Refinement of the Spectral Model of Turbulent Burning Velocity* (In the Case of Stoichiometric Mixtures)

Hiroyuki KIDO**, Toshiaki KITAGAWA**,
Kenshiro NAKASHIMA** and Jun-Hyo KIM**

The formerly proposed spectral model of turbulent mass burning velocity is refined for practical use. The model is expressed by an integral form of the product of two independent functions. One is the turbulence energy spectrum and the other is the characteristic spectrum solely related to the mixture properties, such as laminar burning velocity and laminar flame thickness. Refinements are made in regard to the following three points: (1) a small modification in the characteristic spectrum of the mixture, (2) consideration of turbulence spectrum shape variation with turbulence intensity and (3) a new idea regarding the upper limit of the integral which is strongly related to the small-scale structure of the turbulent flame. The predicted velocities for stoichiometric mixtures are compared with the measured turbulent mass burning velocities, where the laminar burning velocity, laminar flame thickness and turbulence intensity are varied extensively and independently with each other. The comparison shows fairly good quantitative consistency.

Key Words: Internal Combustion Engine, Premixed Combustion, Turbulent Burning Velocity, Spectral Model, Turbulence Energy Spectrum, Laminar Burning Velocity, Laminar Flame Thickness

1. Introduction

The turbulent burning velocity is the most important factor in engine performance. In order to predict adequately the turbulent burning velocity S_T of premixed mixture quantitatively, many studies have been conducted^{(1)~(8)}. Except for some recent cases^{(6)~(8)}, the expressions of turbulent burning velocity are rather simple because they are formulated based on experimental results using only a few mixtures. Predicted velocities by some of the existing expressions show vast differences for a given mixture and turbulence condition.

The authors conducted experimental studies of turbulent burning velocities on more than thirty mixtures⁽⁹⁾, where laminar burning velocity S_L , laminar flame thickness δ , equivalence ratio and turbulence intensity u' were varied extensively and independently with each other. The results show that every mixture and turbulence parameter stated above has a large influence on the turbulent burning velocity.

Experiments where the length scale of turbulence is varied independently keeping the other mixture and turbulence parameters constant, appear not to have been carried out yet, since the mean length scale of turbulence is strongly affected by the turbulence intensity and the configuration of the combustion chamber. The mean length scale of turbulence, however, should be treated as an independent parameter of turbulence in order to discuss measured burning velocities using combustion chambers with different configurations.

Thus, the turbulent burning velocities of premixed mixtures are discussed in regard to five independent parameters. They are the mixture characteristics: laminar burning velocity, laminar flame thickness and equivalence ratio; and turbulence characteristics: intensity and mean length scale.

The influence of the spectrum shape, or the ratio of the Taylor microscale to the mean length scale, is treated as a parameter dependent on the turbulence Reynolds number.

Although the quantitative effects of the equivalence ratio on the turbulent burning velocity are large⁽⁹⁾, only those for stoichiometric mixtures are discussed in this study to show the spectral model clearly.

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** Faculty of Engineering, Kyushu University, 6-10-1 Hakozaiki, Higashi-ku, Fukuoka 812, Japan

Refinements of the formerly proposed spectral model of turbulent mass burning velocity follow three points: (1) a small modification of the characteristic spectrum of the mixture; (2) consideration of the turbulence spectrum shape variation with turbulence intensity; (3) a new idea of the upper limit of the integral strongly related to the small-scale structure of the turbulent flame. The calculation method for mixture properties is simplified, in addition to the above three refinements.

In this study, the measured turbulence field in the constant volume chamber and the experimental results of turbulent mass burning velocity with this chamber are shown first. Refinements of the spectral model and the method used to obtain the turbulence energy spectrum are explained next. The model predicts turbulent mass burning velocity, which is compared with the measured turbulent mass burning velocities, where laminar burning velocity, laminar flame thickness and turbulence intensity are varied extensively and independently with each other under stoichiometric mixture conditions.

The results confirm the quantitative accuracy and availability of the refined model.

2. Nomenclature

- a : thermal diffusivity of mixture
- C : an empirical constant ($=0.26$) in Eq.(9)
- c_1 : a coefficient given by Eq.(16)
- c_2 : a coefficient given by Eq.(18)
- Da : Damköhler number of turbulent flame given by Eq.(8)
- $E(k)$: turbulence energy spectrum
- $F(k)$: characteristic spectrum related only to mixture property given by Eq.(3)
- $G(k)$: a part of $F(k)$ given by Eq.(4)
- k : wave number
- k_c : upper integral limit of wave number
- k_m : wave number at the maximum of $E(k)$
- L_g : lateral integral length scale of turbulence
- M : molecular weight
- n_1 : an exponent fixed at 4.0
- n_2 : an exponent as a function of turbulence Reynolds number given by Eq.(19)
- P : pressure
- P_0 : initial pressure
- S_L : laminar burning velocity
- S_T : turbulent burning velocity
- t : time
- u' : turbulence intensity
- x : molar fraction in Eq.(13)
- y : represents thermal conductivity λ or viscosity μ in Eq.(13)
- δ : laminar flame thickness defined by Eq.(11)

for hydrocarbon mixture and by Eq.(12) for hydrogen mixture

- ε : dissipation rate of kinetic energy
- η : preheat zone thickness in the unburned state given by Eq.(10)
- η_k : Kolmogorov length scale
- λ : thermal conductivity of a mixture
- λ_g : Taylor microlength scale
- μ : viscosity of a mixture
- ν : kinematic viscosity of a mixture
- τ_c : characteristic chemical reaction time given by Eq.(7)
- τ_T : characteristic turbulent mixing time given by Eq.(6)
- ϕ_{ij} : given by Eq.(14)

3. Measured Turbulence Field

The combustion chamber used in this experiment was a constant volume bomb having a ball-like inner space with an approximate diameter of 125 mm⁽⁹⁾. Mixture was ignited at the center of the chamber by an electric spark of charged energy 0.45 J. The turbulence field created by four fans cooperating with four perforated plates surrounding the inner space was steady and virtually isotropic. The turbulence intensity was controlled by fan speed.

The characteristics of the turbulence field was measured and reported elsewhere⁽¹⁰⁾. Some of the results are shown in Fig. 1, where L_g is the lateral integral length scale and λ_g is the Taylor microlength scale. The L_g 's were measured by the cross-correlation method of velocity fluctuation, assuming isotropy of the turbulence field. The λ_g 's were obtained by a curve-fitting method of measured cross-correlation functions near zero separation of two probe volumes. Also shown in the figure is the λ_g/L_g calculated from the turbulence energy spectrum Eq.(15) as a function of turbulence Reynolds number $(u'L_g)/\nu$ using measured u' and L_g and calculated kinematic viscosity of the mixture ν . L_g becomes slightly large as u' increases. The λ_g/L_g decreases with increasing u'

Fig. 1 Measured and calculated spatial scales of turbulence versus turbulence intensity

because the energy spectrum distribution extends to the higher-frequency region as u' increases. In this combustion chamber, λ_θ is larger than L_θ in the relatively weak turbulence range.

4. Composition of Mixture

Three groups of hydrocarbon mixtures and one group of hydrogen mixtures were used in the experiment. These mixtures were concocted in the combustion chamber according to the partial pressures of the components. In each group, the compositions of mixtures were determined experimentally by changing the volumetric proportion of fuels, oxygen and inert gases so that the mixtures had several fixed values of laminar burning velocity S_L . The value of laminar flame thickness δ was varied extensively keeping the laminar burning velocity virtually constant for each group. The hydrocarbons used in the experiment were methane, acetylene and propane, and the inert gases were nitrogen and helium.

The properties of mixtures are shown in Table 1. In this table, δ is laminar flame thickness, η is preheat zone thickness in the unburned state and ν is kinematic viscosity of the mixture. All mixtures are stoichiometric in this study. Hydrocarbon groups I, II and III have about 43, 62 and 73 cm/s of laminar burning velocity S_L , respectively, and hydrogen group IV has about 42 cm/s of S_L .

Table 1 Properties of mixtures used in the experiment
All mixtures are stoichiometric.
 δ is calculated by $\delta=5\eta$ for hydrocarbon mixtures and by $\delta=(5\eta)\times 0.38$ for hydrogen mixtures.

		Mixture	S_L m/s	δ mm	$\eta \times 10^2$ mm	$\nu \times 10^5$ m ² /s
I	43-1A	C ₂ H ₂ +2.5O ₂ +16.55N ₂	0.435	0.232	4.65	1.47
	43-2M	CH ₄ +2O ₂ +7.17N ₂	0.429	0.246	4.93	1.52
	43-3M	CH ₄ +2O ₂ +6.86N ₂ +0.63He	0.442	0.274	5.49	1.62
	43-4A	C ₂ H ₂ +2.5O ₂ +15.74N ₂ +1.77He	0.431	0.285	5.71	1.60
	43-5M	CH ₄ +2O ₂ +6.56N ₂ +1.25He	0.438	0.313	6.26	1.72
	43-6A	C ₂ H ₂ +2.5O ₂ +14.32N ₂ +4.85He	0.434	0.376	7.53	1.86
	43-7A	C ₂ H ₂ +2.5O ₂ +13.75N ₂ +6.10He	0.434	0.416	8.32	1.97
	43-8M	CH ₄ +2O ₂ +5.64N ₂ +3.12He	0.446	0.422	8.44	2.05
	43-9M	CH ₄ +2O ₂ +4.78N ₂ +4.88He	0.428	0.561	11.22	2.42
II	62-1P	C ₃ H ₈ +5O ₂ +15.2N ₂	0.625	0.151	3.01	1.39
	62-2A	C ₂ H ₂ +2.5O ₂ +14.45N ₂	0.632	0.159	3.18	1.47
	62-3M	CH ₄ +2O ₂ +6.09N ₂	0.620	0.170	3.41	1.52
	62-4M	CH ₄ +2O ₂ +5.59N ₂ +1.03He	0.615	0.219	4.38	1.71
	62-5M	CH ₄ +2O ₂ +4.74N ₂ +2.77He	0.623	0.301	6.01	2.05
	62-6M	CH ₄ +2O ₂ +3.39N ₂ +5.53He	0.632	0.444	8.88	2.73
III	73-1A	C ₂ H ₂ +2.5O ₂ +13.7N ₂	0.735	0.137	2.73	1.46
	73-2M	CH ₄ +2O ₂ +5.51N ₂	0.745	0.142	2.84	1.53
	73-3M	CH ₄ +2O ₂ +5.06N ₂ +1.09He	0.730	0.189	3.78	1.73
	73-4M	CH ₄ +2O ₂ +4.66N ₂ +1.92He	0.738	0.223	4.45	1.90
	73-5A	C ₂ H ₂ +2.5O ₂ +10.96N ₂ +5.96He	0.725	0.264	5.27	2.04
	73-6M	CH ₄ +2O ₂ +3.55N ₂ +4.13He	0.742	0.324	6.47	2.43
	73-7M	CH ₄ +2O ₂ +3.11N ₂ +5.16He	0.746	0.371	7.42	2.71
IV	42-1H	H ₂ +0.5O ₂ +4.69N ₂	0.434	0.148	7.81	1.76
	42-2H	H ₂ +0.5O ₂ +4.17N ₂ +1.05He	0.414	0.203	10.70	2.10
	42-3H	H ₂ +0.5O ₂ +3.65N ₂ +2.11He	0.411	0.257	13.54	2.51
	42-4H	H ₂ +0.5O ₂ +3.13N ₂ +3.16He	0.404	0.319	16.77	2.98
	42-5H	H ₂ +0.5O ₂ +2.80N ₂ +3.83He	0.427	0.338	17.77	3.33

The laminar burning velocity was calculated from the rate of pressure rise $(dP/dt)_L$, after ignition at an increased pressure of $P-P_0=14.7$ kPa above the fixed initial pressure of $P_0=0.101$ MPa, when the radius of the spherical laminar flame was about 40 mm and the calculated laminar burning velocity was relatively insensitive to a certain increment of combustion pressure. The maximum pressure attained in the combustion process, which was necessary for the calculation of S_L , was computed for a constant volume adiabatic combustion, taking the dissociation reaction into consideration.

The laminar flame thickness, preheat zone thickness and kinematic viscosity were obtained through procedures stated later.

5. Measured Turbulent Mass Burning Velocity

Combustion experiments were conducted for each mixture under five different turbulence intensities from 0.49 to 3.76 m/s.

The rate of pressure rise $(dP/dt)_T$ in turbulent combustion at $P-P_0=14.7$ kPa was calculated from the pressure history. Then the ratio of it to $(dP/dt)_L$, the rate of pressure rise of laminar combustion at the same increased pressure, was obtained. This measured ratio of mass burning rate is considered to be virtually equal to the ratio of the turbulent burning velocity S_T to the laminar burning velocity S_L ⁽¹¹⁾.

The results are shown in Fig. 2 by open circles and fine lines. The ratio $(dP/dt)_T/(dP/dt)_L$ increases approximately linearly with turbulence intensity in the range of small u' , but it flattens out with increasing u' , as shown, for example, in mixtures where $S_L=43$ cm/s, $\delta=0.313$ mm (43-5M) and $S_L=62$ cm/s, $\delta=0.444$ mm (62-6M). A decrease in the ratio occurs with a further increase in u' , as shown in the mixture where $S_L=43$ cm/s, $\delta=0.561$ mm (43-9M). Finally, the flame is extinguished when the turbulence intensity exceeds a certain value.

The figure also shows that in mixtures with a smaller δ , the rate of increase in the ratio $(dP/dt)_T/(dP/dt)_L$ against u'/S_L is larger in the range of small u' , and the flattening out effect of the ratio itself is smaller in the range of medium to large u' .

6. Spectral Model

6.1 Description of refined model

The spectral model of turbulent burning velocity is expressed by Eq. (1).

$$\frac{S_T}{S_L} = \int_0^{k_c} F(k) \cdot E(k) / \left(\frac{3}{2} u'^2 \right) dk + \int_0^{k_c} G(k) dk. \quad (1)$$

The main part is an integration of the product of two independent functions $E(k)$ and $F(k)$, where $E(k)$

is the turbulence energy spectrum and $F(k)$ is the characteristic spectrum related only to the mixture properties, such as laminar burning velocity and laminar flame thickness. Each spectrum is a function of wave number k . Since the ratio of S_T to S_L depends on the product of the two independent spectra, there must be an optimum spectrum shape of turbulence to a specific mixture. In other words, the turbulent eddy with too small or too large mean length scale is not effective in increasing the burning velocity of a given mixture. In addition, to integrate this product with respect to the wave number indicates that each scale of turbulent vortices contributes to the increase in turbulent burning velocity, depending not on its intensity $E(k)$ solely but on the weighted product $F(k)E(k)$.

The upper limit k_c of the integral sets a limit to the eddy size, below which no effective increase in the turbulent flame surface can be expected. This smallest size of eddy that can form a corrugated flame surface may well correspond to the inner cutoff scale of the fractal analysis of the turbulent flame⁽⁶⁾.

The characteristic spectrum $F(k)$ consists of two antithetic functions of the wave number. One is the

expected net increase in the flame surface area with increasing wave number when a unit mass of turbulent eddies with a specified wave number affects hypothetically a unit surface of the laminar flame. The other, on the contrary, is a decreasing "effective" flame surface area with increasing wave number due to the mutual interference of turbulent eddies. The latter function predominates when eddy size decreases, and appears again in the second term of the right-hand side of Eq.(1) as $G(k)$.

The second term of Eq.(1), which represents the turbulence effects on the original unit area of the laminar flame, is considered separately from the first term which represents the turbulence effects related to the net, increased flame surface. The subtraction of the imaginary unit area of the laminar flame from the total turbulent flame area makes it easy to consider the case with no turbulence. Then, the following equation can be obtained from Eq.(1) because S_T approaches S_L when $E(k)$ approaches zero.

$$\int_0^\infty G(k)dk=1 \quad (2)$$

Some refinements of the original model⁽³⁾ give the following equation for S_T :

Fig. 2 Measured and calculated mass burning velocities

$$F(k) = 2\sqrt{3} \frac{u'}{S_L} (16\delta k)^2 e^{-16\delta k} \quad (3)$$

$$G(k) = 16\delta e^{-16\delta k} \quad (4)$$

$$\frac{S_T}{S_L} = 2\sqrt{3} \frac{u'}{S_L} \int_0^{k_c} (16\delta k)^2 e^{-16\delta k} E(k) / \left(\frac{3}{2} u'^2 \right) dk + 16\delta \int_0^{k_c} e^{-16\delta k} dk, \quad (5)$$

where the factor k^2 shows the increasing flame surface effect with increasing wave number. The factor $\exp(-16\delta k)$ indicates, on the contrary, the increasing mutual interference effect of turbulent eddies with an increasing wave number. These effects are solely expressed in relation to the mixture parameter, the laminar flame thickness δ . This indicates that the relative scale of a turbulent eddy, which is virtually the reciprocal of k , to the laminar flame thickness δ is decisive of the net increase in the flame surface area. The optimum eddy size is assumed to be $1/k = 8\delta$ in Eq.(3). Eddies with too large or too small size compared with 8δ are not effective in increasing S_T . The turbulence energy spectrum $E(k)dk$ is normalized by the kinetic energy in Eq.(5).

6.2 Upper integral limit k_c

The upper limit k_c of the integral in Eq.(5) sets a limit of the turbulent eddy size to the increase in the effective flame surface. The critical eddy size is a function of the mixture and turbulence parameters because the small-scale structure⁽¹²⁾ is mainly influenced by the Damköhler number, which is the ratio of the characteristic turbulent mixing time τ_T to the characteristic chemical reaction time τ_c . Here τ_T is considered to be the turnover time for an eddy with the size of lateral integral length scale L_g at a speed u' , the turbulence intensity, and is given by

$$\tau_T = L_g / u'. \quad (6)$$

The characteristic chemical reaction time τ_c is based on the characteristic transit time through a laminar flame with a thickness δ at a speed S_L , the laminar burning velocity, and is given by

$$\tau_c = \delta / S_L. \quad (7)$$

With τ_T and τ_c , the Damköhler number Da of a turbulent flame is

$$Da = \frac{\tau_T}{\tau_c} = \frac{S_L L_g}{u' \delta}. \quad (8)$$

The critical eddy size ($\approx 1/k_c$) is determined in order that the molecular diffusion over the eddy size within τ_c is nearly equal to the turbulent diffusion over δ within τ_T . Hence,

$$k_c = C \sqrt{Da} / \delta, \quad (9)$$

where C is a constant of the order one, which will be determined later by comparing the calculated S_T with the measured one.

This critical eddy size ($\approx 1/k_c$) is inherently connected to the microstructure of the turbulent flame,

indicating that any eddy smaller than the critical size cannot form an effective flame surface. Therefore, the critical size is considered to correspond to the inner cutoff size of the fractal analysis of the turbulent flame⁽⁶⁾. At mixture and turbulence conditions of large S_L and/or small δ , a small-scale flame surface can be formed. However, when S_L decreases and/or δ increases, the minimum size of the turbulent flame corrugation increases.

6.3 Laminar flame thickness δ

In the original spectral model, the laminar flame thickness was calculated tediously by some procedures taking the variation of mixture properties with a temperature increase near the flame front into consideration. The calculated laminar flame thickness, which was checked by the measured data of Morgan and Kane⁽¹³⁾ was shown to be about 4.5 to 5 times larger than the preheat zone thickness η of the laminar flame. The preheat zone thickness can be easily calculated by

$$\eta = a / S_L, \quad (10)$$

where a is the thermal diffusivity of the unburned mixture and S_L is the laminar burning velocity. Since η can be calculated with no ambiguity, for simplicity, the laminar flame thickness δ is calculated by the following equation in this study.

$$\delta = 5\eta. \quad (11)$$

The above method of calculation of the laminar flame thickness δ is valid for hydrocarbon mixtures, however, hydrogen mixtures require some modification because the reaction of hydrogen is not controlled by the thermal diffusion mechanism but by the molecular diffusion mechanism of the H radical⁽¹⁴⁾. Therefore, the laminar flame thickness of the hydrogen mixture is modified as follows, considering the positive diffusion zone thickness of the H radical.

$$\delta = (5\eta) \times 0.38. \quad (12)$$

The constant 0.38 is the average ratio of the diffusion zone thickness of the H radical to the preheat zone thickness. The δ calculated by Eq.(12) is also used in the calculation of Da by Eq.(8) for every hydrogen mixture.

6.4 Calculation of mixture properties

The thermal conductivity λ and the viscosity μ of a mixture composed of three or four components are calculated by the following semiempirical formula by Wilke⁽¹⁵⁾:

$$y_{\text{mix}} = \sum_i \left(\frac{x_i y_i}{\sum_j x_j \phi_{ij}} \right) \quad (13)$$

$$\phi_{ij} = \frac{1}{\sqrt{8}} \left(1 + \frac{M_i}{M_j} \right)^{-1/2} \left[1 + \left(\frac{\mu_i}{\mu_j} \right)^{1/2} \left(\frac{M_j}{M_i} \right)^{1/4} \right]^2, \quad (14)$$

where y represents the thermal conductivity λ or viscosity μ , x is the molar fraction, M is the molecu-

lar weight of the component, and i and j denote the mixture components.

7. Turbulence Energy Spectrum

Factors, excluding the turbulence energy spectrum $E(k)$, for calculating S_T by Eq. (5) have all been explained and the procedures of calculation indicated. Originality of the spectral model also exists in considering $E(k)$ and its variation with the turbulent Reynolds number. The standardized expression of the turbulence energy spectrum in this paper is

$$E(k) = \frac{c_1}{k_m} \left(\frac{k}{k_m} \right)^{n_1} \exp \left[-c_2 \left(\frac{k}{k_m} \right)^{n_2} \right], \quad (15)$$

where c_1 , c_2 , n_1 and n_2 are coefficients determined later and k_m is the wave number at maximum $E(k)$.

The coefficient c_1 can be determined such that the integration of $E(k)$ from zero to infinite k must be equal to the turbulent kinetic energy:

$$c_1 = \frac{3}{2} u'^2 (n_1 + 1) c_2^{(n_1+1)/n_2} / \Gamma \left(1 + \frac{n_1+1}{n_2} \right), \quad (16)$$

where Γ shows a gamma function.

The coefficient n_1 is fixed at 4.0, independent of the turbulent Reynolds number, since the turbulence energy spectrum in the small wave number range begins in proportion to k^4 , as shown by Hinze⁽¹⁶⁾. Thus we obtain

$$n_1 = 4.0. \quad (17)$$

The third relationship for the remaining two coefficients can be obtained at the condition where the energy spectrum takes a maximum value at $k = k_m$. Then we have

$$n_1 = c_2 n_2. \quad (18)$$

Lastly, the coefficient n_2 is expressed as a function of the turbulence Reynolds number $(u' L_\theta)/\nu$ as follows,

$$n_2 = 16.2 \left(\frac{u' L_\theta}{\nu} \right)^{-2/3}, \quad (19)$$

where L_θ is the lateral integral length scale of turbulence and ν is the kinematic viscosity of the mixture. Equation (19) is checked in order that the calculated ratio of the Taylor microscale λ_θ to L_θ can accurately approximate the measured ratio at the five Reynolds numbers shown in Fig. 1.

As stated above, n_2 , c_2 and c_1 are calculated for a given Reynolds number by Eq. (19), Eq. (18) and Eq. (16), respectively, at $n_1 = 4.0$. Thus the turbulence energy spectrum can be finally determined by Eq. (15) for a given turbulence Reynolds number, as shown in Fig. 3.

For practicality, the expressions for L_θ , the dissipation rate of kinetic energy ε , Taylor microscale λ_θ and Kolmogorov length scale η_k are shown as follows.

$$L_\theta = \frac{\pi}{4 u'^2} \int_0^\infty \frac{E(k)}{k} dk = \frac{3\pi}{8 k_m} \left(\frac{n_1}{n_2} \right)^{1/n_2} \left(\frac{n_1+1}{n_1} \right)$$

$$\times \Gamma \left(1 + \frac{n_1}{n_2} \right) / \Gamma \left(1 + \frac{n_1+1}{n_2} \right). \quad (20)$$

$$\varepsilon = 2\nu \int_0^\infty k^2 E(k) dk = 3\nu k_m^2 u'^2 \left(\frac{n_1+1}{n_1+3} \right) c_2^{-2/n_2} \times \Gamma \left(1 + \frac{n_1+3}{n_2} \right) / \Gamma \left(1 + \frac{n_1+1}{n_2} \right). \quad (21)$$

$$\varepsilon = 15\nu u'^2 / \lambda_\theta^2. \quad (22)$$

$$\eta_k = (\nu^3 / \varepsilon)^{1/4}. \quad (23)$$

8. Comparison between Calculated and Measured Mass Burning Velocities

Calculations, which were simple numerical integrations of Eq. (5), were made first for mixture 43-1A by varying the constant C in Eq. (9). A best fit with the measured velocities was obtained for $C = 0.26$. Thus the upper wave number limit k_c was calculated thereafter by

$$k_c = 0.26 \sqrt{D_a} / \delta. \quad (24)$$

The calculated mass burning velocities for all mixtures using the respective mixture and turbulence parameters are also shown in Fig. 2 by bold lines. In addition, the fine broken lines in this figure represent the calculated results for $k_c = \infty$, which show the effect of the change of spectrum shape with the turbulence intensity alone on the turbulent burning velocities.

The predicted burning velocities show fairly good quantitative agreement with the measured ones for all mixtures. S_T/S_L increases approximately linearly with u'/S_L in the range of small u'/S_L , where the gradient increases with the decrease in laminar flame thickness δ . In the range of medium to rather large u'/S_L , the increasing rate of S_T/S_L is reduced showing a gentle slope. This arises from two reasons. One is the change of spectrum shape, i.e., the broadening effect of the energy spectrum to the range of high wave number with an increase in the turbulence intensity, as illustrated by the fine broken lines in the figure. The other is the decrease in the upper integral limit k_c with an increase in turbulence intensity, as illustrated by the increasing difference between the fine broken and bold solid lines. In the range of large u'/S_L , the gentler

Fig. 3 Turbulence energy spectrum by Eq. (15)

slope becomes a plateau and S_T then slows down.

A small difference between fine broken and bold solid lines assures a greater increase in S_T with u' but the increase in this difference foresees an extinction of the propagating turbulent flame.

It is desirable to apply the model to other existing data of turbulent burning velocities. The recent data by Baritaud⁽¹⁷⁾ bear a close resemblance to those of 43-1 A to 43-4 A. No attempt to apply the model is made at this time due to the lack of information about mixture and turbulence parameters and to nonstoichiometric mixtures used in the experiment. The model, however, estimates that the data must be correlated by at least two lines which account for different S_L and δ .

9. Summary

Refinement of the spectral model of turbulent mass burning velocity was accomplished. The calculated velocities exhibited fairly good quantitative agreement with the measured ones for a large variety of stoichiometric mixtures.

Calculation procedures are summarized as follows.

(1) Measurement of turbulence intensity u' , lateral integral length scale L_g and laminar burning velocity S_L .

(2) Calculation of thermal diffusivity a and kinematic viscosity ν of a mixture in unburned state.

(3) Calculation of laminar flame thickness δ by Eq.(11) for hydrocarbon mixtures or by Eq.(12) for hydrogen mixtures, where preheat zone thickness η is calculated by Eq.(10).

(4) Calculation of the upper wave number limit k_c by Eq.(24), where $Da = S_L L_g / (u' \delta)$.

(5) Calculation of the shape of the turbulence energy spectrum, where n_2 is calculated by Eq.(19) as a function of turbulence Reynolds number $u' L_g / \nu$, $n_1 = 4.0$ by Eq.(17), c_2 by Eq.(18) and c_1 by Eq.(16).

(6) Then, turbulent mass burning velocity S_T can be calculated by a simple numerical integration of Eq. (5).

Further verification of the model through many experimental data is anticipated, and an extension of the model to nonstoichiometric mixtures is necessary considering the Lewis number based on the molecular diffusivity of the deficient reactant in the reaction zone.

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