Turbulent Behavior below an Air-Water Interface Affected by Wall Bursting, Interfacial Shear Stress, and Interfacial Waves*

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The momentum transfer across an air-water interface was experimentally studied in a horizontal rectangular channel. The water flow Reynolds number, based on a hydraulic diameter, was fixed at 11 500, while the air flow Reynolds number was varied from 0 to 5 000. The streamwise air flow velocity was measured by a hot-wire probe. For the water flow, streamwise and vertical velocities were measured by using a split-film probe. The behavior of streamwise turbulence intensities showed their dependence on the interfacial shear stress. On the other hand, the vertical turbulence intensities showed less dependence on the interfacial shear stress. When the interfacial waves were formed, the increased Reynolds stress near the interface was measured. As a possible explanation of this increased Reynolds stress, nonlinear motion caused by the nonuniform distribution of the interfacial shear stress was proposed.

Key Words: Turbulent Flow, Multiphase Flow, Free Surface, Wave, Stratified Flow

1. Introduction

As often seen in various industrial devices, transfer phenomena occurring at a gas-liquid interface can be much more intensive than those occurring at a wall. Many studies have been performed on the heat and mass transfer phenomena and empirical correlations have been proposed as reviewed by Bankoff⁽¹⁾ and Sideman and Moalem-Maron⁽²⁾. Some experimental studies^{(3)~(8)} were conducted to see how the turbulence controls transfer phenomena across the gas-liquid interface. From experiments in an open-channel flow, Komori et al.⁽⁶⁾ discovered that the surface renewal eddies were highly correlated with the bursting events on the wall side. Recently, Banerjee⁽⁸⁾ performed experiments and computations on the tur-

bulence structure below the smooth air-water interface with interfacial shear stress. He observed that a streaky or patchy structure, similar to that in the wall turbulence, appears in the vicinity of the smooth interface. These studies give us an image of turbulent behavior below the interface.

In the field of numerical analysis, the k - ϵ model was developed to simulate the high Reynolds number turbulence and the wall turbulence quantitatively. Furthermore, the limiting behavior of the turbulence quantities to the wall was clarified⁽⁹⁾. However, when the gas-liquid interface is considered as a boundary condition, the k - ϵ model can no longer be applied. This deficiency comes from the unknown turbulent behavior near the interface and the unknown boundary conditions of k and ϵ at the interface. Some attempts^{(10)~(12)} to solve the flow field below the interface numerically by the k - ϵ model were performed; at present, however, the turbulent behavior in the vicinity of the interface is not very well understood. Thus it is necessary to perform experiments to clarify the turbulence near the interface so that we can model the turbulence for numerical analysis.

Very recently, Lam and Banerjee⁽¹³⁾ computed a

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stratified flow with interfacial shear stress by using direct numerical simulation (DNS). Ideally, DNS does not contain any assumption, and the computed result may be regarded as a true picture. By the advancement of computer ability, DNS will be established as a major method for investigating the turbulence near the interface.

The present authors have performed experiments and computations on the gas-liquid interface in a horizontal rectangular channel⁽¹⁴⁾. In this study, the bursting events occurring on the wall were observed to affect the turbulence near the interface. Moreover, the interfacial shear stress could cause a phenomenon similar to the bursting events on the wall. These results are consistent with the studies of Komori et al.⁽⁶⁾ and Banerjee⁽⁸⁾.

In the present study, our attention is focused on the turbulent behavior below the smooth and wavy interfaces. Experiments are performed in an air-water stratified cocurrent flow in a horizontal rectangular channel. The velocities are measured by using a hot-wire probe for an air flow and a split-film probe for a water flow, respectively. The measurement of the two velocity components enables us to see the behavior of the velocity fluctuation which is normal to the interface. The Reynolds stress is also obtained from the two velocity components.

Nomenclature

- a_0 : coefficient
 c_w : wave velocity, m/s
 D : hydraulic equivalent diameter, m
 g : acceleration of gravity, m/s²
 H_w : wave height, m
 h_L : liquid film thickness, m
 Re : Reynolds number $= U_m D / \nu$
 T : wave period, s
 t : time, s
 U : time-averaged velocity in streamwise direction, m/s
 u : velocity fluctuation in streamwise direction, m/s
 u_{rms} : root-mean-square velocity fluctuation in streamwise direction, m/s
 u_w : streamwise velocity oscillation caused by interfacial waves, m/s
 u_* : wall friction velocity, m/s
 u_{i*} : interfacial friction velocity, m/s
 v : velocity fluctuation in vertical direction, m/s
 v_{rms} : root-mean-square velocity fluctuation in vertical direction, m/s
 v_w : vertical velocity oscillation caused by interfacial waves, m/s
 x : coordinate in streamwise direction, m

- y : coordinate in vertical direction, m
 y^+ : nondimensional distance from the wall $= u_* y / \nu$
 y_i : distance from the interface, m
 y_i^+ : nondimensional distance from the interface $= u_{i*} y_i / \nu$
 ν : kinematic viscosity, m²/s
 λ : wavelength, m

Subscripts

- G : gas
 i : interface
 L : liquid
 m : averaged value in a cross section of each phase
 w : wave
 $*$: friction velocities

2. Experimental Apparatus and Procedures

The experimental apparatus is shown in Fig. 1, which is the same as used in Ref. (14). The test section 6 is a rectangular channel, 6.0m in length, 400 mm in width, and 40 mm in height. Air and water at approximately room temperature and atmospheric pressure are used as test fluids. The water is circulated by a pump 2, the capacity of which is controlled by an inverter. The water passes through a 5 μ m filter 3, a flowmeter 4, and then enters the test section. The water level in the test section is controlled by the weir installed in the outlet tank 8. In order to exclude otherwise possible upstream influences of the weir, an expansion part 7, 800 mm wide, is installed (see Fig. 2). The room air is sucked by a vacuum pump 10, which is connected to the outlet tank 8, and the air flows in the same direction as the water. The water temperature, water flow rate, air temperature, air

Fig. 1 The experimental apparatus

Fig. 2 Details of the entrance and outlet part of the apparatus

flow rate, velocity profiles of both phases, water level, interfacial wave heights, and interfacial wave frequencies are measured. The air velocity probe, the water velocity probe, and the wave height gauge are installed at the spanwise center at 5 020, 5 030, and 5 110 mm downstream from the entrance, respectively. In terms of hydraulic diameter, D , these locations are beyond $50D_L$ (about $200h_L$) for the liquid phase, and beyond $150D_G$ for the gas phase, so the fully developed flow is established there. The spanwise uniformity of the velocity profile is confirmed by comparing the measurements at the center line and at the location 5 cm from the side wall. The temperature of water and air is measured by chromel-alumel thermocouples near the location of the velocity measurement. The output of these thermocouples is amplified 500 times using a D.C. amplifier (Kyowa Dengyo DA-510B). Interfacial wave heights and frequencies are measured by a water level gauge. This water level gauge consists of two platinum wires of 0.1 mm in diameter. These wires are mounted vertically in the channel about 10 mm apart in the spanwise direction. The instantaneous water level is proportional to the electric resistance. The resistance variation is recorded by a method similar to that of the strain gauge. Velocity profiles are measured by a CTA unit (KANOMAX MODEL 1010, 1011, 1013, 1008). A velocity probe is set on a traverse stage and is able to move in the vertical direction. Each output is temporarily stored in an A.D. converter (TEAC PS-9351), which is controlled by a microcomputer, and is then transferred to a floppy disk. The split-film probe (TSI MODEL 1 287 W, 152 μm in diameter) is calibrated by using a calibration pool. The calibration carriage on which the probe is mounted is driven by a stepping motor. To determine the angle dependence of the output of the split-film

probe, the angle of the calibration carriage is varied from -30 degrees to $+30$ degrees. The hot-wire probe (KANOMAX MODEL 0248R-T5, 5 μm in diameter) is calibrated by using a probe calibrator (KANOMAX MODEL 1065). The calibration is performed before and after each experiment, and the result is certified to be within a level of unavoidable uncertainty. The ambient fluid temperature varies during the calibration and/or the experiment, and the effect of temperature variation is compensated by using a temperature compensation equation, which is derived by assuming that the heat transfer coefficient at the probe surface is constant⁽¹⁴⁾.

The sampling frequency of the A.D. converter is 100 Hz for the water flow and 1 000 Hz for the air flow. The 16384 sampling data are recorded for each measurement (4096 sampling data for laminar cases of the air flow). Kolmogorov's microscales are estimated to be 0.2 mm for the water flow, and 0.15 mm for the air flow. These scales are larger than the diameter of each probe. Data reduction is performed by a microcomputer. The wall friction velocities of both phases and the interfacial friction velocity of the gas phase are determined through fitting the mean velocity profiles to the universal velocity profile. Uncertainty analysis is performed and the results are indicated by error bars in each figure, whose confidence level is 95% coverage. When measured values are used as nondimensionalizing parameters, the uncertainty of those values is also taken into account. When the size of the error bars is less than the size of the symbols, the error bars are omitted.

3. Results and Discussion

3.1 The experimental condition

In this study, our interest is on how the fully

developed turbulence in the vicinity of the interface is affected by the bursting events which originate from the lower wall. We are also interested in examining the effect of the interfacial shear stress on the turbulence below the smooth and wavy interface. Experimental conditions are summarized in Table 1. The Reynolds number is defined by choosing the hydraulic diameter and the cross-sectional averaged mean velocity as the length and velocity scales, respectively. When $Re_G = 5000$, three-dimensional, pebble-like interfacial waves were formed. The wave amplitude, the frequency, and the wavelength are 0.56 mm, 11 Hz, and 4 cm, respectively. The time trace of the water level is shown in Fig. 3.

3.2 Turbulence intensity

Liquid-phase turbulence intensities are shown in Fig. 4, where turbulence intensities are nondimensionalized by the wall friction velocity. The

Table 1 The experimental conditions

	Reynolds number	Temperature [°C]	Mean velocity [cm/s]	Water level [mm]
Split-film Cocurrent				
Water	11500	20.5–21.1	11.8–13.7	23.4–27.3
Air	0–5000	20.4–21.1	0–229.6

Fig. 3 Time trace of the water level

Fig. 4 Streamwise and vertical turbulence intensities

streamwise turbulence intensity, u_{rms}/u_* , becomes large due to the interfacial shear stress on the interface side. This can be explained by considering the increased turbulence production by the mean shear stress which is caused by the interfacial shear stress. When the interfacial waves are formed, the streamwise turbulence intensity becomes very large.

The vertical turbulence intensity, v_{rms}/u_* , approaches zero when approaching the interface for the smooth interface cases, in agreement with the DNS result of Lam and Banerjee⁽¹³⁾. The disappearance of the vertical velocity fluctuation at the interface is recognized because of the existence of a flat interface. The interfacial shear stress does not affect the vertical turbulence intensities as much as it does the streamwise turbulence intensities. When the interfacial waves are formed, the vertical turbulence intensities are also affected by the periodic interfacial waves. At the nearest point to the interface, the vertical turbulence intensity has almost the same value as the streamwise turbulence intensity. The contribution of the interfacial waves to the transport phenomena will be discussed in the next section.

3.3 Reynolds stress

The Reynolds stress is obtained in two different ways. One is based on the linear distribution of shear stress. For the fully developed flow, the Reynolds stress is expressed as below⁽⁴⁾;

$$-\overline{uv} = u_*^2 \left(1 - \frac{y_L}{h_L} \right) + u_*^2 \frac{y_L}{h_L} - \nu \frac{\partial U_L}{\partial y_L}, \quad (1)$$

where the velocity gradient is calculated from the experimental results. In the other method, the Reynolds stress is directly obtained from the measured correlation between velocity fluctuations.

In Fig. 5, the Reynolds stress for the smooth interface is shown. The uncertainty is large, but the

Fig. 5 The Reynolds stress profile for the case of the smooth interface with interfacial shear stress; $Re_L = 11500$, $Re_G = 3260$

Reynolds stress from the velocity fluctuations and that from the linear distribution of shear stress coincide qualitatively. The results for the wavy interface are shown in Fig. 6. In this case, the Reynolds stress obtained from the velocity correlation is larger than that estimated from the linear distribution of shear stress. The deviation from the linear distribution will be explained below. It should be noted here that the periodic motion of fluid caused by the interfacial waves does not contribute to the transfer phenomena by itself. For example, the third-order analysis of finite amplitude waves in deep water gives the periodic velocities as below⁽¹⁵⁾;

$$u_w = \frac{2\pi a_0}{\lambda} c_w \exp(-2\pi y_i/\lambda) \cos\left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t\right) \quad (2)$$

$$v_w = \frac{2\pi a_0}{\lambda} c_w \exp(-2\pi y_i/\lambda) \sin\left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t\right) \quad (3)$$

where

$$a_0 = \frac{H_w}{2} \left[1 - \frac{3\pi^2}{8} \left(\frac{H_w}{\lambda} \right)^2 \right] \quad (4)$$

$$c_w = \sqrt{\frac{g\lambda}{2\pi} \left[1 + \left(\frac{2\pi a_0}{\lambda} \right)^2 \right]^{0.5}} \quad (5)$$

$$\lambda = \frac{gT^2}{2\pi} \left[1 + \left(\frac{2\pi a_0}{\lambda} \right)^2 \right]. \quad (6)$$

These are the velocity oscillations in the streamwise and vertical directions caused by the periodic interfacial waves. However, because these oscillations do not have the component in phase, the product of the two velocity oscillations vanishes in time average ;

$$-\overline{u_w v_w} = 0. \quad (7)$$

In the previous experiments⁽¹⁶⁾⁽¹⁷⁾ on the heat transfer at a steam-water interface, the increased transfer has been reported to be caused by the effect of interfacial waves. Some of the previous modelings of promoted transport by interfacial waves were based on the scale consideration analogous to the mixing length hypothesis of turbulent flows. However, the fact that the periodic fluid movement itself does

Fig. 6 The Reynolds stress profile for the case of the wavy interface with interfacial shear stress ; $Re_L = 11\,500$, $Re_c = 5\,000$

not contribute to the transfer indicates that those kinds of the previous modelings of interfacial waves cannot explain the actual mechanism for the promoted transport observed in the experiments.

One feasible way to explain the increased Reynolds stress is given below. Okuda et al.⁽¹⁸⁾ observed a nonlinear motion below a wavy interface. The motion was caused by the nonuniform distribution of interfacial shear stress. As shown in Fig. 7, on the windward side of interfacial waves, the shear stress is large and the fluid is accelerated. On the leeward side, the separation can occur and the shear stress becomes less than that on the windward side. The accelerated fluid on the windward side goes into the weakly sheared region on the leeward side and is blocked by the leeward fluid. The blocked fluid may go downwards. Okuda et al. observed this motion in their experiments. If this structure is formed below the interface, the turbulent structure below the wavy interface may be synchronized to the configuration of the wavy interface.

Near the wavy interface, the probe from the lower wall approaches closer to the trough of the interfacial waves than their crest. If the contribution to the Reynolds stress is locally large below the trough, the measured large Reynolds stress can be explained. The nonlinear motion explained above is synchronized to the configuration of interfacial waves, so it is expected that there exists a region where the contribution to the Reynolds stress is locally large according to the phase of the interfacial waves.

3.4 Influence of wall bursting and interfacial shear stress

It has been reported that the bursting events affected the turbulence below the interface in an open-channel flow by Komori et al.⁽⁶⁾, Banerjee⁽⁸⁾, and Murata et al.⁽¹⁴⁾. In those studies, the bursting event is identified as the abrupt and intermittent lifting-up of fluid from the wall side toward the interface. This upward motion reaches the interface and affects the turbulence below the interface.

When the interfacial shear stress is imposed on the interface, Banerjee observed that the streaky

Fig. 7 The nonlinear motion caused by the nonuniform distribution of interfacial shear stress along the wavy interface

structure could occur below the interface. His study is supported by the study of Lee et al.⁽¹⁹⁾ who showed that only the mean shear stress is important in order to form the streaky structure.

In this section, to see the effect of the wall bursting events on the turbulence below the interface, the time series of the velocity fluctuations (u, v) are plotted on the four quadrants, where the horizontal and vertical axes indicate the streamwise and vertical velocity fluctuations, respectively. The effects of the interfacial shear stress and the interfacial waves are also discussed.

Figure 8 shows the wall side case without the interfacial shear stress. In the figure, the hyperbolas correspond to $|uv|=8(-\overline{uv})$; $-\overline{uv}$ is the measured local Reynolds stress. The points plotted outside the hyperbolas correspond to the large contribution to the Reynolds stress. Kim et al.⁽²⁰⁾ showed that the ejection event ($u < 0, v > 0$) and the sweep event ($u > 0, v < 0$) dominate the Reynolds stress near the walls. In Fig. 8, the contribution from these two quadrants dominates the Reynolds stress and the ejection event shows a larger contribution than does the sweep event at this location ($y^+ = 46.7$).

In Fig. 9, the result of the interface side with no interfacial shear stress is shown. It is seen that the ejection and sweep events are predominant on the interface side as well. However, by the existence of the flat interface, the fluctuation normal to the interface is restricted. The distribution becomes very dense as compared with that of the wall side (see Fig. 8). In the case of a smooth interface with interfacial shear stress (Fig. 10), this tendency is also seen. From the comparison between Fig. 9 and Fig. 10, it seems that the interfacial shear stress makes the distribution scatter.

In this study, the interfacial shear stress is

Fig. 8 Distribution of (u/u_* , v/u_*) on the wall side with no interfacial shear stress; $Re_L = 11\,500$, $Re_G = 0$, $y^+ = 46.7$

imposed by the cocurrent air flow. In this case, the directions of the ejection event and the sweep event on the interface side become reversed as compared with that on the wall side. As explained in Fig. 11, the second quadrant represents the ejection events on the

Fig. 9 Distribution of (u/u_* , v/u_*) on the smooth interface side with no interfacial shear stress; $Re_L = 11\,500$, $Re_G = 0$, $y^+ = 208.4$, $y_i^+ = 0.5$ mm

Fig. 10 Distribution of (u/u_* , v/u_*) on the smooth interface side with interfacial shear stress; $Re_L = 11\,500$, $Re_G = 5\,000$, $y^+ = 219.9$, $y_i^+ = 2.8$

Fig. 11 Explanation for reversed phases in bursting events on the wall side and the interface side

wall side, and the sweep events on the interface side.

On the wall side, the sweep event is predominant in the vicinity of the wall⁽²⁰⁾. If the suppression of turbulence by the wall is not important in the structure of bursting events as pointed out by Lee et al.⁽¹⁹⁾, the interfacial sweep event (the second quadrant in Fig. 10) must be predominant in the vicinity of the sheared interface. This tendency is seen in Fig. 10.

When the interfacial waves are formed, the tendency in Fig. 12 becomes completely different from the case of smooth interfaces. The turbulence on the interface is dominated by the periodic interfacial waves and the tendency of predominance of the ejection and sweep events disappears.

3.5 Quadrant analysis

To know the contribution to the Reynolds stress from four quadrants, Wallace et al.⁽²¹⁾ used the quadrant analysis. This analysis divides the Reynolds stress into four components according to the sign of velocity fluctuations. As described in § 3.4, the ejection and sweep events are predominant in the wall bursting events. This analysis is conducted in order to investigate the difference between the smooth and wavy interface cases in the Reynolds stress here.

The results for the smooth interface are shown in Fig. 13. These data correspond to the case of Figs. 5 and 10. On the wall side, the contribution from the ejection events is larger than that from the sweep events. This result agrees with the result of Kim et al. As explained in § 3.4, the ejection and sweep events on the interface side become reversed in the present coordinates (see Fig. 11). If the interfacial bursting is similar to the wall bursting, the interfacial sweep events ('Ejection' in Fig. 13) should predominate in the vicinity of interfaces and the interfacial ejection events ('Sweep' in Fig. 13) should predominate in a

Fig. 12 Distribution of $(u/u_*, v/u_*)$ on the wavy interface side with the interfacial shear stress; $Re_L=11\,500$, $Re_c=5\,000$, $y^+=209.5$, $y_i^+=5.5$

farther region from the interfaces. At about $y^+=180$ in Fig. 13, the sweep datum (the interfacial ejection) increases and has almost the same value as the ejection datum. In the region closer to the interface, the ejection (the interfacial sweep) predominates. This tendency is not seen in the result with no interfacial shear stress (not shown here). As seen in Fig. 9, the wall bursting events affect the interface, so the effect of interfacial shear stress is not clear in Fig. 13. These results, however, qualitatively support the similarity between the wall bursting events and the interfacial bursting events.

In Fig. 14, the results of the wavy interface are shown. For the wavy interface, the tendency is almost similar to the results of Fig. 13. It is seen that the large Reynolds stress near the interface (see Fig. 6) is caused by the interfacial sweep events ('Ejection' in Fig. 14). This seems to indicate the existence of interfacial sweep events below the trough of interfacial waves as discussed in § 3.3. A further investigation is

Fig. 13 Reynolds stress from each quadrant normalized by the local Reynolds stress, for the case of a smooth interface with interfacial shear stress; $Re_L=11\,500$, $Re_c=3\,260$

Fig. 14 Reynolds stress from each quadrant normalized by the local Reynolds stress, for the case of a wavy interface with interfacial shear stress; $Re_L=11\,500$, $Re_c=5\,000$

necessary in order to clarify the turbulent motion below the wavy interface through visualizing the turbulence.

4. Conclusions

Experiments on the momentum transfer across the air-water interface in a horizontal channel were performed. By using a split-film probe, the vertical velocity fluctuation was measured. Our interest was focused on how the turbulence below the interface is affected by the wall bursting, the interfacial shear stress, and the interfacial waves. The results showed that when the interface was smooth, the turbulence below the interface was highly affected by the wall bursting. However, when the interfacial waves were formed, the turbulent behavior seemed to change. The Reynolds stress from the two velocity components showed a larger value below the interface than that of the linear distribution of shear stress. The periodic interfacial wave does not contribute to the transfer phenomena by itself. Thus, this increased Reynolds stress can be explained by a nonlinear movement. One feasible explanation was given by considering the nonuniformity of interfacial shear stress caused by the configuration of wavy interfaces.

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