Pipe Flow Problems of Magnetic Fluids*

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Recent theoretical and experimental studies on pipe flow problems of magnetic fluids in a magnetic field are reviewed. Pipe flow resistance increases largely with the application of a magnetic field. A theoretical model of particle aggregation is presented to explain the flow characteristics obtained in the experiments. As specific examples of pipe flows, gas-liquid two-phase flow of a magnetic fluid and oscillatory flow of a magnetic fluid plug are investigated.

Key Words: Magnetic Fluid, Pipe Flow, Magnetic Field, Oscillatory Flow, Two-Phase Flow, Flow Resistance

1. Introduction

Magnetic fluids are well known to be a colloidal suspension of many fine particles of a solid ferromagnetic material in a carrier liquid such as water, hydrocarbon, ester or fluorocarbon. The most important feature is that the liquid can respond to a magnetic field. This characteristic results mainly from the magnetic body force occurring in a magnetic field^{(1)~(3)}.

The pipe flow problems of magnetic fluid in an applied magnetic field are very important as the basis for the development of engineering applications, such as new energy conversion system⁽⁴⁾, viscous dampers^{(5),(6)} and actuators⁽⁷⁾ utilizing magnetic fluids.

Some basic studies to clarify the pipe flow characteristics of magnetic fluids have therefore been performed in our laboratory.

To analyze pipe flow characteristics of magnetic

fluid, it is necessary to formulate the basic equations. Basic equations for magnetic fluids were first proposed by Neuringer and Rosensweig⁽⁸⁾. They considered the limiting case that the relaxation time for magnetization is zero; that is, the magnetization M is collinear with the magnetic field H . Collinearity is a good approximation for sufficiently small-size particles which behave superparamagnetically.

If a magnetic fluid contains relatively large particles, the relaxation time for magnetization of the fluid is determined by Brownian rotation of the particles. In the hydrodynamic description of such a suspension as a homogeneous medium, it is necessary to treat the suspension as a fluid having internal angular momentum. The basic equations of magnetic fluids with internal rotation were first derived by Shliomis⁽⁹⁾, under the assumption that the particles are uniformly dispersed in a carrier liquid without particle-particle interaction. Recently, Shizawa and Tanahashi⁽¹⁰⁾ presented new basic equations based on the micropolar theory and thermodynamical method.

However, recent investigations on magnetic parti-

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cle behavior in a magnetic fluid have clarified that the particles partially aggregate and form needlelike clusters with the application of a magnetic field, especially in the case of water-based magnetic fluids^{(11)~(13)}. It is, therefore, necessary to investigate the pipe flow characteristics by taking into account the aggregate of magnetic particles.

In the present paper, theoretical and experimental studies of steady and unsteady pipe-flow problems of magnetic fluids are reviewed. That is, (1) the effect of a magnetic field on pipe flow resistance and (2) gas-liquid two-phase flow characteristics are first explained. Then, (3) the oscillatory flow characteristics in a nonuniform magnetic field are discussed in detail.

Nomenclature

- B : magnetic flux density
 C_m : pressure increase ratio
 D : shear rate
 d : particle diameter
 d_{eff} : particle diameter including the surfactant layer
 d_1 : pipe diameter
 f : frequency
 H : magnetic field strength
 $L(\xi)$: Langevin function
 l : pipe length
 M : magnetization
 m : magnetic moment of particle
 N_g : number density of vapor bubble generation
 N_u : number of particles in a cluster
 n : number density of particles
 p : pressure
 r : bubble radius
 T : temperature
 v : velocity
 W_0 : Womersley number
 α : void fraction
 β : rotational friction per unit volume
 Γ_g : rate of vapor bubble production
 γ : aspect ratio of particle
 η : viscosity
 η_c : viscosity of carrier liquid
 λ : coefficient of pipe resistance
 μ_0 : permeability in vacuum
 ν : kinematic viscosity
 ξ : $\mu_0 m H / k T$
 ρ : density
 σ : variance
 τ : shear stress
 ω : angular frequency
 ϕ : volumetric density of magnetic particles

2. Steady Pipe Flow of Magnetic Fluid

2.1 Pipe flow resistance

Theoretical and experimental studies on the steady pipe flow of magnetic fluid have been conducted to clarify the flow characteristics of magnetic fluids in an applied magnetic field^{(14)~(18)}.

The study shows that a peculiar increment in pipe flow resistance arises especially for water-based magnetic fluid, which cannot be explained by Shliomis' theory.

Figure 1 shows the test results obtained using the test pipe (inner diameter $d_1=5.5$ mm) in uniform magnetic flux density⁽¹⁵⁾.

Water- and kerosene-based magnetic fluids with 20% mass concentration of particles are used. The coefficient of pipe resistance λ is plotted vertically. The horizontal axis is the modified Reynolds number Re^* .

The coefficient of pipe resistance λ and the modified Reynolds number Re^* are defined as follows :

$$\lambda = \Delta p' \frac{2d_1}{\rho v_0^2 l} \quad (1)$$

$$Re^* = Re \frac{\eta}{\eta + \Delta\eta} \quad (2)$$

where $\Delta p'$ denotes the pressure drop due to flow resistance over the pipe length l , v_0 the average velocity and $\Delta\eta$ the increments in apparent viscosity induced by the magnetic field.

The experimental data for laminar flow, however, deviate from the analytical prediction of $\lambda = 64/Re^*$, taking into account the effective viscosity in both cases of water and kerosene carriers. The deviation suggests that the chain formation of fine particles causes a further increase in the flow resistance of magnetic fluids in a transverse magnetic field, especially in the case of a water-based magnetic fluid.

The same results have been obtained by viscosity measurements in a strong magnetic field⁽¹³⁾.

To explain such a large deviation between theory

Fig. 1 Resistance coefficient λ versus modified Reynolds number Re^*

and experiment, some theoretical investigations have been conducted which assume that particles aggregate in the shape of an elongated ellipsoid^{(11),(19)} and in needlelike clusters⁽²⁰⁾.

Figure 2 shows an example of calculated results of the effective viscosity obtained using a needlelike cluster model⁽²⁰⁾; that is, the apparent viscosity is a function of the number of particles forming a cluster N_u for two effective particle diameters $d_{\text{eff}}=2d$ and $d_{\text{eff}}=d$, where d is the solid particle diameter, d_{eff} the diameter of a particle including the surfactant layer, η_c the viscosity of the carrier liquid, and η_0 the zero-shear rate viscosity which is given by

$$\eta_0 = \lim_{D \rightarrow 0} \tau/D, \quad (3)$$

where τ is the shear stress and D the shear rate.

In Fig. 2, RBM indicates rotary Brownian motion and asymptotic results are obtained by calculating the asymptotic line of η_0 for $N_u \rightarrow \infty$.

It is seen in Fig. 2 that the apparent viscosity increases with the effective particle diameter and with the number of particles in a cluster.

Thus, the large increment in pipe flow resistance due to the applied magnetic field is explained by taking into account the cluster formation of particles.

2.2 Gas-liquid two-phase flow

An energy conversion system using magnetic fluids proposed first by Resler and Rosensweig⁽⁴⁾ was based on the principle that the magnetization of magnetic fluids changes with temperature. However, significant results have not been obtained to date. One of the reasons is that it is not easy to prepare temperature-sensitive magnetic fluids, the susceptibility of

which changes strongly with temperature below the boiling point of the carrier liquid. In order to overcome this limitation, we have contrived a new energy conversion system using two-phase flow produced by vaporization of the carrier liquid due to heat addition⁽²¹⁾. This idea is based on the two-phase liquid metal MHD power generation system proposed by El-Boher et al.⁽²²⁾, who have utilized only the buoyancy force of two-phase flow. However, if we use a temperature-sensitive magnetic fluid in which magnetization changes sharply with temperature, we can expect a driving force larger than that of conventional systems to induce the magnetic fluid flow because the properties of magnetization change not only by temperature rise but also by gas inclusion.

Figure 3 shows schematically the system used in the theoretical analysis, as well as the experiments⁽²³⁾. The magnetic field H has a nonuniform distribution along the z -axis. The magnetic fluid is heated downstream past the point of maximum magnetic field strength and boiled to form a gas-liquid two-phase flow. The vapor bubbles are assumed to be produced in the region of $z=0 \sim 2z_{\text{max}}$ with the minimum bubble radius r_{min} and a normal distribution of generation rate as

$$\Gamma_g = \frac{4}{3} \pi r_{\text{min}}^3 \rho_g N_g, \quad (4)$$

and

$$N_g = \frac{n_{ga}}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(z-z_{\text{max}})^2}{2\sigma^2}\right\}, \quad (5)$$

where Γ_g is the rate of vapor bubble production, N_g the number density of vapor bubble generation, n_{ga} the total number of bubble generation and σ the variance.

We can obtain the flow characteristics of such a two-phase fluid by simultaneous solution of the equations of continuity and motion for gas- and liquid-

Fig. 2 Relationship between viscosity and number of particles forming a cluster (RMB : rotary Brownian motion)

Fig. 3 Analytical model of two-phase flow in a nonuniform magnetic field

phases, the energy equation for two-phase flow, and the equation of contraction and expansion of the bubbles. We have solved these equations numerically for one-dimensional steady flow.

Figure 4 shows the calculated result of two-phase flow characteristics along the pipe axis where Δp_t is the pressure rise from the static pressure in the state of a single-phase flow without heating and without an applied magnetic field.

$$\Delta p_t = p_t - p_{t(\alpha=0, H=0)} \quad (6)$$

A large pressure increase is obtained by the application of a magnetic field which is mainly due to the magnetic body force. It is also shown that the generated bubble is at first accelerated by the action of the magnetic force as well as by the buoyancy force and then decelerated by the drag force on the downstream side near the exit of the magnetic field region.

A decrease in gas-phase velocity v_g causes an increase in the void fraction α , which is effective in increasing the pressure difference at the exit ($z^* = z/l$

Fig. 4 Calculated results of one-dimensional two-phase flow characteristics

Fig. 5 Theoretical and experimental values of pressure increase ratio as a function of mean void fraction $\bar{\alpha}$

$= 1.0$). In the experiment, we used, as a temperature-sensitive magnetic fluid, hexane-based magnetic fluid with 50% mass concentration of manganese-zinc ferrite particles.

To show the pressure increase at the exit ($z^* = 1.0$), we define the pressure increase ratio as

$$C_m = \frac{\Delta p_t(\alpha, H)}{\Delta p_t(\alpha)} \quad (7)$$

where $\Delta p_t(\alpha)$ is the pressure rise due to gas inclusion without a magnetic field and $\Delta p_t(\alpha, H)$ is the pressure rise with an applied magnetic field.

Figure 5 shows the pressure increase ratio C_m obtained by changing the maximum field strength compared with the theoretical prediction calculated using the values of $r_{\min} = 0.5$ mm and $\sigma = 0.8 \times 10^{-2}$ m. Due to the uncertainty of the bubble generation rate in the experimental study, the experimental results agree only qualitatively with the theoretical prediction.

3. Unsteady Pipe Flow of Magnetic Fluid

3.1 Oscillatory pipe flow characteristics

Oscillatory pipe flow problems of magnetic fluid in an applied magnetic field are very important as the basis for the development of engineering applications such as viscous dampers and actuators⁽⁵⁾⁻⁽⁷⁾.

Theoretical and experimental studies have, therefore, been conducted to clarify the fundamental oscillatory pipe flow characteristics of magnetic fluid in a nonuniform magnetic field⁽²⁴⁾.

Theoretical analysis is conducted assuming that the shape of aggregated particles in a magnetic fluid is prolate spheroid. As shown in Fig. 6, the number of aggregated particles is N_u , and the prolate spheroid has an aspect ratio of $\gamma (= a/b, a < b)$.

The equation of motion for the oscillatory flow in a longitudinal magnetic field is expressed as follows.

$$W_0^2 \frac{\partial v_z^*}{\partial t^*} = -\frac{\partial p^*}{\partial z^*} + \left(1 + \frac{\Delta \eta}{\eta}\right) \times \left(\frac{\partial^2 v_z^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial v_z^*}{\partial r^*}\right) \quad (8)$$

where $W_0 = r_0 \sqrt{\omega/\nu}$ is the Womersley number, r_0 the pipe radius, ω the angular frequency and ν the kinematic viscosity. All the variables are nondimensionalized as $t^* = \omega t$, $v_z^* = v_z/v_{z,0}$, $r^* = r/r_0$, $z^* = z/r_0$, p^*

Fig. 6 Model of aggregated particles in a magnetic field

$= \rho r_0 / \eta v_{z,0}$ where $v_{z,0}$ is the amplitude of the oscillating mean flow velocity.

The increment in effective viscosity $\Delta\eta$ in the case of longitudinal magnetic field is also given as

$$\Delta\eta = \frac{\beta}{4} \left\{ \frac{\xi - \coth\xi}{\xi + \coth\xi} + \left(\frac{1-\gamma^2}{1+\gamma^2} \right) \left(\frac{L}{\xi} - 1 + \frac{\xi}{3} - \frac{4}{3\xi} + \frac{3}{\xi^2} \right) \right\} \quad (9)$$

and

$$\beta = \frac{4\phi\eta_c}{3} \frac{1-\gamma^4}{(2-\gamma^2)(\gamma^2/\sqrt{1-\gamma^2} \ln\left(\frac{1+\sqrt{1-\gamma^2}}{\gamma}\right) - \gamma^2)}, \quad (10)$$

where β is rotational friction per unit volume, ϕ the volumetric density of magnetic particles, L the Langevin function ($= \coth\xi - \xi^{-1}$), $\gamma = 1/Nu$, and $\xi = \mu_0 m H / kT^{(24),(25)}$. The velocity distribution is obtained by solving Eq. (8) with differential calculus when the fluctuating flow rate of the n -mode ($n \geq 1$) is given. Also, the fluctuating pressure difference between two

points in a pipe is expressed by the following equation.

$$\Delta p^* = - \int_0^l \frac{1}{\pi} \int_0^{2\pi} \int_0^1 U_z^* r^* dr^* d\theta^* \quad (11)$$

where $U_z^* = v_z^* / (\partial p^* / \partial z^*)$, $l^* = l/r_0$.

Figure 7 shows the experimental apparatus used to examine oscillatory pipe flow of a magnetic fluid in a stationary nonuniform magnetic field⁽²⁴⁾. The oscillatory flow is produced by the vibration exciter in the range of frequency from 1 Hz to 20 Hz, and the pressure difference between two pressure transducers ($\Delta p^* = p_a^* - p_b^*$) is measured.

The magnetic flux distribution is shown in Fig. 8, which is applied transversely or longitudinally to the pipe axis by an electromagnet and a permanent magnet, respectively.

The test results obtained using the water-based magnetic fluid with 20% mass concentration of particles in a longitudinal magnetic field are shown in Fig. 9 compared with the theoretical predictions. The experimental data in the case of transverse field are also shown. The increasing rate of pressure difference of the first-mode oscillatory flow with the applied magnetic field is plotted vertically. The abscissa is the Womersley number. Also, the Reynold's number Re is defined as $2r_0 v_{z,0} / \nu$ where $v_{z,0}$ is the amplitude of mean velocity. The test results of the fluctuating pressure difference show large increases especially at low- W_0 ranges compared to the theoretical prediction with the assumed Nu -number.

The number Nu for the longitudinal field is much larger than that for the transverse field ($Nu = 10 \sim 50$), which is assumed to explain the experimental data.

Fig. 7 Scheme of experimental apparatus for oscillatory pipe flow

Fig. 8 Magnetic field distribution

Fig. 9 Relationship of increase ratio of pressure difference to Womersley number

Fig. 10 Scheme of experimental apparatus

Fig. 11 Frequency response curve ($l_m=40$ mm, $l_f=100$ mm, $z_{20}=0$ mm, $M=0.48$ g)

In Fig. 9, a large discrepancy appears between theory and experiment in the low-frequency range. This is mainly due to the model of particle aggregates proposed here, which is insufficient to explain the complicated behavior of particles in the low frequency range.

3.2 Oscillation of a magnetic fluid plug induced by an alternating magnetic field

As a specific example of oscillatory pipe flow of a magnetic fluid, the oscillation of a magnetic fluid plug in a U-tube is investigated here by applying an alternating magnetic field as shown in Fig. 10⁽²⁶⁾, which is useful in clarifying the frequency response characteristics of magnetic fluid actuators.

In Fig. 10, the magnetic fluid column, i.e., the magnetic fluid plug, is supported by a nonuniform magnetic field which is generated by a constant supply current to the electromagnet. Furthermore, when a harmonically oscillating current is applied to the magnet, a pulsating magnetic field is generated, which induces the oscillating motion of the plug, which is then transferred to a piston through the water column.

Measurements of the amplitude of piston displacement against the frequency of the alternating field are performed.

As an example of the experimental results, the effect of the spring constant k_c of the spring-piston system on the frequency response curve is shown in Fig. 11, where z_{20} is the initial height of the upper interface of the plug from the center of the applied field, and M is the mass of the piston. Maximum magnetic field strength in the stationary state is $H_{\max}=26.5$ kA/m and the maximum amplitude of the alternating field is $\Delta H_{\max}=26.0$ kA/m.

Kerosene-based magnetic fluid with 20% mass concentration is used as a working fluid.

In the figure, the analytical results of Ref. (26) are also shown. The values of theoretical prediction are high compared with the experimental values because of neglect of the friction loss of the piston in the analysis. However, they agree qualitatively with the experimental values; that is, the resonance frequency increases and the maximum amplitude decreases with the increase in the spring constant.

4. Conclusions

This review deals with the effect of a magnetic field on the pipe flow characteristics of a magnetic fluid.

Several experimental studies show that a large increase in effective viscosity occurs in an applied magnetic field and suggest that the small particles in a strong magnetic field partially aggregate to form a chainlike cluster. The basic equations of a magnetic fluid which take into account the particle aggregate are, therefore, solved for steady and unsteady pipe flows.

As a specific example of pipe flows, gas-liquid two-phase flow is investigated theoretically and experimentally for the development of a new energy conversion system using a magnetic fluid.

Finally, the oscillatory motion of a magnetic fluid plug induced by an alternating magnetic field is investigated as the basis for the development of magnetic fluid actuators.

However, the proposed theoretical models of particle aggregates explain the experimental results only qualitatively. It is, therefore, necessary to investigate particle behavior in an applied magnetic field to a greater extent from the microscopic viewpoint, and to formulate a constitutive equation to explain the experimental results more precisely.

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