A Premixed Turbulent Flame Structure Model Having Reactant Islands and Fractal Flame Surfaces* (Formulation and Test of the Model)

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A model for the premixed turbulent flame structure has been developed in which the flame zone of a turbulent flame is composed of two parts: a wrinkled flame front and some island flamelets behind it. Their fractions are supposed to vary continuously according to a proposed function of u'/S_{Lo} , where u' is the turbulence intensity and S_{Lo} is the laminar burning velocity. All flamelets were assumed to be fractal ones. The concepts of "flame thickness" and "transition time" for laminar flames were introduced into the model for turbulent flames. The model provides not only the structural parameters, but also the burning velocities of turbulent flames.

Key Words: Combustion, Premixed Turbulent Flame, Flame Structure Model, Fractal surface, Reactant Islands, Turbulent Burning Velocity, Damköhler Number

1. Nomenclature

B : Characteristic thickness of premixed turbulent flames, analogous to $2\delta_{Lo}$ for premixed flames;
 C : Model constant, $C=4.8$;
 C_1 : Model constant, $C_1=0.5$;
 Da : Damköhler number, defined as $Da=L_f S_{Lo}/(\eta_0 u')$;
 Da_η : Damköhler number, defined as $Da=L_f S_{Lo}/(4\eta_0 u')$;
 f_D : Fraction of reactant in unit volume consumed by islandlike flamelets during the characteristic time τ_D ;
 f_w : Fraction of reactant in unit volume, consumed by the wrinkled flame front during the characteristic time τ_w ;
 Ka : Karlovitz number, defined by Eq.(4);
 L_f : Longitudinal integral length scale of turbulence;

Pr : Prandtl number of unburned mixture;
 Re : Reynolds number, defined as $Re=L_f u'/\nu$;
 Sc : Burning velocity of islandlike flamelets in the radial direction;
 S_L : Burning velocity of strained, local laminar flame;
 S_{Lo} : Burning velocity of unstrained laminar flame;
 S_T : Turbulent burning velocity;
 T_b : Adiabatic flame temperature at constant pressure combustion;
 u' : Turbulence intensity, taken to be the root-mean-square of turbulent fluctuation velocities;
 ν_K : Kolmogorov velocity scale, $\nu_K \equiv \nu/\eta_K$
 X_C : Mean size (diameter) of initially formed reactant islands;
 γ : Flamelet straining rate;
 δ_{Lo} : Thickness of reaction zone of unstrained laminar flamelet;
 ϵ : Dissipation rate of turbulence energy;
 ϵ_i : Inner cutoff of fractal flamelets;
 ϵ_0 : Outer cutoff of fractal flamelets;
 η : Preheat zone thickness of local laminar flamelets in a turbulent flame;

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- η_0 : Preheat zone thickness of an unstrained, laminar flame, defined as $\eta_0 = \nu / (Pr S_{Lo})$;
 η_K : Kolmogorov length scale, $\eta_K = (\nu^3 / \epsilon)^{0.25}$;
 λ_θ : Taylor microscale;
 ν : Kinematic viscosity of unburned mixture;
 ξ : Ratio of ξ_D to ξ_w ;
 ξ_D : Combustion fraction by island flamelets;
 ξ_w : Combustion fraction by the wrinkled flame front;
 τ_c : Characteristic chemical reaction time (or transition time) of a laminar flame, defined as $\tau_c = 2\delta_{Lo} / S_{Lo}$;
 τ_D : Characteristic burn-up time of reactant islands, taken to be $\tau_D = X_c / (2S_c)$;
 τ_T : Characteristic reaction time (or transition time) of a turbulent flame, analogous to τ_c for a laminar flame;
 τ_w : Characteristic reaction time of wrinkled flamelets, taken to be the same as τ_c ;
 Φ : Equivalence ratio of mixture.

2. Introduction

Recently, remarkable progress has been made in experimental studies of the structure of premixed turbulent flames. Information on the flamelet thickness has been provided by the Rayleigh scattering technique⁽¹⁾ and microprobe method⁽²⁾⁻⁽⁴⁾. The curvature and wrinkling of flamelets have been revealed by shadowgraph⁽⁵⁾, the schlieren method⁽⁶⁾ and laser tomography⁽⁷⁾⁻⁽⁹⁾. The islandlike features of the turbulent flame zone have been indirectly shown by shadowgraph⁽⁵⁾, schlieren method⁽⁶⁾ as well as the micro-probe method⁽⁴⁾, and have been confirmed by laser tomography⁽⁹⁾. Moreover, analysis of flamelet two-dimensional images shows that turbulent flame surfaces have fractal properties^{(7),(8),(10)}.

On the other hand, in modeling the structure of premixed turbulent flames, theoretical studies have proposed different combustion regimes on a plane whose coordinates are dimensionless parameters, such as the $Re-Da$ plane⁽⁷⁾ or the $u'/S_{Lo} - L_f/\eta_0$ plane⁽¹¹⁾, according to certain characterization criteria. Truly, such a plane provides substantial comprehensive knowledge on the nature of premixed turbulent combustion but little quantitative information on the flame structure.

The purpose of this work is to develop a quantitative model for the premixed turbulent flame structure on the basis of recent experimental results. For this purpose, the experimental results on the flame structure were first summarized, then a number of assumptions on the basis of the experimental results were proposed, and several parameters for the flame structure were derived. As the turbulent burning velocity is

closely related to the flame structure, it is also involved in the model as an important model-provided parameter. The model was tested by its predictions of turbulent burning velocities for sixteen mixtures at different turbulence levels (u'/S_{Lo} up to about 10), and has been confirmed by the good agreement of the predictions with measurements.

3. Basic Experimental Results

The model has been constructed on the basis of the following experimental results:

(1) Chemical reactions in premixed turbulent flames are mainly confined within thin flamelets although the thickness of the flamelets may be influenced by turbulence. As shown in Fig. 1, the flame potential signal obtained by a passive electrostatic probe⁽⁴⁾ shows very sharp spikes, indicating that thin flamelets were present in the flame. In our experiment, such sharp spikes were observed until the flame failed to propagate for mixtures with small laminar burning velocities. For mixtures with large laminar burning velocities, such sharp spikes were detected until the u'/S_{Lo} ratio was increased up to the order of 10, which is the limit of our experiment devices. Similar results were obtained by Rajan et al.⁽¹⁾ using the Rayleigh scattering technique.

(2) The higher the turbulence intensity, the more the islandlike flamelets are formed and the more the flamelets (including the flame front) are curved. This is suggested by the fact that either the fringes on the schlieren photographs of premixed turbulent flames in a constant-volume combustion chamber obtained by the authors⁽⁶⁾ or those on the microshadowgraphs of premixed-charge engine flames by Smith⁽⁵⁾ show that the number and curvature of the fringes increased rapidly as the turbulence was intensified.

(3) Both the thickness of the turbulent flame zone and the number of flamelets in the zone increase

Fig. 1 Flame potential signal of premixed turbulent flame

as the u'/S_{LO} ratio increases^{(3),(4)}. Although this result has been obtained by the authors using a microprobe method, two-dimensional visualization of the flame structure by zur Loye and Bracco⁽¹²⁾ also shows that the thickness of the turbulent flame zone increased as the engine speed increased. The result that the number of flamelets in the turbulent flame zone increases with turbulence intensity implies that the turbulent flame structure changes from a single, continuous flame front to a distributed islandlike pattern as the u'/S_{LO} ratio increases.

(4) The average separation distance between adjacent flamelets in the turbulent flame zone shows little change with the u'/S_{LO} ratio, being about two times the longitudinal integral length scale of the turbulence⁽²⁾⁻⁽⁴⁾. This result suggests that the sizes of "reactant islands" depend strongly on the integral scale of the turbulence.

(5) As the u'/S_{LO} ratio increases to a certain extent, the structure of turbulent flames transits from combustion regime I to combustion regime II, and finally the flames fail to propagate^{(2),(3)}. By definition⁽²⁾, in combustion regime I, a turbulent flame has only one flamelet—the flame front, and in combustion regime II, islandlike flamelets are formed in the turbulent flame zone. Detailed definitions of the two combustion regimes and the method of determining the critical ratio of u'/S_{LO} for a turbulent flame to transit from one regime to another were given in Ref. (2). In Fig. 2, the curves from the upper left to the lower right, indicated by "85-P10-14", and so on, are the lines for different mixtures whose properties are given in Table 1. The "○" marks in Fig. 2 indicate the boundary between the single spike and multiple spikes

in the flame potential signals detected by a passive electrostatic probe⁽²⁾⁻⁽⁴⁾; the "●" marks are the boundary between one or two and three or more spikes in the flame potential signals, taken to be the boundary between combustion regimes I and II⁽²⁾; the "□" marks are where the flames failed to propagate at a probability of 50% in the experiment when the ignition energy was 0.8 J. From Fig. 2, one can see the flame structure changing from a single, continuous flame front to a distributed islandlike pattern as the turbulence intensity is increased.

(6) Flamelets in premixed turbulent flames are fractallike surfaces, with their fractal dimension increasing with an increase in the u'/S_{LO} ratio, and gradually leveling off as the u'/S_{LO} ratio is further increased to some extent. This result was obtained by Mantzaras, et al.⁽⁷⁾⁽⁸⁾ from the laser tomograms of stoichiometric premixed turbulent flames in a research engine, and was supported by the experiments of Murayama et al.⁽¹⁰⁾ and Ando and Yoshida⁽¹³⁾, respectively, using lean mixtures.

(7) The turbulent burning velocity significantly decreases as the laminar flame thickness (thus, the preheat zone thickness) increases for mixtures with the same laminar burning velocity and at the same turbulence intensity^{(14),(15)}. If the total area of the flamelets in a turbulent flame is supposed to be determined mainly by the turbulence characteristics and the laminar burning velocity of the mixture and not significantly influenced by the laminar flame thickness, the above result is indicative of the effect of flame strain on the local laminar burning velocity of flamelets, because an increase in the laminar flame thickness increases the strain rate, which in turn decreases the local, strained laminar flame burning velocity⁽¹⁶⁾⁻⁽¹⁹⁾. If the local, strained laminar burning velocity of the flamelets is not supposed to be influenced by the laminar flame thickness, the same result can also imply the effect of laminar flame thickness on the inner cutoff of the fractallike sur-

Fig. 2 Experimentally determined combustion regimes of premixed turbulent combustion

Table 1 Properties of mixtures used

MIXTURE	MOLAR FRACTION				δ_{LO}	S_{LO}	T_b	η_0	ν
	C_2H_6	O_2	N_2	He					
85-P10-14	1.0	5.0	12.0	2.00	.146	.858	2496	.284	1.52
85-P10-18	1.0	5.0	11.0	4.50	.177	.846	2472	.365	1.72
79-P10-21	1.0	5.0	10.5	6.50	.213	.791	2440	.454	1.88
49-P10-20	1.0	5.0	17.0	0.36	.198	.494	2329	.415	1.41
50-P10-22	1.0	5.0	16.5	2.00	.219	.501	2310	.476	1.51
49-P10-24	1.0	5.0	16.0	3.20	.240	.493	2299	.534	1.59
46-P10-29	1.0	5.0	15.4	5.60	.289	.464	2262	.675	1.74
33-P10-30	1.0	5.0	20.0	2.00	.298	.331	2168	.712	1.51
22-P10-36	1.0	5.0	24.0	0.00	.358	.224	2060	.898	1.41
17-P10-45	1.0	5.0	26.0	0.00	.451	.169	1986	1.19	1.42

faces of flamelets because the outer cutoff does not seem to be largely changed by the laminar flame thickness, while an increase in the laminar flame thickness is expected to increase the inner cutoff⁽³⁾ which, in turn, decreases the total area of the flamelets⁽²⁰⁾. Therefore, the decrease in the turbulent burning velocity caused by an increase in the laminar flame thickness can be attributed to either a decrease in the local laminar burning velocity of the flamelets or an increase in the inner cutoff of the fractallike flamelet surfaces, or probably to both factors.

4. Assumptions

On the basis of the experimental results mentioned in the preceding section, the following assumptions are proposed for the model.

(1) The flamelets in the turbulent flame zone consist of two components: a wrinkled, continuous flame front and some islandlike flamelets behind the flame front. Of a unit volume of mixture, the fraction consumed by the island flamelets within their characteristic time, τ_D , is f_D , and f_W by the wrinkled flame front within its characteristic time, τ_W . f_D is assumed to take the form of

$$f_D = \exp\left(-C \frac{S_{Lo}}{u'}\right); \quad (1)$$

thus,

$$f_W = 1 - f_D. \quad (2)$$

Here, C is an experimentally determined constant greater than zero, and will be discussed later. Equation (1) has been based on experimental results No. 2, No.3 and No.5 which show that an increase in the u'/S_{Lo} ratio results in an increase in the fraction of combustion in the form of island flamelets. When u'/S_{Lo} approaches zero, f_D approaches zero, which means that the turbulent flame is thus reduced to a laminar flame; while u'/S_{Lo} approaches infinity, f_D approaches unity, which means that the mixture is completely consumed by the island flamelets.

(2) The flamelets are strained at a strain rate of $\gamma (=u'/\lambda_\theta)$, and the local laminar burning velocity S_L can be estimated by a formula proposed by Law⁽²⁰⁾. When the Lewis number is unity for stoichiometric mixtures dealt with in this report, the formula is reduced to

$$S_L = S_{Lo}(1 - Ka), \quad (3)$$

where Ka is the Karlovitz number. By definition,

$$Ka = \gamma\nu/(PrS_{Lo}^2). \quad (4)$$

As it is very difficult to measure the local laminar burning velocity of flamelets in a turbulent flame, Eq. (3) has not been directly confirmed. However, the decrease in the local laminar burning velocity derived from this equation is directly proportional to $(\eta_0/\eta_K)^2$ (cf. Eq.(14)), which is qualitatively consistent with

experimental result No.7. Replacement of this equation is possible if better formulas are available.

(3) The preheat zone of the local laminar flamelets is thickened by the Kolmogorov vortices, and its thickness, η , can be estimated by

$$\eta = \eta_0 + C_1 \nu_K \eta_0 / S_{Lo}, \quad (5)$$

where C_1 is a constant of order one, ν_K is the Kolmogorov velocity scale, and η_0/S_{Lo} is the transition time of the preheat zone. $C_1 \eta_0/S_{Lo}$ is considered to be the effective time for the Kolmogorov eddies to transfer mass and heat from the zone, resulting in thickening of the zone.

(4) All flame surfaces involved in a turbulent flame are fractal surfaces with an outer cutoff, ε_0 , being equal to L_f , the longitudinal integral scale of turbulence, and an inner cutoff, ε_i , being determined by the Kolmogorov length scale, η_K , as well as the local laminar flame preheat zone thickness, η , in the following way:

$$\varepsilon_i = \frac{\eta_K}{\eta_K + \eta} \eta_K + \frac{\eta}{\eta_K + \eta} \eta = \frac{\eta_K^2 + \eta^2}{\eta_K + \eta}, \quad (6)$$

where $\eta_K/(\eta_K + \eta_0)$ and $\eta/(\eta_K + \eta_0)$ are the weights of η_K and η , respectively.

For the inner cutoff ε_i , it is reported that ε_i is close to the Kolmogorov scale η_K for interfaces in nonreacting flows⁽²¹⁾, and Gouldin⁽²⁰⁾ has basically used the Kolmogorov scale as the inner cutoff in a turbulent burning velocity model. However, if $\varepsilon_i = \eta_K$ applies to the turbulent flame surface, at sufficiently high turbulence intensities, it would result in $\varepsilon_i < \eta_0$. This seems unlikely because the definition of the flame surface would become ambiguous in this case. Therefore, for turbulent flame surfaces, intuitively, the inner cutoff is likely to be influenced by a certain thickness scale of the flamelets, for which the local laminar flame preheat zone thickness seems to be a good choice.

For the outer cutoff ε_0 in nonreacting flows, Screenivasan and Meneveau⁽²¹⁾ found ε_0 to be about three times the transverse integral scale of turbulence in a turbulent boundary layer. In reacting flows, an experimental estimate performed on the surfaces of premixed turbulent engine flames by Mantzaras et al.⁽⁷⁾ reveals that ε_0 is of the order of the longitudinal integral length scale of the turbulence. Goix et al.⁽²²⁾ found ε_0 to be a function of the integral length scale of the turbulence for a V-shaped premixed flame. As accurate measurements of the outer cutoff are not available, $\varepsilon_0 = L_f$ is assumed in this model, as in Gouldin's model⁽²⁰⁾.

(5) The fractal dimension of the flamelets can be approximated by

$$D_3 = 2 + 0.5 \exp\left\{\frac{-0.53}{\ln(1 + u'/S_{Lo})}\right\}. \quad (7)$$

This assumption is based on the measured fractal dimensions of flamelets in stoichiometric mixtures by Mantzaras et al.^{(7),(8)}, which shows the fractal dimension D_3 to increase as u'/S_{Lo} increases. Such fractal dimensions of turbulent flamelets in nonstoichiometric mixtures were obtained by Murayama et al.⁽¹⁰⁾ and Ando and Yoshida⁽¹³⁾, respectively. The values of D_3 calculated by Eq.(7) are plotted in Fig. 3 and compared with the measured results of Refs. (8) and (10), showing that the approximation is in good agreement with measurements.

(6) The mean size (diameter) of the initially engulfed reactant islands, X_c , is of the order of the longitudinal integral scale of turbulence, L_f , i.e., $X_c \approx L_f$. According to experimental result No.4, the average separation distance between adjacent flamelets measures about two times the longitudinal integral scale of turbulence. Considering the distances between successive reactant islands to be increased because of burned gas expansion, the mean size of the reactant islands is expected to be smaller than the average separation distance between adjacent flamelets.

5. Formulation

(1) Dissipation rate of turbulence energy, ε

According to the cascade model⁽³⁾ for turbulence energy transfer, most of the energy exchanged across eddies with a given scale comes from the next larger eddies and goes to the next smaller eddies, and finally, the energy is dissipated into heat at eddies with the Kolmogorov length scale. If no energy is dissipated by eddies with intermediate scales between the largest scale, where energy is supplied by the main flow, and the smallest scale, where the energy is dissipated, the energy dissipation rate can be estimated by the rate at which it is supplied. This is the so-called inviscid estimate for the dissipation rate. An inviscid estimate

for the energy dissipation rate, ε , is given as

$$\varepsilon = A_0 u'^3 / L_f, \quad (8)$$

where A_0 is a constant of order one⁽²³⁾. Abdel-Gayed et al.⁽²⁴⁾ reviewed the centerline turbulence properties in pipe flow and recommended $A_0 = 0.37$. Based on the turbulence properties measured from an in-cylinder turbulence field, Kido et al.⁽²⁵⁾ recommended the following equation:

$$\varepsilon = A_1 \left(\frac{u'^3}{L_f} \right) \left(\frac{\eta_K}{L_f} \right), \quad (9)$$

where A_1 is a constant of about 11⁽²⁶⁾.

Either Eq.(8) or Eq.(9) can be adopted in this model with little difference in the predicted turbulent burning velocities, but replacing one with the other requires a slight adjustment of the two model constants C and C_1 correspondingly. Here, Eq.(9) was adopted.

(2) The Kolmogorov length scale η_K and speed scale v_K

By the definition of the Kolmogorov length scale, $\eta_K \equiv (\nu^3/\varepsilon)^{0.25}$. From Eq.(9) and this definition, we have

$$\eta_K = \frac{L_f}{A_1^{0.2} Re^{0.6}}. \quad (10)$$

Therefore,

$$v_K \equiv \nu/\eta_K = u' A_1^{0.2} / Re^{0.4}. \quad (11)$$

(3) The Taylor microscale λ_θ

The Taylor microscale λ_θ can be calculated from⁽²³⁾

$$\varepsilon = 15 \nu u'^2 / \lambda_\theta^2. \quad (12)$$

Substitution of Eq.(9) into Eq.(12) yields

$$\lambda_\theta = \frac{15^{0.5} L_f}{A_1^{0.4} Re^{0.2}}. \quad (13)$$

(4) The Karlovitz number Ka

The Karlovitz number is defined by Eq.(4). Substitution of Eqs.(10) and (13) into Eq.(4) yields

$$Ka = \frac{Pr}{15^{0.5}} \left(\frac{\eta_0}{\eta_K} \right)^2, \quad (14)$$

where η_0 is the thickness of the unstrained laminar flame preheat zone, defined as $\eta_0 = \nu / (Pr S_{Lo})$.

(5) The wrinkled flame front burning velocity S_w and the burning velocity of island flamelets in the radial direction, S_c .

By assumption No.4, the wrinkled flame front is a fractal surface with an inner cutoff of ε_i and an outer cutoff of ε_0 . After Gouldin's reasoning⁽²⁰⁾ for turbulent burning velocity, S_w can be estimated by

$$S_w = S_L \left(\frac{\varepsilon_0}{\varepsilon_i} \right)^{D_3-2}, \quad (15)$$

where S_L is the local laminar burning velocity calculated by Eq.(3), and D_3 is the fractal dimension of the flame front estimated by Eq.(7). As all flamelets in the turbulent flame can be expected to be wrinkled in the same way, Eq.(15) applies to the burning velocity

Fig. 3 Comparison of Eq.(7) with experimental results^{(8),(10)}

of island flamelets in the radial direction, S_c ; that is, $S_c = S_w$. The relationship between S_w , S_c and S_L is shown in Fig. 4.

(6) The turbulent burning velocity S_T

By assumption No. 1, the turbulent flame zone is composed of two parts: a wrinkled flame front and some island flamelets, therefore, the turbulent burning velocity S_T is the sum of the burning velocities of these two parts. Because, of the unit volume of reactants, the fraction consumed by the wrinkled flame front within its characteristic time τ_w is f_w (assumption No. 1); during an infinitesimal time dt , the quantity of mixture consumed by the flame front, dF_{w1} , is

$$dF_{w1} = (f_w/\tau_w) dt, \quad (16)$$

where the subscript "1" refers to unit volume. Similarly, the quantity of mixture, dF_{D1} , consumed by the island flamelets during dt is

$$dF_{D1} = (f_D/\tau_D) dt. \quad (17)$$

Suppose that for a unit area normal to the S_T direction, there are B unit volumes of reactants burning simultaneously; therefore, the turbulent burning velocity S_T can be calculated by the sum of the burning rates in the B unit volumes, i.e.,

$$S_T = B \frac{d(F_{w1} + F_{D1})}{dt} \\ = B \frac{f_w}{\tau_w} + B \frac{f_D}{\tau_D}, \quad (18)$$

where $B(f_w/\tau_w)$ is the component of the total turbulent burning velocity contributed by the wrinkled flame front, and $B(f_D/\tau_D)$ is that by the island flamelets behind the flame front.

In order to clarify the physical meaning of B , we take a laminar flame (where $u' = 0$) for example. In this model, the characteristic thickness of a laminar flame is taken to be $2\delta_{Lo}$ and the characteristic time τ_w is $2\delta_{Lo}/S_{Lo}$. On the other hand, when $u' = 0$, from Eqs. (1) and (2), $f_D = 0$ and $f_w = 1$ are obtained. Therefore, Eq. (18) is reduced to $S_T = B/\tau_w = BS_{Lo}/(2\delta_{Lo})$. For a laminar flame, $S_T = S_{Lo}$; thus $B = 2\delta_{Lo}$. In other words, B in this case is reduced to the characteristic thickness of the laminar flame. Therefore, B is a characteristic thickness of turbulent flames, being analogous to $2\delta_{Lo}$ for laminar flames. It should be noted that although B is related to the physically measured thickness of turbulent flame zones, it is different from the latter. In this work, B is termed as the characteristic thickness of turbulent flames.

(7) The characteristic thickness of turbulent flames, B

As indicated above, B is a characteristic thickness of the turbulent flames. In order to determine B , note that in Eq. (18), $B(f_w/\tau_w)$ is the component of the total turbulent burning velocity performed by the

wrinkled flame front; that is, $B(f_w/\tau_w)$ is the burning velocity of the wrinkled laminar flame front, which has been denoted by S_w . Thus, $S_w = B(f_w/\tau_w)$, or

$$B = (\tau_w/f_w) S_w, \quad (19)$$

with S_w being calculated from Eq. (15).

(8) The characteristic time of turbulent flames, τ_T

Equation (18) can be rewritten as

$$S_T = B/\tau_T, \quad (20)$$

where

$$\frac{1}{\tau_T} = \frac{f_w}{\tau_w} + \frac{f_D}{\tau_D}. \quad (21)$$

From Eq. (20), one can find that the characteristic time of turbulent flames, τ_T , is analogous to the characteristic reaction time of laminar flames, τ_c . This is why τ_T is termed the characteristic time of turbulent flames.

(9) The combustion fractions of the wrinkled flame front and island flamelets, ξ_w and ξ_D

By assumption No. 1, in the unit volume of reactants, the fraction f_w takes the wrinkled flame front the characteristic time of τ_w to burn, and the fraction f_D takes the island flamelets the characteristic time of τ_D . In fact, the time permitting the two components to burn is the same, and their combustion fractions should be calculated by

$$\xi_w = \frac{f_w/\tau_w}{f_D/\tau_D + f_w/\tau_w} = \left(\frac{\tau_T}{\tau_w} \right) f_w \quad (22)$$

and

$$\xi_D = \frac{f_D/\tau_D}{f_D/\tau_D + f_w/\tau_w} = \left(\frac{\tau_T}{\tau_D} \right) f_D, \quad (23)$$

where ξ_w is the combustion fraction of the wrinkled flame front and ξ_D is that of the island flamelets. In addition, the ratio of ξ_D to ξ_w is

$$\xi = \frac{\xi_D}{\xi_w} = \left(\frac{\tau_w}{\tau_D} \right) \left(\frac{f_D}{f_w} \right). \quad (24)$$

ξ is an important parameter for the structure of a turbulent flame.

6. Determination of Model Constants and Test of the Model

6.1 Determination of model constants

The two model constants, C in Eq. (1) and C_1 in Eq. (5), have been determined so as to make the predictions of S_T/S_{Lo} best fit the measurements for flames of a given mixture (mixture 43-1A, refer to Table 2), as shown in Fig. 5, where the data⁽¹⁵⁾ of $(dp/dt)_T/(dp/dt)_L$ were used as measurements of S_T/S_{Lo} because the latter can be well approximated by the former⁽²⁷⁾. Here dp/dt is the rate of pressure rise and the subscripts "T" and "L" refer to turbulent flame and laminar flame, respectively. The influences of the values of C and C_1 on the predicted results are shown in Fig. 5, from which $C = 4.8$ and $C_1 = 0.5$ were

Table 2 Properties of mixtures⁽¹⁵⁾ for model test

MIX-TURE	COMPOSITION	δ_{L0} ■	S_{L0} ■/s	T_b K	η_0 ×10 ■	ν ×10 ⁶ ■ ² /s
43-1A	C ₂ H ₂ +2.5O ₂ +16.55N ₂	.227	.435	2052	.465	1.45
43-2M	CH ₄ +2O ₂ +7.17N ₂	.232	.429	2262	.493	1.51
43-3M	CH ₄ +2O ₂ +6.86N ₂ +0.63He	.250	.442	2253	.549	1.60
43-4A	C ₂ H ₂ +2.5O ₂ +15.74N ₂ +1.77He	.269	.431	2033	.571	1.59
43-5M	CH ₄ +2O ₂ +6.56N ₂ +1.25He	.277	.438	2244	.626	1.70
43-6A	C ₂ H ₂ +2.5O ₂ +14.32N ₂ +4.85He	.338	.434	2000	.752	1.84
43-7M	CH ₄ +2O ₂ +5.64N ₂ +3.12He	.350	.446	2217	.844	2.06
43-8A	C ₂ H ₂ +2.5O ₂ +13.75N ₂ +6.10He	.367	.434	1987	.832	1.95
62-1P	C ₃ H ₈ +5O ₂ +15.2N ₂	.160	.625	2415	.301	1.38
62-2A	C ₂ H ₂ +2.5O ₂ +14.45N ₂	.170	.632	2189	.318	1.45
62-3M	CH ₄ +2O ₂ +6.09N ₂	.173	.620	2375	.341	1.51
62-4M	CH ₄ +2O ₂ +5.59N ₂ +1.03He	.209	.615	2360	.438	1.69
62-5M	CH ₄ +2O ₂ +4.74N ₂ +2.77He	.268	.623	2334	.601	2.04
73-1M	CH ₄ +2O ₂ +5.51N ₂	.150	.745	2437	.284	1.51
73-3M	CH ₄ +2O ₂ +5.06N ₂ +1.09He	.186	.730	2414	.378	1.72
73-4M	CH ₄ +2O ₂ +4.66N ₂ +1.92He	.211	.738	2399	.445	1.88
73-6M	CH ₄ +2O ₂ +3.55N ₂ +4.13He	.287	.742	2370	.647	2.41

Fig. 4 Wrinkled flame burning velocity and local laminar flame burning velocity

adopted.

6.2 Test of the model

With the model constants C and C_1 determined, the model was tested by comparing the predicted S_T/S_{L0} ratios for sixteen mixtures (refer to Table 2) with the measured ones⁽¹⁵⁾. The results are given in Fig. 6, showing that the predictions of the S_T/S_{L0} ratio are in quantitative agreement with the measurements for most of the mixtures tested. Although for a few mixtures with extremely large laminar flame thickness such as mixtures 43-8A and 73-6M, the predicted burning velocities are larger than the measured ones, considering the good quantitative agreement for most of the mixtures, the validity of this model is confirmed.

In order to show the model-predicted influences of mixture properties on the flame structure, the predicted results for two mixtures with different laminar flame thicknesses (refer to Table 2) are

Fig. 5 Determination of model constants C and C_1

shown in Fig. 7, where Ka , ϵ_i and η given in Fig. 7(a) are concerned with the local properties of the flame structure, while τ_T , B and ξ_D in Fig. 7(b) characterize the global feature of the flame structure.

7. Discussion of the Model

7.1 Basic considerations of the model

Premixed turbulent combustion can be regarded as a rapid chemical reaction taking place in a turbulence field. Therefore, the structure of premixed turbulent flames is influenced by both the chemical properties of the mixture and the characteristics of the turbulence. In order to model the flame structure, it is important to choose some dimensionless quantities appropriate to comprehensively represent many concerned quantities involved in the chemical, and heat and mass transfer properties of the mixture and the characteristics of the turbulence. Moreover, it is also important to take account of how each affecting

Fig. 6 Comparison of model predictions with measurements of turbulent burning velocities (A-Measurements: B-Model predictions)

Fig. 7 Model-predicted influences of mixture properties on flame structure and turbulent burning velocity

factor influences the global feature of the flame structure as well as the local geometry of the flamelets in the turbulent flame, and to relate the quantities concerning the microscales of both the turbulence and the flamelets, which are often difficult to measure, to those quantities which are large in scale and easier to measure.

In this work, the Damköhler number Da_η , Reynolds number Re , Prandtl number Pr and the ratio of δ_{Lo}/δ_0 were used to represent the mixture properties and turbulence characteristics comprehensively. For hydrocarbon mixtures, the Prandtl number and the ratio of δ_{Lo}/η_0 vary within narrow ranges. When these two dimensionless variables are taken to be constants, all variables involved in the model can be expressed as functions of only Re and Da_η .

In the model, the influences from the mixture properties and turbulence characteristics on the global feature of the turbulent flame were mainly represented by the ratio of u'/S_{Lo} and the fractal properties of the flamelets. The influences on the local, microscale geometry of the flamelets were considered such that the inner cutoff and the preheat zone thickness of the flamelets were affected by the Kolmogorov scale of the turbulence. Moreover, the structure of turbulence was characterized by the integral length scale, the Taylor microscale and the Kolmogorov scale, and described by the cascade theory for turbulence energy dissipation⁽²³⁾, which made it possible to express the microscale quantities in large-scale quantities such as the turbulence integral scale.

7.2 Confirmation of the model

The model was tested by comparing the predictions of turbulent burning velocities with the measurements. As shown in Fig. 6, except for two or three mixtures with extreme properties, the predicted turbulent burning velocities are in good agreement with the measured ones for the remainder of the sixteen mixtures with wide-ranging properties. This indirectly confirmed the validity of the model.

7.3 Characteristic thickness and characteristic time of turbulent flames

The concepts of flame thickness and characteristic chemical reaction time (or transition time) for laminar flames were introduced into the model for turbulent flames. The characteristic thickness B and the characteristic time τ_r for turbulent flames are analogous to $2\delta_{Lo}$ and $\tau_c (=2\delta_{Lo}/S_{Lo})$, respectively, for laminar flames. In other words, if a laminar flame consumes a thickness of $2\delta_{Lo}$ of reactants within its characteristic reaction time τ_c , then a turbulent flame will consume a thickness of B of reactants within a time of τ_c . Although the value of B is concerned with the physically measured value of the thickness of the

turbulent flame zone⁽⁴⁾, they are of different quantities. Further investigations are required to clarify the relationship between them.

7.4 Unification of turbulent flame structure with laminar flame structure

One of the major features of the model is that the turbulent flame structure and the laminar flame structure are unified by the introduction of the concepts of characteristic flame thickness and characteristic reaction time. When the ratio u'/S_{Lo} approaches zero, all parameters describing turbulent flames are reduced to the corresponding parameters describing laminar flames. To show this, one can set u'/S_{Lo} to zero, which results in $f_D=0$, $S_L=S_{Lo}$, $D_3=2$, $S_w=S_L$, $\tau_r=\tau_c$, $B=2\delta_{Lo}$ and $S_T=S_{Lo}$.

7.5 Compatibility with Gouldin's model

On the basis of the fractal properties of the turbulent flame surface, Gouldin has proposed a model for the turbulent burning velocity⁽²⁰⁾. When both of the two model constants C and C_1 in the present model are given to be zero, the model is reduced to Gouldin's model logically, except that in Gouldin's model, the Karlovitz number was estimated to be $(\eta_0/\eta_K)^2$, the fractal dimension D_3 was taken to be a constant of 2.37, and the inner cutoff of fractal flamelets was basically taken to be the Kolmogorov scale of the turbulence.

7.6 Limitation and future modification of the model

The model has been developed on the basis of a number of assumptions. Some of the assumptions can become limitations to the model. First, the mixture was assumed to be stoichiometric (as the Lewis number Le was taken to be unity in assumption No.2). Modification of Eq.(3) is required in order to apply the model to nonstoichiometric mixtures. Second, as the two model constants C and C_1 have been determined according to the measured turbulent burning velocities with the u'/S_{Lo} ratio ranging in 0~10, the condition of $u'/S_{Lo} \leq 10$ may become a limitation to model applications. Further confirmation of the model's validity in the case of $u'/S_{Lo} > 10$ is expected.

It is worth mentioning that although the predictions of turbulent burning velocities shown in Fig. 6 are consistent with the measurements, the former are somewhat higher than the latter in most cases. One of the possible causes for such a result is attributed to the use of Eq.(3), which may have underestimated the decrease in the local laminar burning velocity caused by the flame strain. A further decrease in the local laminar burning velocity by the flame strain is expected to make the predicted turbulent burning velocities agree with the measured ones.

8. Summary

In the above-described model for the structure of premixed turbulent flames, the flame zone of a turbulent flame was supposed to be composed of two parts: a wrinkled flame front and some island flamelets behind it. Their fractions were assumed to vary continuously according to a proposed function of the ratio of u'/S_{Lo} and their respective characteristic times. All flamelets were assumed to be fractal ones, with the inner cutoff being determined by the Kolmogorov scale and the thickness of the local laminar flame preheat zone. The effects of turbulence on the local flamelets were considered such that the eddies on the size of the Taylor microscale strained the flamelets, and thus decreased the local laminar burning velocity, while the smallest eddies (with the Kolmogorov scale) thickened the preheat zone of the flamelets, increasing the inner cutoff.

This work aimed at modeling the structure of premixed turbulent flames quantitatively, and resulted in obtaining a quantitative model for both the structure and turbulent burning velocity of the flames because of their close connection. The turbulent burning velocities predicted by the model were in good agreement with those measured for most of the mixtures tested, except for two or three mixtures with extremely large flame thicknesses.

In addition, when both of the two model constants C and C_1 are set to zero, the model is reduced into Gouldin's turbulent burning velocity model formally. In this sense, the present model is an extension of Gouldin's model. Moreover, by introducing the concepts of flame thickness and characteristic reaction time (or transition time) for laminar flames into the model for turbulent flames, a turbulent flame can be considered to be the extension of a laminar flame (or a laminar flame to be a particular turbulent flame where no turbulence exists).

Finally, it is worth mentioning that, when the Prandtl number and the ratio of δ_{Lo}/η_0 are taken to be constants, the dimensionless turbulent burning velocity S_T/S_{Lo} and the parameter ξ , one of the important parameters characterizing the flame structure, can be expressed by only two variables, the Reynolds number and the Damköhler number, making it possible to show the turbulent burning velocity and the parameter ξ in the plane with Re and Da_η being the coordinates. Such a plane has so far been used to show the so-called "combustion regimes" by Abraham et al.⁽²⁸⁾ and other researchers^{(7),(8)}. Therefore, adding the turbulent burning velocities and the ξ parameters to such a plane can be expected to show the nature of premixed turbulent combustion more comprehensively.

Details of the results were presented in another report⁽²⁹⁾ of this work.

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