Deformation Behavior of a Viscoelastic Fluid Jet*

Yukio TOMITA** and Tsutomu TAKAHASHI***

When a viscoelastic fluid jet falling vertically encounters a deformation from the horizontal direction, the jet shows an anomalous behavior very different from that of a Newtonian fluid jet. We observe the deformation behavior of the jet in detail and find that the jet behavior is divided into two conditions. Moreover, we derive an empirical formula of the transition point, making use of the dimensional analysis method.

**Key Words**: Viscoelastic Fluid, Jet, Transition, Dimensional Analysis Method

1. Introduction

Many polymer melts and solutions have viscoelastic properties which affect the flow in a drastic way. Due to their elasticity, viscoelastic fluids exhibit many peculiar and very complex phenomena that are not observed in Newtonian fluids. Formation operations of polymer materials are generally difficult because of the complexity and the peculiarity of the behavior of viscoelastic fluids. In order to develop polymer engineering, it is necessary to clarify the flow behavior of viscoelastic fluids.

We observe the deformation behavior of a viscoelastic fluid jet falling vertically from a round nozzle in free air after it encounters a deformation from the horizontal direction in this investigation.

The deformations are caused in the following two ways: i) impulsively hitting the jet with a stick (dynamic deformation). ii) giving a displacement quasi-statically by an edge (quasi-static deformation). Figures 1 and 2 show the jet behavior of Newtonian fluids and a viscoelastic fluid after the dynamic deformation is inflicted on them. In the case of Newtonian fluids, the jet flies to bits or is stretched at the inflection point, but the other parts of the jet are not affected by the deformation, as shown in Fig. 1. However, in the case of the viscoelastic fluid jet, it is

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* Received 21th June, 1991. Paper No. 89-1181A
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Fig. 1 Jet behavior of Newtonian fluids subjected to dynamic deformation
observed in Fig. 2 that the deformation transmits either upstream or downstream depending on the position of the inflection. The jet upon which quasi-static deformation is inflicted also shows two types of behavior. One is a behavior wherein the jet is stretched and curved by the edge as if it were a string, and the other type is a behavior in which the jet above the edge is unaffected by the edge and maintains the same shape. In cases of both dynamic and quasi-static deformations, the jet is divided into two regions having widely different properties of the transmission of deformation at a peculiar point. We call this peculiar point "the transition point of deformation."

In this study, we observe in detail the behavior of a jet subjected to deformation caused in the above two ways, and experimentally investigate the position of the transition point of deformation under various conditions. Moreover, we determine some factors which affect this position and derive an empirical formula through the use of the dimensional analysis method.

**Nomenclature**

- \( D \) : nozzle diameter, mm.
- \( h \) : distance from the nozzle exit to the position subjected to deformation, m.
- \( H \) : jet length (distance from the nozzle exit to the tilted plate), m.
- \( k_c \) : transition point of dynamic deformation, m.
- \( k'_c \) : transition point of quasi-static deformation, m.
- \( L \) : nozzle length, mm.
- \( n, g, s, A \) : rheological constants of Denn model.
- \( V_a \) : mean velocity in the nozzle, m/s.
- \( f \) : shear rate in the nozzle \((=8V_aD), 1/s.\)

2. **Experimental Apparatus**

A schematic diagram of the experimental apparatus is shown in Fig. 3. A test fluid in a supply tank is pressurized by compressed air in a regulated-pressure flow out from the nozzle exit into free air. The liquid jet formed downstream of the nozzle exit falls vertically onto a tilted plate placed below the nozzle. The liquid then flows down the tilted plate and into a beaker. The distance from the nozzle exit to the tilted plate is adjusted over the range of 0 mm to 1000 mm in order to change the jet length. Two inflection devices in Fig. 3 are used to inflict deformation on the jet. One of them, (a) in Fig. 3, is used to give a dynamic deformation and the other, (b), is used to give a quasi-static deformation to the jet. Device (a) for inflicting dynamic deformation has a stick which

![Diagram](image1)

(a) Type A: The jet behavior in the case where dynamic deformation is inflicted in the region above the transition point of deformation

![Diagram](image2)

(b) Type B: The jet behavior in the case where dynamic deformation is inflicted in the region below the transition point of deformation

Fig. 2 Jet behavior of a viscoelastic fluid subjected to dynamic deformation

can be swung in the horizontal plane. The stick is a round bar made of acrylic resin of which the diameter and length are 7 mm and 250 mm, respectively. A rubber band is used to swing this stick. As the stick swings and collides into the jet, it inflicts an impact onto the jet. Device (b) consists of an edged plate made of acrylic resin and a microtraverse gear. The edged plate can be moved precisely and slowly on the horizontal plane by the microtraverse gear. With this device, we can inflict quasi-static deformation onto the jet by pushing the edged plate to the jet.

The reservoir is made of an acrylic resin pipe with an inner diameter of 100 mm. Circular pipes with various diameters D and lengths L are used as the nozzle. They are made of copper or brass pipe. The dimensions of the nozzles are shown in Table 1.

In the present study, 2.0%, 3.0% and 3.5% aqueous solutions of polyacrylamide (trade name: SEPARAN AP-30) and a 3.0% aqueous solution of polyethylene oxide (trade name: PEO) are used; here we call them SEP 2.0% SEP 3.0%, SEP 3.5% and PEO 3.0%, respectively. These are viscoelastic fluids well known to exhibit non-Newtonian viscosity, shear elasticity, and normal stress effects.

We use the Denn model for the rheological equation of viscoelastic fluids. The Denn model has four rheological constants: m, n, s, l. They are shown in Table 2. Here, m and n are calculated from the pressure loss measured in the nozzle, and s and l are determined from the first normal stress difference measured by a parallel-plate rotational rheometer.

3. Experimental Results

3.1 Dynamic response

Figure 1 shows the jet behavior of Newtonian fluids when the jet is subjected to dynamic deformation by the stick from the horizontal direction. With water, the jet is cut by the stick at the collision point and spray is sent flying through the air, as shown in Fig. 1(a). With glycerin (shown in Fig. 1(b)), the region of the jet with which the stick collided is stretched by stick without breaking. However, in both cases, other parts of the jet are unaffected by the stick.

Figure 2 shows the jet behavior of a viscoelastic fluid when the jet subjected to dynamic deformation by the stick from the horizontal direction. The photographs in Figs. 2(a) and (b) are taken continuously, at about 1/9 second intervals, by a motor-driven camera. In both cases, the test fluid is SEP 3.5%. As shown in a sketch of Fig. 2, the deformation is transmitted upward as a wavelike motion when deformation is inflicted on the jet at a point (h) in the region above a certain point h. This type of jet behavior is termed "Type A" and is shown in Fig. 2(a). However, when deformation is inflicted at h in the region below h, that is, h > h, a different jet behavior occurs. In this case, the deformation is transmitted downstream only; that is, the flow behavior in the region above the inflection point is unaffected by the inflection and the jet maintains the same shape there. The jet behavior of this type is termed "Type B" and is shown in Fig. 1(b).

The transmission phenomenon of the deformation is not observed in Newtonian fluids. A jet of viscoelastic fluid can be divided into two regions according to the deformation behavior. We call the dividing point, h, between the two regions of the jet "the transition point of dynamic deformation" and call the distance between the nozzle exit and the transition point of deformation "the falling length to the transition point."

Figure 4 shows the relationship between the jet behavior subjected to dynamic deformation and the apparent shear rate in the nozzle. The ordinate axis in Fig. 4 represents the falling length, h, of the jet from

<table>
<thead>
<tr>
<th>Nozzle No.</th>
<th>Diameter [mm]</th>
<th>Length [mm]</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
<td>120</td>
</tr>
<tr>
<td>3</td>
<td>3.0</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td>3.0</td>
<td>120</td>
</tr>
<tr>
<td>5</td>
<td>3.0</td>
<td>160</td>
</tr>
<tr>
<td>6</td>
<td>3.0</td>
<td>120</td>
</tr>
<tr>
<td>7</td>
<td>3.0</td>
<td>140</td>
</tr>
<tr>
<td>8</td>
<td>3.0</td>
<td>160</td>
</tr>
<tr>
<td>9</td>
<td>4.0</td>
<td>80</td>
</tr>
<tr>
<td>10</td>
<td>6.0</td>
<td>80</td>
</tr>
<tr>
<td>11</td>
<td>6.0</td>
<td>120</td>
</tr>
<tr>
<td>12</td>
<td>6.0</td>
<td>160</td>
</tr>
</tbody>
</table>

Table 2 Material constants of test fluids

<table>
<thead>
<tr>
<th>FLUID</th>
<th>m</th>
<th>n</th>
<th>s</th>
<th>l</th>
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<tbody>
<tr>
<td>SEP 2.0%</td>
<td>0.273</td>
<td>14.97</td>
<td>0.650</td>
<td>0.888</td>
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<tr>
<td>SEP 3.0%</td>
<td>0.176</td>
<td>36.44</td>
<td>0.633</td>
<td>0.805</td>
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<tr>
<td>SEP 5.0%</td>
<td>0.157</td>
<td>43.41</td>
<td>0.608</td>
<td>0.868</td>
</tr>
<tr>
<td>PEO 3.0%</td>
<td>0.136</td>
<td>51.32</td>
<td>0.575</td>
<td>1.510</td>
</tr>
</tbody>
</table>

Fig. 3 Schematic diagram of the experimental apparatus
the nozzle exit to the position subjected to deformation. Here, the test fluid is SEP 3.5% and the nozzle dimensions are 6 mm in diameter and 80 mm in length. The jet length, $H$, is set to 1 000 mm by the tilted plate. The apparent shear rate $\Gamma$ is calculated by $\Gamma = 8V_{A}/D$: $V_{A}$ is mean velocity in the nozzle and $D$ is the nozzle diameter. The plots in this graph represent the falling length to the transition point of deformation. If the dynamic deformation is inflicted in the region above the curve that links these plots, the jet shows the behavior of "Type A", whereas, the behavior of "Type B" occurs when the deformation is inflicted in the region below the curve. It is clarified in Fig. 4 that the position of the transition point of the dynamic deformation moves upward with increasing shear rate.

Figure 5 shows the relationship between the position of the transition point of the dynamic deformation and apparent shear rate in each test fluid. The ordinate axis in Fig. 5 represents the position of the transition point of dynamic deformation. The nozzle dimensions in Fig. 5 are 6 mm diameter and 80 mm length. It is clarified by Fig. 5 that all test fluids show the same tendency; that is, the position of the transition point moves upward with increasing shear rate. For this reason, all the following photographs and numerical values only show data for SEP 3.5% to simplify the description.

Next, we explain the effect of the nozzle dimensions for the position of the transition point of the dynamic deformation. Figure 6 shows the relationship between the position of the transition point of the dynamic deformation and nozzle diameter. The nozzle length $L$ is 80 mm and jet length $H$ is 1 000 mm. It is found that the falling length to the transition point decreases, that is, the position of the transition point moves upward with increasing nozzle diameter, as shown in Fig. 6. Figure 7 shows the relationship between the position of the transition point of dynamic deformation and nozzle length. In this figure, the nozzle diameter and the jet length are 6 mm and 1 000 mm, respectively. As far as we investigated, the nozzle length does not affect the position of the transition point of the dynamic deformation as shown in Fig. 7.

Figure 8 shows the relationship between the jet length $H$ and the position of the transition point of dynamic deformation. The nozzle dimensions are 6 mm diameter and 80 mm length. The figure shows that the falling length to the transition point is unaffected
by the jet length. When the tilted plate is moved upward to above the transition point and the jet length is short as compared with the falling length to the transition point, there is no transition point for dynamic deformation. In this case, the jet shows only Type A behavior when it is subjected to dynamic deformation at any point.

From the above experimental results, it is found that the position of the transition point of dynamic deformation is affected by the nozzle diameter and fluid properties, but not by the nozzle length or the jet length.

3.2 Quasi-static response

In this section, we investigate the behavior of the jet subjected to quasi-static deformation caused by giving a displacement to the jet quasi-statically by an edge.

For a Newtonian fluid jet subjected to quasi-static deformation, the jet sends spray flying through the air at the edge or adheres to the underwall of the edge, as shown in Fig. 9. Figure 10 shows the behavior of the viscoelastic fluid jet when it is subjected to quasi-static deformation. In quasi-static deformation also, two types of jet behavior are observed. One of them is the behavior in which the jet is stretched and curved as if it were a string, as is shown in Fig. 10(a), and that behavior is termed “Type C.” This type of jet behavior occurs when the deformation is inflicted on the jet at a point \( h \) in the region above a certain point \( k_c \), that is, \( h < k_c \). The other type of jet behavior is shown in Fig. 10(b). It occurs when the quasi-static deformation is inflicted on \( h \) in the region below \( k_c \) \( (h > k_c) \). In this case, the jet behavior above the plate is unaffected by the plate and maintains the same shape. This behavior is similar to that of Newtonian fluids, but the jet behavior below the plate is different; that is, the jet bounces at the edge of the plate, as shown in Fig. 10(b). It is termed “Type D” behavior. The point dividing the jet behavior into these two types is termed “the transition point of the quasi-static deformation.”

Figure 11 shows the relationship between the apparent shear rate in the nozzle and the jet behavior subjected to quasi-static deformation. In order to compare the positions of the transition point of quasi-static and dynamic deformations, we also show the data of the transition point of dynamic deformation which was measured under the same conditions in
Fig. 11. In this figure, the nozzle dimensions are 6 mm diameter and 80 mm length, and the jet length is 1,000 mm. As shown in Fig. 11, the positions of the transition point by the two different deformations almost agree. Similar results are obtained in all experiments that we carried out. These results lead to the following conclusion: there is a transition point of deformation in a viscoelastic fluid jet which divides the jet into two regions having different properties. Moreover, the position of the transition point of deformation is unaffected by the deformation type, as far as we have studied.

4. Consideration by the Dimensional Analysis Method

From the experimental results shown in the previous sections, we infer that the position of the transition point of deformation is affected by many factors. We try to account for the position of the transition point of deformation as follows.

We use the Denn model, which has four material constants, for the rheological equation of viscoelastic fluids. Then, it is inferred that the density of the fluids $\rho$, material constants $\eta, n, s, \lambda$, acceleration of gravity $g$, nozzle diameter $D$, and mean velocity in the nozzle $V_s$ are physical quantities which influence the position of the transition point of deformation, $h_c$. Thus, through consideration by the dimensional analysis method, we obtain the following dimensionless numbers related to the transition point of deformation $h_c$.

$$Fr = \frac{V_s}{\sqrt{gD}}$$  

($=$ inertial force 
 gravitational force): Frue number

$$Re^* = \frac{\rho V_s^{1+n}D^n}{\eta}$$

Therefore, the next equation is satisfied at the position of the transition point of deformation.

$$f[(h_c/D), Re^*, N_w, Fr] = \text{const.} \quad (1)$$

The result which correlates the position of the transition point of deformation with Eq. (1) is shown in Fig. (12). From Fig. 12, we obtain the following empirical formula for the position of the transition point of deformation:

$$(h_c/D)^{0.22} \times Re^{0.30} \times N_w^{0.20} \times Fr^{-0.52} = 3.71 \quad (2)$$

or

$$\frac{h_c}{D} = 387 \times Re^{-1.36} \times N_w^{-0.91} \times Fr^{2.59} \quad (3)$$

5. Conclusions

We studied the behavior of a viscoelastic fluid jet subjected to deformation. As a result, the following conclusions were obtained.

1) A viscoelastic fluid jet subjected to deformation shows anomalously reacting behavior. That is, the deformation transmits either upstream or downstream depending on the position of the inflection when dynamic deformation is inflicted. In the case of quasi-static deformation, the jet is displaced by the edge or not displaced depending on the position of the inflection. The behavior of the viscoelastic fluid jet does not vary continuously along its entire length. In both cases of deformation, dynamic and quasi-static deformations, the jet is divided into two regions having different properties of the transmission of deformation by the same peculiar point. We term this peculiar point "the transition point of deformation."

2) The influences of various factors, such as the physical properties, nozzle dimensions and jet length, on the position of the transition point of deformation were clarified experimentally, that is, the position of it is affected by the nozzle diameter and fluid properties, but not by the nozzle length or the jet length.

3) The empirical formula for the position of the transition point of deformation was determined through use of the dimensional analysis method.

Acknowledgment

We would like to thank Mr. Hidehito Ito and Mr. Toshinari Yamaguchi for their help in the experiments.