Analytical Study on Plate Edge Noise*

(Trailing Edge Noise Caused by Vorticity Waves)

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An analysis is performed on the trailing edge noise which is one of the important mechanisms of noise generation in flow machines. An acoustic field is treated where a semi-infinite flat plate is placed parallel to the inviscid uniform flow with incident vorticity waves convected from the upstream direction. Applying the Wiener-Hopf technique, we obtain an exact solution to the sound pressure proportional to the amplitude of the incident vorticity wave without restriction of frequency or velocity. The calculated acoustic field, which varies with flow velocity, exhibits general features of the sound pressure level (SPL) in a cardioid pattern with the constant phase surface distorted by the main flow. The relationship between flow velocity and SPL is ascertained to be dependent on the 5th law at low Mach numbers. However, the results show that such dependence does not hold at higher Mach numbers where the radiated noise level rises progressively as the flow velocity increases.

Key Words: Noise, Subsonic Flow, Aerodynamic Acoustics, Trailing Edge Noise, Vorticity Wave, Wiener-Hopf Technique

1. Introduction

Although it is recognized that aerodynamic sound originates from flow turbulence itself and also from the interaction between flow turbulence and body surface, the detailed mechanism has not been fully clarified. The trailing edge noise treated in this paper is representative of this sound and can be found in compressor cascade, struts in ducts, propellerfans, helicopter rotors, open wind tunnel walls, and so forth.

In general, it is supposed that trailing edge noise is generated by turbulence convected in the boundary layer along the wing surface. The turbulence interacts with the wing trailing edge and couples with unsteady vortex shedding from the edge. Although the problem may be influenced by the edge shape and details of the viscous boundary layer structure, we believe the phenomenon is primarily explained by the inviscid situation, where a semi-infinite flat plate of zero thickness is placed parallel in a uniform flow. That is to say, we

deal with the trailing edge only. In the past, the problem of trailing edge noise has been studied by Ffowcs-Williams and Hall⁽¹⁾, Crighton and Leppington⁽²⁾ and Levine⁽³⁾ based on Lighthill's theory. These authors treated this noise as radiation from quadrupoles located or convected near the trailing edge, where the quadrupoles represent flow turbulences. Crighton⁽⁴⁾ and Davis⁽⁵⁾ solved the problem of radiation of sound waves by adequate modeling of the shed vortex. On the other hand, Howe⁽⁶⁾ concluded that on the perfect imposition of the Kutta condition at the trailing edge, the shed vortex is generated so as to cancel out the sound pressure from the convected turbulent vortex. This tendency is partly confirmed by the experiment of Heavens⁽⁷⁾. Another experiment⁽⁸⁾ reports achieving noise reduction by replacing the solid trailing edge portion with porous materials. Thus, it can be said that the relationship between flow structure and noise generation involving the Kutta condition is far from being comprehensively understood. Other analyses by Chase⁽⁹⁾ and Chandiramani⁽¹⁰⁾ treated the sound spectrum from the turbulent boundary layer in a limited span range. Goldstein⁽¹¹⁾ investigated the structure of vorticity waves in a shear

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layer and examined the acoustic directivity in the far field from the leading edge or the trailing edge on the assumption of sufficiently large or sufficiently small frequencies.

To summarize these analyses, it can be reliably concluded⁽¹²⁾ that trailing edge noise has characteristics dependent on the 5th law of velocity and of dipolelike directivity, as ascertained by several experiments⁽¹³⁾⁽¹⁴⁾ in low Mach number flows. However, the characteristics in high Mach number flows have never been investigated.

In this paper, we introduce the vorticity wave, which is accompanied by no pressure fluctuation, as the turbulence in the flow. On the solid boundary, the vorticity wave is coupled with the potential field generated at the trailing edge, i.e., the radiated acoustic field. From this theory we obtain an exact solution to the sound pressure which is proportional to the amplitude of the incident vorticity wave. There is no restriction on flow velocity in the analysis. Therefore, we can examine the level and directivity of the generated acoustic field comprehensively.

2. Formulation

We investigated a two-dimensional flow field where a semi-infinite flat plate is placed parallel to the mean flow (see Fig. 1). On both sides of the plate there is a uniform and inviscid flow of density ρ_0 and velocity U . In the uniform flow, the vorticity waves without pressure fluctuations are assumed to be convected from infinite upstream which corresponds to the initial disturbances to the flow field. The sound radiation is produced so as to negate the normal velocity induced by the vorticity waves on the plate. Assuming small and isentropic disturbances, the governing equations are linearized as,

Fig. 1 Idealized model for the analytical problem

$$\frac{\partial \rho}{\partial t} + U \frac{\partial \rho}{\partial x} + \rho_0 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} \quad (2)$$

$$\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} \quad (3)$$

$$p = a^2 \rho \quad (4)$$

where a is the speed of sound, (u, v) , p , ρ are the disturbance components of the velocity, the pressure and the density. From these equations we have

$$(1 - M^2) \frac{\partial^2 \phi_d}{\partial x^2} + \frac{\partial^2 \phi_d}{\partial y^2} - \frac{1}{a^2} \frac{\partial^2 \phi_d}{\partial t^2} - \frac{2M}{a} \frac{\partial^2 \phi_d}{\partial x \partial t} = 0 \quad (5)$$

where $M = U/a$. This is the wave equation for the perturbation velocity potential ϕ_d where $u = \partial \phi_d / \partial x$, $v = \partial \phi_d / \partial y$. Since the actual sound wave propagation decreases in the viscous media, we shall treat this wave equation by adding a damping factor.

$$(1 - M^2) \frac{\partial^2 \phi_d}{\partial x^2} + \frac{\partial^2 \phi_d}{\partial y^2} - \frac{1}{a^2} \frac{\partial^2 \phi_d}{\partial t^2} - \frac{2M}{a} \frac{\partial^2 \phi_d}{\partial x \partial t} - \epsilon \frac{\partial \phi_d}{\partial t} = 0, \quad (6)$$

where ϵ is a small positive number, which is necessary in the mathematical treatment. We accordingly consider the limit of reducing the factor ϵ to zero. The velocity potential can be deformed as

$$\phi_d = \phi_0 e^{-i\omega t} = \phi_0 e^{-\frac{ikM}{1-M^2}x} e^{-i\omega t}, \quad (7)$$

where ω is the circular frequency and k is the wave number ($= \omega/a$). By applying this deformation to (6), we obtain

$$(1 - M^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + l^2 \phi = 0, \quad (8)$$

where

$$l^2 = \frac{k^2}{1 - M^2} + i\omega\epsilon. \quad (9)$$

Since l^2 has a positive imaginary part, we can write

$$l = l_1 + il_2; \quad l_1, l_2 > 0. \quad (10)$$

The incident vorticity wave is expressed by

$$v_i = \bar{v}(k_y) e^{-i\omega(t - \frac{x}{U})} e^{ik_y y}, \quad (11)$$

where k_y is the wave number in the y -direction and $\bar{v}$ is the amplitude of this wave. From Eqs. (2) and (3), the equation for the vorticity is

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \zeta = 0.$$

The solution of this equation is

$$\zeta = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \zeta_0 e^{-i\omega(t - \frac{x}{U})} e^{ik_y y}$$

$$u = u_0 e^{-i\omega(t - \frac{x}{U})} e^{ik_y y}$$

$$v = u_0 e^{-i\omega(t - \frac{x}{U})} e^{ik_y y}$$

$$u_0 = -\frac{k_y U}{\omega} v_0, \quad \zeta_0 = -\frac{iU}{\omega} \left(k_y^2 + \frac{\omega^2}{U^2} \right) v_0.$$

This vorticity wave surely has no pressure fluctuation. Furthermore, we introduce a small positive real number l_3 in the incident vorticity wave which should have a damping factor with respect to time.

$$v_i = \bar{v}(k_y) e^{-i(\frac{x}{\bar{v}} - it_3)(U_t - x)} e^{ik_y y} \quad (12)$$

The boundary conditions are as follows.

- (1) The normal velocity on the plate is zero.

$$\frac{\partial \phi_d}{\partial y} + v_i = 0; \quad y=0, \quad -\infty < x \leq 0,$$

from Eq.(12),

$$\frac{\partial \phi_0}{\partial y} = -\bar{v} e^{i(\frac{x}{\bar{v}} - it_3)x}. \quad (13)$$

- (2) The pressure is continuous on the wake surface.

$$p_{y=+0} = p_{y=-0}; \quad y=0, \quad 0 < x < \infty$$

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) (\phi_{d,y=+0} - \phi_{d,y=-0}) = 0$$

$$\phi_{0,y=+0} - \phi_{0,y=-0} \equiv \Delta \phi_{T.E.} e^{i(\frac{\omega}{\bar{v}})x} \quad (14)$$

That is to say, the potential jump at the trailing edge ($\Delta \phi_{T.E.}$) is convected downstream by the mean flow. This quantity should be related to $\bar{v}$.

- (3) The displacement of the material lines, η_y , are equal on the wake surface, i.e.,

$$\left. \frac{\partial \phi_d}{\partial y} \right|_{y=+0} = \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \eta_{y=+0}$$

$$\left. \frac{\partial \phi_d}{\partial y} \right|_{y=-0} = \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \eta_{y=-0}$$

$$\therefore \left. \frac{\partial \phi_0}{\partial y} \right|_{y=+0} = \left. \frac{\partial \phi_0}{\partial y} \right|_{y=-0}; \quad 0 < x < \infty. \quad (15)$$

- (4) The farthest radiation field from the trailing edge is obtained by the asymptotic expansion of the Hankel function.

$$|\phi| \sim C \exp \left\{ -l_2 \left(\frac{x^2}{1-M^2} + y^2 \right)^{1/2} \right\} \quad (16)$$

On treating the semi-infinite boundary-value problem of the wave equation (8), it is useful to apply the Wiener-Hopf technique⁽¹⁵⁾ based on the Fourier integral method. Let us take the Fourier transform as follows,

$$\left. \begin{aligned} \Phi(\alpha, y) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(x, y) e^{i\alpha x} dx \\ &= \Phi_+(\alpha, y) + \Phi_-(\alpha, y) \\ \alpha &= \sigma + i\tau \\ \Phi_+(\alpha, y) &= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \phi(x, y) e^{i\alpha x} dx \\ \Phi_-(\alpha, y) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 \phi(x, y) e^{i\alpha x} dx \end{aligned} \right\} \quad (17)$$

From Eq.(16), we see that Φ_+ is bounded in the semi-infinite field for $\tau > -l_2(1-M^2)^{1/2}$ and Φ_- is bounded for $\tau < l_2(1-M^2)^{1/2}$ of the α -plane. Since Φ_+ is regular in the upper half plane and Φ_- is regular in the lower half plane, Φ becomes regular in the following strip region.

$$-\frac{l_2}{\sqrt{1-M^2}} < \tau < \frac{l_2}{\sqrt{1-M^2}} \quad (18)$$

Applying Eq.(17) to (8), we find that

$$\frac{\partial^2 \Phi}{\partial y^2} - (1-M^2)\alpha^2 \Phi + l^2 \Phi = 0. \quad (19)$$

In the strip region of Eq.(18), the solution to Eq.(19) satisfying the boundary condition (3) is

$$\begin{aligned} \Phi(\alpha, y) &= \Phi_+ + \Phi_- \\ &= \begin{cases} A(\alpha) e^{-\gamma y} (y \geq 0) \\ -A(\alpha) e^{\gamma y} (y \leq 0), \end{cases} \end{aligned} \quad (20)$$

where

$$\gamma = \{(1-M^2)^{1/2} \alpha + l\}^{1/2} \cdot \{(1-M^2)^{1/2} \alpha - l\}^{1/2}. \quad (21)$$

The branch cut of this function γ is shown in Fig. 2.

From Eq.(14) we find

$$\begin{aligned} \Phi_+(+0) - \Phi_+(-0) &= \frac{\Delta \phi_{T.E.}}{\sqrt{2\pi}} \int_0^{\infty} e^{i(\frac{\omega}{\bar{v}} + i\frac{KM}{1-M^2} + \alpha)x} dx \\ &= \frac{-\Delta \phi_{T.E.}}{\sqrt{2\pi} i \left\{ \frac{\omega}{aM(1-M^2)} + \alpha \right\}} \equiv F_+(\alpha), \end{aligned} \quad (22)$$

where $\Phi_{\pm}(\pm 0)$ means $\Phi_{\pm}(\alpha, y = \pm 0)$. The integrand must be in $\tau > 0$ region for its convergence and the function $F_+(\alpha)$ defined in Eq.(22) is regular in this upper half plane. From Eq.(15), we find

$$\Phi'_+(+0) = \Phi'_+(-0) \equiv \Phi'_+(0), \quad (23)$$

where ' means $\partial/\partial y$. By differentiating Eq.(22) and by applying Eq.(22) we find

$$\Phi'_-(+0) = \Phi'_-(-0) \equiv \Phi'_-(0). \quad (24)$$

By applying Eqs.(23) and (24) we find from Eq.(20)

$$\Phi'_+(0) + \Phi'_-(0) = -\gamma A. \quad (25)$$

From Eqs.(13) and (24), we find

$$\begin{aligned} \Phi'_-(0) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 \left\{ -\bar{v} e^{i(\frac{\omega}{\bar{v}} - i\frac{KM}{1-M^2} + \alpha)x} \right\} dx \\ &= \frac{-\bar{v}}{\sqrt{2\pi} i \left\{ \frac{\omega}{aM(1-M^2)} - i\frac{KM}{1-M^2} + \alpha \right\}} \equiv G_-(\alpha), \end{aligned} \quad (26)$$

where the defined function $G_-(\alpha)$ is regular in the lower half plane $\tau > l_3$. Applying Eqs.(22)~(26) and eliminating $A(\alpha)$, we obtain the next two equations.

$$\Phi_+(-0) = -S_-(\alpha) - \frac{F_+(\alpha)}{2} \quad (27)$$

$$\Phi'_+(0) + G_-(\alpha) + \frac{\gamma}{2} F_+(\alpha) = -\gamma D_-(\alpha), \quad (28)$$

Fig. 2 Branch-cut

where

$$2S_-(\alpha) = \Phi_- (+0) + \Phi_- (-0)$$

$$2D_-(\alpha) = \Phi_- (+0) - \Phi_- (-0).$$

Both $S_-(\alpha)$ and $D_-(\alpha)$ are regular in $\tau < l_2(1-M^2)^{1/2}$.

In the following formulation, we will solve Eq.(28). Dividing Eq.(28) by $\{(1-M^2)^{1/2} + l\}^{1/2}$, we obtain

$$\frac{\Phi'_+(0)}{\{(1-M^2)^{1/2}\alpha + l\}^{1/2}} + \frac{G_-(\alpha)}{\{(1-M^2)^{1/2}\alpha + l\}^{1/2}} + \{(1-M^2)^{1/2}\alpha - l\}^{1/2} \frac{F_+(\alpha)}{2} = \{(1-M^2)^{1/2}\alpha - l\}^{1/2} D_-(\alpha) \quad (29)$$

Each term of the left-hand side of Eq.(29) is regular in the following region respectively,

$$\tau > -\frac{l_2}{\sqrt{1-M^2}}, -\frac{l_2}{\sqrt{1-M^2}} < \tau < l_3, 0 < \tau < \frac{l_2}{\sqrt{1-M^2}}, \tau < \frac{l_2}{\sqrt{1-M^2}}.$$

Applying Eqs.(26) and (22), we decompose the second and the third term of the left hand side of Eq.(29) to one part which is regular in a certain upper half plane and the other part which is regular in a certain lower half plane.

$$\begin{aligned} \frac{G_-(\alpha)}{\{(1-M^2)^{1/2}\alpha + l\}^{1/2}} &= H_+(\alpha) + H_-(\alpha) \\ H_+(\alpha) &= \frac{i\bar{v}}{\sqrt{2\pi} \left\{ \frac{\omega}{aM(1-M^2)} - il_3 + \alpha \right\}} \cdot \left[\frac{1}{\{(1-M^2)^{1/2}\alpha + l\}^{1/2}} - \frac{1}{\{(1-M^2)^{1/2} \left(-\frac{\omega}{aM(1-M^2)} + il_3 \right) + l\}^{1/2}} \right] \\ H_-(\alpha) &= \frac{i\bar{v}}{\sqrt{2\pi} \left\{ \frac{\omega}{aM(1-M^2)} - il_3 + \alpha \right\}} \cdot \left[(1-M^2)^{1/2} \left\{ -\frac{\omega}{aM(1-M^2)} + il_3 \right\} + l \right]^{1/2} \\ \{(1-M^2)^{1/2}\alpha - l\}^{1/2} \frac{F_+(\alpha)}{2} &= I_+(\alpha) + I_-(\alpha) \\ I_+(\alpha) &= \frac{i\Delta\phi_{T.E.}}{2\sqrt{2\pi} \left\{ \frac{\omega}{aM(1-M^2)} + \alpha \right\}} \cdot \left\{ -\frac{\omega}{aM(1-M^2)^{1/2}} - l \right\}^{1/2} \\ I_-(\alpha) &= \frac{i\Delta\phi_{T.E.}}{2\sqrt{2\pi} \left\{ \frac{\omega}{aM(1-M^2)} + \alpha \right\}} \cdot \left[\{(1-M^2)^{1/2}\alpha - l\}^{1/2} - \left\{ -\frac{\omega}{aM(1-M^2)^{1/2}} - l \right\}^{1/2} \right], \end{aligned}$$

where the defined function H_+ is regular in $\tau > -l_2(1-M^2)^{1/2}$, H_- is regular in $\tau < l_3$, I_+ is regular in $\tau > 0$ and I_- is regular in $\tau < l_2(1-M^2)^{1/2}$. By these decompositions, Eq.(29) can be divided into one part which is regular in the upper half plane and another part which is regular in the lower half plane. Then we can define $J(\alpha)$ as follows:

$$J(\alpha) \equiv \frac{\Phi'_+(0)}{\{(1-M^2)^{1/2}\alpha + l\}^{1/2}} + H_+ + I_+ = -H_- - I_- - \{(1-M^2)^{1/2}\alpha - l\}^{1/2} D_-, \quad (30)$$

where each term has an overlapping region of regularity in $0 < \tau < l_3$ or $0 < \tau < l_2(1-M^2)^{1/2}$. By use of the analytical continuation process, $J(\alpha)$ is confirmed to be regular throughout the α -plane and thus to be constant. Since each term of Eq.(30) can be considered to be zero for $\alpha \rightarrow \pm\infty$, $J(\alpha)$ must be zero. Eventually the function Φ'_+ is obtained from Eq.(30) and we can write $A(\alpha)$ from Eq.(25) by applying Eq.(26).

$$A(\alpha) = -\frac{1}{\gamma} \{ \Phi'_+(0) + \Phi'_-(-0) \} \quad (31)$$

Now we must relate $\Delta\phi_{T.E.}$ to $\bar{v}$. From Eq.(14) and the Fourier inversion of Eq.(17), we can write

$$\begin{aligned} \phi_{0,y=+0} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty+is}^{\infty+is} A(\alpha) d\alpha \\ \phi_{0,y=-0} &= -\frac{1}{\sqrt{2\pi}} \int_{-\infty+is}^{\infty+is} A(\alpha) d\alpha \\ \Delta\phi_{T.E.} &= \lim_{x \rightarrow 0} (\phi_{0,y=+0} - \phi_{0,y=-0}) \\ &= \frac{2}{\sqrt{2\pi}} \int_{-\infty+is}^{\infty+is} A(\alpha) d\alpha, \end{aligned} \quad (32)$$

where an arbitrary constant s must be in $0 < s < l_3$ or $0 < s < l_2/(1-M^2)^{1/2}$. Hereafter, we consider the formulation by $l_2, l_3 \rightarrow 0$ since the factor ε in Eq.(6) can set to zero. From Eqs.(31) and (32) we obtain

$$\Delta\phi_{T.E.} = \frac{2I_0\bar{v}}{\frac{k}{M}I_0 - 2\pi k \left(\frac{1-M}{M} \right)^{1/2}} \quad (33)$$

$$I_0 = \int_{-\infty}^{\infty} \frac{d\tilde{a}}{\left\{ \tilde{a} + \frac{1}{M(1-M^2)} \right\} \left(\tilde{a} - \frac{1}{1-M^2} \right)^{1/2}}, \quad (34)$$

where $\tilde{a} = a/k$, I_0 is a constant number determined only by Mach number. The relationship between $\Delta\phi_{T.E.}$ and $\tilde{v}$ is shown in Fig. 3. $A(a)$ is obtained as follows:

$$A(a) = \frac{\sqrt{2\pi} \{M(1-M^2)\}^{1/2} \tilde{v}}{\left\{ \frac{k}{M(1-M^2)} + a \right\} \left\{ a - \frac{k}{1-M^2} \right\}^{1/2} \left\{ \frac{k(1-M)}{M} \right\}^{1/2} [I_0 - 2\pi \{M(1-M)\}^{1/2}]}. \quad (35)$$

By applying Eqs. (35) and (20) and the Fourier inversion of Eq. (17), we can express the sound pressure p in $y \geq 0$ as follows:

$$p = -\rho_0 \left(\frac{\partial \phi_d}{\partial t} + U \frac{\partial \phi_d}{\partial x} \right) \\ \frac{p}{\rho_0 \tilde{v} a} = \frac{iM^2 e^{-i\frac{kM}{1-M^2}x}}{k^{1/2} [I_0 - 2\pi \{M(1-M)\}^{1/2}]} \cdot \int_{-\infty}^{\infty} \frac{e^{-iax - \tau y}}{\left(a - \frac{k}{1-M^2} \right)^{1/2}} d\alpha \quad (36)$$

Equation (36) can be nondimensionalized by the wave number k and the sound pressure can be calculated from the following equation.

$$\frac{p}{\rho_0 \tilde{v} a} = \frac{iM^2 \exp\left(-i\frac{M}{1-M^2}X\right)}{[I_0 - 2\pi \{M(1-M)\}^{1/2}]} \cdot \int_{-\infty}^{\infty} \frac{\exp\left[-i\tilde{a}X - \left\{(1-M^2)\tilde{a}^2 - \frac{1}{1-M^2}\right\}^{1/2}Y\right]}{\left(\tilde{a} - \frac{1}{1-M^2}\right)^{1/2}} d\tilde{a}, \quad (37)$$

where $X = kx$ and $Y = ky$.

3. Characteristics of trailing edge noise

One of the most interesting feature of trailing edge noise is the frequency distribution of the generated noise, which is expected to be dependent on the thickness of the plate and the shape of the edge, the boundary layer, and so forth. However, in this analysis nothing other than the wave number has the dimension of length since we assume that the semi-infinite flat plate of zero thickness is placed in the idealized inviscid uniform flow field. Thus, we cannot estimate the frequency dependence from Eq. (37) which is nondimensionalized by the wave number. Here we discuss only the sound pressure distribution around the trailing edge and its dependence on the mean flow Mach

number. Although there is also the important problem of how the Kutta condition should be applied, we do not explicitly introduce this condition and leave the trailing edge as a singularity.

As described in Eq. (11), the incident vorticity wave has the wave number in the y -direction, but the induced sound pressure is not influenced by it. Since the acoustic field can be represented by the summation of each sound for each k_y , we can estimate its general aspects by the result for a single k_y . The integral of Eq. (37) can be calculated accurately because the integrand decays exponentially.

3.1 Acoustic field around the trailing edge

The sound pressure distribution for three Mach numbers of the mean flow is shown in Fig. 4, where the upper figures are the amplitude contours of the sound pressure and the lower figures are the phase contours on the wave fronts. We omit the calculation and the illustration of the lower half field because it is clear from Eq. (20) that the lower half field has the same amplitude but the opposite phase to the upper half field. The amplitude of the sound pressure on the wake is zero since the pressure on it is continuous. The directivity shows a cardioid pattern like the radiation field of a dipole source on the edge of a semi-infinite rigid plate. The azimuth of the strongest pressure amplitude point is constant at any distance from the trailing edge. As the Mach number increases, the pressure amplitude shows less variation in the y -direction, a fact which can be understood through the transformation of the coordinate system in the wave equation. The constant phase surface becomes more distorted from the spherical pattern. This distortion

Fig. 3 Variation of potential jump at the trailing edge with mean flow Mach number

Fig. 4 Calculated acoustic field by Eq.(37) (upper : amplitude, lower : phase)

Fig. 5 Sound pressure on the plate ($M=0.1, 0.5, 0.9$)

indicates that the sound originated from the trailing edge propagates upstream is forced back by the mean flow and carried downstream.

The wall pressure on the plate is shown in Fig. 5 for three different Mach numbers. It is ascertained

Fig. 6 Transformed coordinate

from the phase variation that the sound propagates at the speed of $(a - U)$ on the plate. The amplitude of the sound pressure shows the singular characteristics of the trailing edge and decreases in proportion to the inverse square root of $(-X)$.

3.2 Comparison with a particular solution

From basic equations (1) ~ (4), we derive the following wave equation to describe sound pressure p ,

$$(1 - M^2) \frac{\partial^2 p}{\partial x^2} - 2ikM \frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial y^2} + k^2 p = 0. \quad (38)$$

One particular solution to it on the transformed coordinate in Fig. 6 is

$$p = \frac{1}{\sqrt{lr}} e^{i \frac{kM}{1-M^2} x} e^{-i l r} \cos \frac{\theta}{2}. \quad (39)$$

The amplitude contour and the phase surface at $M=0.5$ is calculated by Eq.(39) and shown in Fig. 7. We conclude that the acoustic field (Fig. 4) approximately follows this particular solution alone. That is to say, the sound radiated from the trailing edge decreases by the inverse square root of r and has a directivity dependent on $\theta/2$.

3.3 Dependence on the mean flow Mach number

Equation (39) describes only the appearance of

Fig. 7 Calculated acoustic field by Eq.(39) ($M=0.5$)

the acoustic field. On the other hand, Eq.(37) is quite useful because it gives the intensity of the generated sound in relation to the incident velocity disturbance. To investigate the relationship between the flow velocity and the sound strength, we calculate the amplitude of the sound pressure at each 5-degree azimuthal angle on the semicircle with its center on the trailing edge and with a nondimensional radius of 1. Then, to express the sound strength, we integrate the square of the sound pressure on the circle. This integral is considered to represent the average sound pressure level. The amplitude of the vorticity wave $\bar{v}$ must be known beforehand, but now the assumption that it is a quantity proportional to the mean flow velocity is introduced. By this appropriate assumption, the averaged SPL is obtained and shown in Fig. 8. The dashed line in it has an inclination of 5 which means $p^2 \sim M^5$. Figure 8 shows that this analysis confirms the 5th law at low Mach numbers as indicated by other analyses and experiments. In addition to this, we must notice here the difference between the calculated SPL line and the dashed line at higher Mach numbers. The variation of the n -th dependence $p^2 \sim M^n$ versus the mean flow Mach number is shown in Fig. 9. From this, it is clear that the exponent of n -th dependence is almost 5 near the $M=0$ region, but that it increases significantly near $M=1$ and the dependence of the fifth law no longer holds there. The experimental data by Brooks and Hodgson⁽¹³⁾ of $n=5.07$ at $U \leq 73.4$ m/s can be thought to represent the increasing tendency of the exponent of the n -th law. This fact is naturally

Fig. 8 Variation of the averaged sound pressure level with mean flow Mach number

Fig. 9 Variation of the index- n with mean flow Mach number

supported by our analysis, and more data for higher Mach numbers should be prepared for comparison. Incidentally, this Mach number dependence never changes due to the distance from the trailing edge. This tendency is also suggested also in 3.1 and 3.2.

4. Conclusions

In this paper, the sound generation at the trailing edge has been formulated in a simple model wherein a semi-infinite plate is placed parallel to the inviscid compressive uniform flow. The sound is thought to be caused by the convected vorticity waves as the initial

velocity disturbance. We apply the Wiener-Hopf technique based on the two-dimensional linear theory and obtain an exact solution without any restriction on frequency or mean flow velocity. The potential jump introduced in this theory is found to be proportional both to the mean flow velocity and the amplitude of the vorticity waves.

The calculated acoustic field shows the characteristics of sound generation and radiation at the trailing edge. The outline of the acoustic field shows no dependence on the distance from the trailing edge and can be approximately represented by a simple particular solution whose directivity is $\cos(\theta/2)$ and whose decay rate is $1/\sqrt{r}$.

The SPL's dependence on the 5th power of the mean flow velocity, which has been pointed out by other researchers, is also ascertained in this analysis at low Mach numbers. In addition to this, it is found that the SPL increases more progressively than the 5th law, and that the velocity law no longer holds at higher Mach numbers.

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