Resonance of Circular Shock Waves*

Myeong-Kwan PARK**, Shuzo OSHIMA** and
Ryuichiro YAMANE**

In supersonic centrifugal compressor or disc MHD power generators, oscillation modes of the circular shock wave can be caused by the static change or the dynamic change of the back pressure. While previous studies, which included only the former effects, were carried out both experimentally and theoretically, the latter effects of forced oscillation are considered here. The excitation of the back pressure fluctuation in a downstream stagnation chamber was accomplished by rotating a butterfly valve. The pressure fluctuation was measured to investigate the resonance of the circular shock wave at concentric positions in the vicinity of the first shock and in the downstream stagnation chamber. A-D converted signals were statistically correlated to extract the spacial oscillation modes. Resonance was confirmed to occur when the excitation frequency was near the natural frequency of mode 0. Furthermore, the small perturbation of the higher harmonic modes in the downstream stagnation chamber influenced the oscillation modes of the circular shock wave.

Key Words: Fluid Dynamics, Compressible Flow, Shock Wave, Oscillating Shock, Pressure Fluctuation, Forced Vibration, Resonance

1. Introduction

Radial supersonic flow between parallel plates occurs in gas bearings, disc valves, centrifugal compressors, disc MHD power generators and so on. With the increasing need to achieve high speeds, high pressure ratios, and high performance in these machines, an encounter with supersonic flow is close at hand. In radial supersonic flows, a circular shock wave, which becomes a pseudoshock by interaction with a boundary layer, appears⁽¹⁾, but the pseudoshock usually tends to oscillate, which causes vibration and instability in the performance of the machinery. In the previous report⁽²⁾, oscillation of a circular shock wave was found to consist of various dynamic modes. Also in the previous report⁽³⁾, the shock wave was confirmed to be statically deformed from a true circle by nonaxisymmetric boundary conditions, and, furthermore, to be oscillating with various dynamic modes. This oscillation is, so to speak, free oscillation.

In practical machinery, these machines are not used by themselves but generally are connected to a discharge pipe, an exhaust pipe and other equipment in which pressure pulsations and flow instabilities occur, which act as a back pressure fluctuation in radial flow. If the resonance with the back pressure occurs at the natural mode of the circular shock wave, it may lead to large fluctuations and instability of the flow.

Similar experiments were conducted by Bogar et al.⁽⁴⁾ and Sajben et al.⁽⁵⁾⁽⁶⁾ on the flow oscillation in ramjet inlets induced by combustion instabilities. They simulated this situation with a rotating rotor, and no resonance effects were found at the natural frequencies of the shock wave. This may have been due to the fact that the rotor was too far from the shock position for the perturbation to reach it, and that the perturbation which was produced by a small area change in the main flow by rotor blockage was too weak.

This study examines resonance effects both experimentally and theoretically. The behavior and oscillation modes of a circular shock wave are investigated by applying a dynamic change in the back

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** Tokyo Institute of Technology, Ookayama 2-12-1, Meguro-ku, Tokyo 152, Japan

pressure, with a prescribed frequency as in forced vibration.

2. Nomenclature

$Coh^2(f)$: coherence

f : frequency

M : Mach number

n : mode number

P : pressure

P_0 : stagnation pressure

$Ph(f)$: mean phase function

r : radius

$S(f)$: power spectrum

t : time

u : velocity

x : ratio of specific heats

ρ : mean density

Subscript

1: ahead of shock wave

2: behind shock wave

e : exit of duct

r : r directional component

θ : directional component

$'$: fluctuation component

3. Experimental Apparatus and Procedure

3.1 Apparatus

As in the previous reports^{(2),(3)}, experiments were carried out with an intermittent blowdown supersonic wind tunnel using air as the working fluid, except that here a butterfly valve was installed as the exciter to induce a downstream back pressure perturbation. The butterfly valve was placed just at the exit of the downstream stagnation chamber in order to allow the perturbation to be as near as possible to the position of the shock wave. It was rotated by an electric motor, and the frequency could be varied continuously by a 2-pole, 200 V inverter, which generated 1.5 kW of power. Although the nominal rotation speed of the motor was 2870 rpm, it could be varied up to the maximum frequency of the inverter.

A schematic view and the size of the test section are shown in Fig. 1. The top and bottom walls (test section) of the passage were made of a 20-mm-thick acrylic plate. The top wall of the passage was nearly square, 1 080 mm long and 1 060 mm wide. The diameter of the bottom wall, with a friction bearing supporting the circular plate at the outer periphery, was altogether 900 mm. This circular plate could be turned inside the bearing.

The static pressure holes were placed in radial directions of 0° , 30° , 45° and 60° at 10-mm intervals. The pressure fluctuation measuring holes were situated in the radial directions of 90° , 180° and 270° , along

with 0° , 30° , 45° and 60° . The reference points were at 5-mm intervals: 65, 70, 75, 80, 85 and 90 mm in radius on both sides of the measuring hole groups to obtain a precise measurement. Measurements in all directions were performed by turning the circular plate four times, each time by 90° , inside the bearing.

Also, the five pressure fluctuation measuring holes were positioned at $0=30^\circ$, 15° , 0° , -15° and -30° on the side wall of the downstream stagnation chamber, as shown in Fig. 1.

3.2 Measurements and experimental procedure

The same measuring method as in the previous reports^{(2),(3)} was employed. The stagnation pressure was held constant at 0.160 MPa in absolute pressure, and the butterfly valve was rotated. After the position of the shock wave was determined by the distributions of the static pressures, the pressure fluctuation in each position was measured, and was statistically analyzed to investigate the oscillation components of the circular shock wave.

In this experiment, the radius of the circular shock wave was nearly the same, 75 mm, as in previous reports^{(2),(3)}, where the natural frequency of mode 0 was about 100 Hz. Therefore, the rotating speed of the butterfly valve was chosen to be 50, 100 and 150 Hz on the basis of the mode 0 natural frequency. The maximum experimental frequency was 150 Hz, considering the safety of the apparatus, the air column resonance, and furthermore considering that even if the excitation frequency were lower, the perturbation propagating to the shock wave was the superposition of the various higher frequencies produced in the downstream stagnation chamber.

It is supposed that the air column resonance consists of three components in this experimental

Fig. 1 Size of test section and pressure hole position

apparatus: (I) the air column between the disc of the butterfly valve and the upstream stagnation chamber; (II) the air column between the shock wave and the exit of the radial passage; (III) the air column between the butterfly valve and the exit of the discharge pipe. The calculated resonance frequencies⁽⁷⁾ for each type were 154, 228, and 154 Hz, respectively, which were higher than the maximum excitation frequency of 150 Hz.

4. Results and Discussion

4.1 Distribution of static pressure

The distribution of the static pressure in the four radial directions, for an excitation frequency of 100 Hz, is shown in Fig. 2, where an isentropic curve is given as a reference. They are similar to those in a conventional supersonic nozzle, and demonstrate that the flow is axisymmetric. Also, an abrupt pressure rise due to the pseudoshock, and a subsequent subsonic gradual pressure rise occur, as in typical radial supersonic flow.

4.2 Circular shock wave oscillation

4.2.1 Pressure fluctuation in the downstream stagnation chamber

A pressure fluctuation of

Fig. 2 Radial distribution of static pressure (excitation frequency of 100 Hz)

frequencies other than that of the rotation of the butterfly valve is superposed in the downstream stagnation chamber, and propagates to the shock wave as the back pressure. Therefore, it is necessary to investigate the pressure fluctuation in the downstream stagnation chamber. Figure 3 shows the power spectra of the pressure fluctuation at five stations in the downstream stagnation chamber. In the case of an excitation frequency of 50 Hz, a clear higher harmonic mode, as high as 300 Hz of mode 6, is seen, and for 100 and 150 Hz, a clear peak occurs up to a maximum of 600 Hz for mode 6 and 750 Hz for mode 5, respectively. This shows that at a given excitation frequency, not only does the pressure fluctuate at the same frequency as the excitation, but a higher harmonic mode occurs as well. For excitation frequencies of 50 and 100 Hz, the lower frequency components are stronger than those of the higher frequencies. However, in the case of an excitation frequency of 150 Hz, the lower frequency components are nearly of the same order as the higher frequency components.

4.2.2 Power spectra in the vicinity of the shock wave

Figure 4 shows the power spectra of the pressure fluctuations in the vicinity of the shock wave for the 0° direction and for frequencies of 50, 100 and 150 Hz. The result of the previous report⁽²⁾ (no excitation) is shown for comparison. On the whole, the excitation increases the oscillation of the shock wave. A high peak is seen at about 100 Hz for an excitation frequency of 100 Hz. Also, a peak is seen at 50 Hz, along with a small peak at 100 Hz, for an excitation frequency of 50 Hz. No clear peaks are seen for an excitation frequency of 150 Hz, only small oscillations, although the rate of the higher frequency components is relatively greater. The peak at the excitation frequency of 100 Hz is more distinguishable than those at any other exciting frequencies. The components of about twice as many as 100 and 50 Hz are high, which suggests that resonance in this experimen-

Fig. 3 Power spectra of pressure fluctuation in downstream stagnation chamber

tal condition occurs at these frequencies. The components of the basic mode corresponding to the rotational frequency of the butterfly valve are highest among the perturbation in the downstream stagnation chamber, and the influence of these components on the shock wave is large. For an excitation frequency of 150 Hz, the power spectra of the pressure fluctuations in the downstream stagnation chamber have the same level for the components of each mode, but their values are lower than those for an excitation frequency of 100 Hz, and resonance with the shock wave does not seem to occur with the low level at the lower frequency. However, the power spectra of the higher frequency band are higher than those of the other due to the effects of the higher frequency components.

Fig. 4 Power spectra of pressure fluctuation in vicinity of the shock wave

From the fact that the power spectra of the shock wave show the same tendency as those of the downstream stagnation chamber, it is thought that the components of the perturbation of the higher harmonic mode in the downstream stagnation chamber propagate to the shock wave, which causes each oscillation mode of the circular shock wave, and the resulting pressure fluctuation propagates in turn downstream, leading to resonance.

4.2.3 Correlation of pressure fluctuation In the above power spectra, the oscillation mode of the shock wave is not distinguishable. Therefore, in order to make this clear the circumferential pressure fluctuation along the circular shock wave was measured, and the coherence and phase function described in the previous reports^{(2),(3)} were calculated. The result for an excitation frequency of 100 Hz is shown in Fig. 5. Sharp peaks in the coherence and phase function of mode 0 are seen at 100 Hz. A correspondence between the peaks of the mean phase function and the mean coherence is seen at 180 Hz for mode 1, 430 Hz for mode 3, and 500 Hz for mode 4.

For an excitation frequency of 100 Hz, the oscillation of mode 0 was prominent, which shows that resonance was occurring, but the resonance of mode 4 was not noticeable due to the small excitation pressure fluctuation. When the excitation frequency was 50 Hz, mode 1 was at 200 Hz and mode 3 was at 300

Fig. 5 Mean coherence and mean phase functions

Hz, and when the excitation frequency was 150 Hz, mode 2 was at 300 Hz. Thus, the shock wave was slightly influenced by the perturbation components of the higher harmonic mode. To sum up, a circular shock wave oscillates with the natural frequency of each mode with a basic mode of 100 Hz, and resonance occurs at 100 Hz selectively among the excitation fluctuation components due to the prominent effect of the basic mode in this experimental configuration.

5. Analysis

Since the fluid is a continuous medium, its oscillation is highly complicated with infinite degrees of freedom, and so it is not as clear as the simple mechanical oscillation in the experiment. Now, a simple model for the oscillation of a circular shock wave is developed to show that resonance occurs. The oscillation model is shown in Fig. 6. There are various oscillation components in the downstream stagnation chamber as mentioned above, and the pressure perturbation propagates to the upstream shock wave, where the resonance with the natural frequency of each mode occurs. Here, the frequency response to the shock oscillation amplitude is investigated. The following assumptions are made:

- (1) The velocity profile is uniform.
- (2) The pseudoshock is replaced by a single normal shock wave.
- (3) The mass of air between the shock wave and the duct exit oscillates due to the difference in the pressure behind the shock wave and the exit, with the latter changing periodically.
- (4) The wall shear stress is determined from $\tau = \frac{\rho C_f U^2}{2}$, where the friction coefficient C_f is used for convenience, because the effect of the wall shear stress is relatively small. The quantity U is the mean downstream velocity.

5.1 Modes above 1

From the above assumptions, and neglecting the 2nd order and higher order terms, the equations of motion and continuity for the small fluctuation components are as follows:

Fig. 6 Oscillation model

$$\left. \begin{aligned} \frac{\partial u_r'}{\partial t} &= -\frac{1}{\rho} \frac{\partial p'}{\partial r} - 2\alpha u_r' \\ \frac{\partial u_\theta'}{\partial t} &= -\frac{1}{\rho} \frac{1}{r} \frac{\partial p'}{\partial \theta} - \alpha u_\theta' \\ \frac{\partial}{\partial r}(ru_r') + \frac{\partial u_\theta'}{\partial \theta} &= 0 \end{aligned} \right\} \quad (1)$$

where $\alpha = \frac{C_f \rho U}{h}$ and h is the height of the passage.

Considering the time-averaged flow in the radial direction, we substitute U and $U/2$ for the mean velocity components in r and θ directions to represent the wall shear stress of each direction. Eliminating u_r' and u_θ' in Eq. (1), we obtain

$$\left(\frac{\partial}{\partial t} + 2\alpha \right) \left(\frac{\partial}{\partial r} \left(r \frac{\partial p'}{\partial r} \right) + \frac{1}{r} \frac{\partial^2 p'}{\partial \theta^2} \right) - \alpha \frac{\partial}{\partial r} \left(r \frac{\partial p'}{\partial r} \right) = 0. \quad (2)$$

The solutions of Eq. (1) are assumed to be of the form

$$\left. \begin{aligned} p' &= e^{i\omega t} g_p(r) \cos n\theta \\ u_r' &= e^{i\omega t} g_u(r) \cos n\theta \\ r' &= e^{i\omega t} \varepsilon \cos n\theta \end{aligned} \right\} \quad (3)$$

Substituting p' into Eq. (2), we get

$$\left(i\omega + 2\alpha \right) \left(\frac{d}{dr} \left(r \frac{dg_p}{dr} \right) - \frac{n^2}{r} g_p \right) - \alpha \frac{d}{dr} \left(r \frac{dg_p}{dr} \right) = 0, \quad (4)$$

and after rearranging Eq. (4), we obtain

$$\frac{d}{dr} \left(r \frac{dg_p}{dr} \right) - \frac{2\alpha^2 + \omega^2 - i\alpha\omega}{\alpha^2 + \omega^2} \frac{n^2}{r} g_p = 0. \quad (5)$$

Letting $g_p = r^\beta$, $\beta = x + iy$, we get

$$\beta^2 = \frac{n^2 (2\alpha^2 + \omega^2 - i\alpha\omega)}{\alpha^2 + \omega^2} \quad (6)$$

$$\left. \begin{aligned} x &= \pm n \sqrt{\frac{1}{2} \left(\frac{W_2}{W_1} + \sqrt{\frac{W_2^2 + W_3^2}{W_1^2}} \right)} \\ y &= \pm n \sqrt{\frac{1}{2} \left(-\frac{W_2}{W_1} + \sqrt{\frac{W_2^2 + W_3^2}{W_1^2}} \right)} \end{aligned} \right\} \quad (7)$$

where

$$\left. \begin{aligned} W_1 &= \alpha^2 + \omega^{*2} \omega_n^2 \\ W_2 &= 2\alpha^2 + \omega^{*2} \omega_n^2 \\ W_3 &= \alpha \omega^* \omega_n \end{aligned} \right\} \quad (8)$$

and $\omega^* = \omega/\omega_n$ where ω_n is the natural frequency. The solution of Eq. (5) and p' is

$$g_p = C_1 e^{(x_1 + iy_1) \ln r} + C_2 e^{(x_2 + iy_2) \ln r} \quad (9)$$

$$p' = e^{i\omega t} (C_1 e^{(x_1 + iy_1) \ln r} + C_2 e^{(x_2 + iy_2) \ln r}) \cos(n\theta), \quad (10)$$

where x_1, x_2, y_1 and y_2 are each value of x and y in Eq. (7), and C_1 and C_2 are constants. Taking $C_1 = a_1 e^{i\varphi_1}$ and $C_2 = a_2 e^{i\varphi_2}$, Eq. (10) becomes

$$p' = [a_1 e^{x_1 \ln r + i(\omega t + y_1 \ln r + \varphi_1)} + a_2 e^{x_2 \ln r + i(\omega t + y_2 \ln r + \varphi_2)}] \cos(n\theta). \quad (11)$$

The back pressure fluctuation exciting the shock wave at the duct exit ($r = r_e$) is assumed to be of the form

$$p_{re}' = |\Delta p_e| e^{i\omega t} \cos(n\theta). \quad (12)$$

At the shock position ($r=r_e$), we have

$$\left. \frac{\partial u_r'}{\partial t} \right|_{r=r_1} = -\frac{1}{\rho} \frac{\partial p'}{\partial r} - 2\alpha u_r' \Big|_{r=r_1} \quad (13)$$

$$u_r' \Big|_{r=r_1} = \frac{dr'}{dt} \Big|_{r=r_1}$$

Representing the pressure rise and Mach number due to a pseudoshock by the relations for a normal shock wave, and modifying the value by a correction factor η , results in

$$p' = -Kr', \quad (14)$$

where $K = \eta P_1 \frac{2\kappa M_1^2}{(\kappa+1)(M_1^2-1)} (M_1^2 - \frac{\kappa+3}{2}) \frac{1}{r_1}$.

Letting $\varepsilon = \varepsilon_r + i\varepsilon_i$, from Eq.(13), gives

$$-\omega^2(\varepsilon_r + i\varepsilon_i) = -\frac{1}{\rho} \frac{dg_p}{dr} \Big|_{r=r_1} - 2\alpha\omega(\varepsilon_r + i\varepsilon_i)i. \quad (15)$$

Separating the real and imaginary parts yields

$$\left. \begin{aligned} -\omega^2\varepsilon_r &= -\frac{1}{\rho} \frac{dg_{pr}}{dr} \Big|_{r=r_1} + 2\alpha\omega\varepsilon_i \\ -\omega^2\varepsilon_i &= -\frac{1}{\rho} \frac{dg_{pi}}{dr} \Big|_{r=r_1} - 2\alpha\omega\varepsilon_r \end{aligned} \right\} \quad (16)$$

From Eqs. (3), (12)–(14) and (16), we have

$$\left. \begin{aligned} \varepsilon_r(KL_1 + 2S\alpha\omega) + \varepsilon_i(KL_2 - S\omega^2) &= 2|\Delta p_e|T_1 \\ \varepsilon_r(KL_2 - S\omega^2) - \varepsilon_i(KL_1 + 2S\alpha\omega) &= 2|\Delta p_e|T_2 \end{aligned} \right\} \quad (17)$$

Calculating the amplitude $|\varepsilon|$ of the shock wave from the above equations, we obtain

$$|\varepsilon| = \frac{2|\Delta p_e|\sqrt{(T_1^2 + T_2^2)}}{\sqrt{(KL_2 - S\omega^2)^2 + (KL_1 + 2S\alpha\omega)^2}}, \quad (18)$$

After nondimensionalizing Eq.(18) by the natural frequency $\omega_n \left(= \sqrt{\frac{K}{\rho r_1 \ln(r_e/r_1)}} \right)$ of mode 0 for the

oscillation system of the circular shock wave, the magnification factor $R_n = K|\varepsilon|/|\Delta p_e|$, and the ratio of the amplitude of the shock wave and the back pressure fluctuation, Eq.(18) becomes

$$R_n = \frac{2\sqrt{(T_1^2 + T_2^2)}}{Z_0 \sqrt{(Z_1 - \omega^{*2})^2 + \left(Z_2 + \frac{2\alpha\omega^*}{\omega_n}\right)^2}}, \quad (19)$$

Fig. 7 Theoretical result

$$\left. \begin{aligned} a &= l \cos k_1, \quad b = l \sin k_1 \\ c &= m \cos k_2, \quad d = m \sin k_2 \\ k_1 &= y_1 \ln(r_1/r_e), \quad k_2 = y_2 \ln(r_1/r_e) \\ l &= (r_1/r_e)^{x_1}, \quad m = (r_1/r_e)^{x_2} \\ \Pi_1 &= a - c, \quad \Pi_2 = b - d \\ J_1 &= ac - bd, \quad J_2 = ad + bc \\ H_1 &= a\Pi_1 + b\Pi_2, \quad H_2 = a\Pi_2 - b\Pi_1 \\ H_3 &= c\Pi_1 + d\Pi_2, \quad H_4 = c\Pi_2 - d\Pi_1 \\ L_1 &= (H_2 + H_4)x_1 - (H_1 + H_3)y_1 \\ L_2 &= -(H_1 + H_3)x_1 - (H_2 + H_4)y_1 \\ S &= \rho r_1(\Pi_1^2 + \Pi_2^2) \\ T_1 &= (\Pi_1 J_2 - \Pi_2 J_1)x_1 + (\Pi_1 J_1 + \Pi_2 J_2)y_1 \\ T_2 &= (\Pi_1 J_1 + \Pi_2 J_2)x_1 - (\Pi_1 J_2 - \Pi_2 J_1)y_1 \\ Z_0 &= \frac{(\Pi_1^2 + \Pi_2^2)}{\ln(r_e/r_1)} \\ Z_1 &= \frac{L_2 \ln(r_e/r_1)}{(\Pi_1^2 + \Pi_2^2)} \\ Z_2 &= \frac{L_1 \ln(r_e/r_1)}{(\Pi_1^2 + \Pi_2^2)} \end{aligned} \right\} \quad (20)$$

5.2 Mode 0

When the θ -direction component in Eq.(1) is zero, we have

$$\left. \begin{aligned} \frac{\partial u_r'}{\partial t} &= -\frac{1}{\rho} \frac{\partial p'}{\partial r} - 2\alpha u_r' \\ \frac{1}{r} \frac{\partial}{\partial r}(ru_r') &= 0 \end{aligned} \right\} \quad (21)$$

In Eq.(21), u_r' , p' and r' can be put in the form

$$\left. \begin{aligned} u_r' &= f_{u0}(r)\cos(\omega t) + g_{u0}(r)\sin(\omega t) \\ p' &= f_{p0}(r)\cos(\omega t) + g_{p0}(r)\sin(\omega t) \\ r' &= \varepsilon_{01}\cos(\omega t) + \varepsilon_{02}\sin(\omega t) \end{aligned} \right\} \quad (22)$$

whereupon we obtain

$$\frac{\partial}{\partial r}\left(r \frac{\partial p'}{\partial r}\right) = 0. \quad (23)$$

Calculating the solution of Eq.(23) with the assumption that the back pressure fluctuation is $p_{re}' = |\Delta p_e|\cos(\omega t)$ at the position $r=r_e$, the pressure fluctuation is given by

$$p' = (C_0 \ln(r/r_e) + |\Delta p_e|)\cos(\omega t) + D_0 \ln(r/r_e)\sin(\omega t) \quad (24)$$

where C_0 and D_0 are constants. The amplitude ε_0 is obtained by the same method as above, and after nondimensionalizing by ω_n , R becomes

$$R = \frac{1}{\sqrt{(1 - \omega^{*2})^2 + \left(\frac{2\alpha\omega^*}{\omega_n}\right)^2}}. \quad (25)$$

5.3 Analytical results

Figure 7 displays the theoretical results calculated for the present experimental conditions. Resonance occurs at the natural frequency of each mode with high-amplitude peaks. Also, resonance is large at mode 0, and becomes smaller for the higher modes.

The fluctuation at the position of the shock wave was not measured in the present experiment, and instead the similarly rearranged pressure fluctuation in the vicinity of the shock wave is shown in Fig. 8.

Fig. 8 Experimental result

Resonance is prominent at mode 0 and is not noticeable at the higher mode, but qualitatively the same tendency as in the theoretical result is seen.

6. Conclusions

(1) The pressure fluctuation components of the higher harmonic modes of the excitation frequency were observed in the downstream stagnation chamber by rotating the valve.

(2) In the power spectra of the pressure fluctuation, a distinct peak was found for an excitation frequency of 100 Hz in the present experimental apparatus, and resonance of mode 0 occurred.

(3) Resonance effects were found with the perturbation components of the downstream higher harmonic mode at the natural frequency (especially,

mode 0) of the shock wave oscillation.

(4) Resonance was theoretically confirmed to occur when the excitation frequency was equal to the natural frequency of the shock wave.

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