta$ directions, respectively, and that of the H-grid points is $62 \times 26 \times 25$.

At the upstream boundary, which is 1.7 times the chord length upstream from the leading edge, the total pressure and the total temperature are held constant and the inlet flow angle is prescribed. The other variables are extrapolated. At the downstream boundary, which is 2.5 times the chord length downstream from the trailing edge, the static pressure is specified. On the blade surfaces and the walls, nonslip and adiabatic conditions are used. A permeable condition is used at the wake boundary.

3. Results and Discussions
3.1 Comparison with experimental data
Computed results were compared with the corresponding experimental data first in order to verify the effectiveness of the developed method. In the calculation, the inlet Mach number was set to be 0.5, although the corresponding Mach number in the experiment was 0.1. This is because the convergence of the computation becomes poor in the case of low Mach number.

Figure 3 shows the normal force coefficient $C_n$ against the inlet flow angle $\beta$. The computed results are compared with the experimental results in this figure and show good agreement. Because the blade load is 0 at $\beta=50^\circ$ in the calculated result, the angle of attack $\alpha$ is determined to be 0° at this angle. The slight difference of zero-lift angle between the computed and experimental results is considered to be caused by the difference of Mach number and the assumption of a laminar flow.

The velocity vectors on the plane perpendicular to the mean flow are shown in Fig. 4 for the position of 20 mm (one-third of the blade chord length) downstream from the blade trailing edge. The angle of attack $\alpha$ is 5 degrees (the cascade works as a compressor) and the tip clearance $\delta$ is 5 mm. Figure 4(a) shows the experimental results measured with 5-hole pitot tube. It can be said that the calculated results (Fig. 4(b)) well describe the experimental results.

![Fig. 2 Grid for tip clearance region](image)

![Fig. 3 Variation of the normal force at the midspan due to the inlet flow angle](image)
In addition, the computational results were compared with the experimental results about the trace of the tip vortex and the distribution of the static pressure on blades. Since all of the computed results showed good agreement with the experimental data, it can be concluded that the developed method is effective for the qualitative analysis of the flow through cascades with tip clearance.

3.2 Detailed view of the flow field around the blade tip

By the use of the developed method, the flow field around a blade tip was investigated in detail.

Figure 5 shows the velocity vectors on a plane close to the tip surface of a blade. The plane is parallel to the tip surface. The flow through the tip clearance is seen to separate at the edge between the tip surface and the pressure surface of a blade and reattach to the tip surface. As a result, a separation bubble is formed on the tip surface. Near the leading edge, a region of reverse flow is found. It is due to the flow separation at the leading edge of the tip surface as will be shown in the next figure.

Figure 6 shows the velocity vectors near the leading edge of the tip surface. In Fig. 6, the vectors on the plane including the centerline of the blade profile are seen from the pressure side of the blade. The velocity component toward the casing wall emerges near the leading edge of the tip surface. The flow separates at this edge and forms a separation bubble on the tip surface. In this figure, the wall boundary layer is seen to be very thin (the thickness is about 2

![Fig. 5 Vector diagram for tip leakage flow (α=5°, δ=5 mm, upper right side is the suction side of the blade)]

![Fig. 6 Velocity vectors around the leading edge of the tip surface (α=5°, δ=5 mm)]
mm) because the specified inlet condition is uniform in the spanwise direction. The influence of the wall boundary layer to the tip leakage flow should be investigated further.

Figure 7 is a velocity vector diagram of the tip leakage flow. It shows the vectors on the plane perpendicular to the blade chord. The location of the plane is 60% of the chord length from the leading edge. Vectors are viewed from the upstream side. The angle of attack is 5 degrees and the tip clearance is 5 mm. It is clearly found that the leakage flow rolls up into a tip vortex on the suction side of the blade. The generation of a separation bubble on the tip surface is also captured clearly.

As for the case of the small tip clearance of 0.1 mm shown in Fig. 8, no remarkable phenomenon is seen, although a small quantity of leakage flow is observed.

To study the roll-up process of the tip vortex, velocity vectors of the tip leakage flows are shown in Fig. 9 for the chordwise position of $c/c_o=0.2$ and 0.9. From Fig. 9(a), the tip vortex is found to roll up near the leading edge of the blade. As the vortex goes downstream, it grows larger and its center moves away from the blade suction surface and toward the hub.
Figure 10 shows the computed limiting streamlines on the tip surface for the case of $\alpha=5^\circ$ and $\delta=5\text{ mm}$. A reverse flow region due to the separation bubble on the leading edge can be clearly seen. Another region of the reverse flow is observed in the pressure side. It is owing to the separation bubble formed by the tip leakage flow. The formed separation bubble covers a wide area of the pressure side on the tip surface.

3.3 Spanwise distributions of the normal force on blades

Figure 11 shows the spanwise distribution of the normal force coefficient $C_N$ for various angles of attack $\alpha$. The tip clearance is 5 mm. When $\alpha=5^\circ$, $C_N$ has a flat distribution over the blade span. However, when $\alpha=10^\circ$, $C_N$ decreases in the midspan region because of the flow separation at the leading edge. In this case, $C_N$ has a peak on the blade tip. This is due to a strong tip vortex as was observed experimentally in Ref. (7).

When the tip clearance is small, the circulation around a blade is known not to diminish at the tip. Figure 12 shows the spanwise distribution of the normal force coefficient $C_N$ near the tip. $C_N$ on the tip is about zero when the tip clearance is 1 mm (0.6% of the blade span). However, when the clearance is under 1 mm, $C_N$ on the tip has a finite value; that is, a finite circulation is retained at the tip. The value of $C_N$ on the tip increases with a decrease in the tip clearance.

The static pressure distributions on the middle plane between the tip surface and casing are shown in Fig. 13. In the case of $\delta=5\text{ mm}$ (Fig. 13(a)), there seems to be little pressure difference between the suction side and the pressure side of the blade. In the small tip clearance case of $\delta=0.7\text{ mm}$ (Fig. 13(b)), on the other hand, the pressure distribution is just like the one in the case with a blade in this clearance region.

![Fig. 10 Computed limiting streamlines on the tip surface](image1)

![Fig. 11 Spanwise distribution of the normal force](image2)

![Fig. 12 Spanwise distribution of the normal force (near the blade tip)](image3)

![Fig. 13 Static pressure distribution in the tip clearance region (on the middle plane between tip surface and casing wall)](image4)
4. Conclusions

Three-dimensional subsonic flow through a cascade with tip clearance was analyzed numerically by solving the Navier-Stokes equations with a finite difference method. The computation was performed for a linear cascade composed of flat plates. The computational grid was generated by a zonal method in which a C-grid was generated around a blade and an H-grid was inserted in the space between the blade tip surface and the outer casing wall. The conclusions are summarized as follows:

1. By the zonal method of grid generation developed in this study, smooth solutions of the tip leakage flow could be obtained with the correct representation of the shape of the blade tip.
2. The validity of the developed solution method was verified through the comparison of the results with the experimental data.
3. The details of flow phenomena in the tip clearance region, such as the early stage of the process of vortex generation at the tip and the formation of the separation bubbles on the tip surface, were clearly revealed.
4. The normal force on the blade tip is found not to diminish when the tip clearance is under 0.6% of the blade span and to increase with decrease in tip clearance.

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