Diffusion of Fluid Particles by the Combined Model of Random Fourier Modes and Random Flight*

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To investigate the motion of fluid particles around a cylinder in turbulent flows, we developed a new model for simulating the trajectory of particles: the combined model of kinematic simulation (KS) and random flight (RF). The large-scale turbulence is simulated by a sum of random Fourier modes varying in space and time, and the small-scale turbulent fluctuation is simply modelled by an ITO type of stochastic differential equation with a memory time comparable to the Lagrangian integral time scale T_s^L of small-scale motion. The Lagrangian properties of fluid particles in homogeneous isotropic turbulence with uniform mean flow have been examined. Some new results have been obtained using this new model: (a) the cross-correlation of the displacements due to the large- and small-scale motions is negligibly small; (b) random small-scale motion causes decorrelation of the large-scale velocity; (c) the value of T_s^L used in RF can be reasonably estimated from pure KS.

Key Words: Diffusion, Turbulence, Fluid Particle, Random Fourier Modes, Kinematic Simulation, Random Flight, Stochastic Equation, Decorrelation

1. Introduction

Understanding the diffusion mechanism of matter and particles in turbulent flows is very important not only in engineering (for instance, dispersion of solid particles and droplets in coal- and oil-fire burners, formation and dispersion of bubbles in cooling circuits of nuclear plants and mixing of matter in reactor vessels, etc.) but also in meteorology and oceanography in association with environmental pollution. In light of these industrial and environmental importance, we have studied a fundamental diffusion problem, i.e., particle diffusion around a circular cylinder set up in a high-Reynolds-number turbulent flow. For simplicity, it is assumed that the flow field upstream

from the cylinder is composed of the uniform mean and homogeneous fluctuating velocities U_0 and u_0 respectively, and also that the mean flow direction is perpendicular to the axis of the cylinder.

The purpose of this study is the development of a new model for approximate simulation of particle diffusion around the circular cylinder in high-Reynolds-number turbulent flow, which has not yet been simulated to date. In this report, we will deal with only a "fluid particle" which is completely passive to the flow field for the simplification of modeling.

The numerical study of the Lagrangian behavior of particles in a turbulent flow has been approached broadly in two different ways. One is the method in which the trajectories of particles are tracked on the basis of the data of the flow field determined by Eulerian simulation in space and time in a certain geometrical frame of reference. Here we will call this method the "Eulerian particle tracking method". The other is the method in which motions of particles are computed along the trajectories on the basis of the Lagrangian statistics of turbulence, which are assumed to be known (by computer simulation or

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experiments). We will call this method the “**Lagrangian particle tracking method**”. For the former, there have been two different approaches to simulating the Eulerian velocity field, i.e., dynamical simulation (DS) and kinematic simulation (KS) (see Ref. 1, for example). The basic idea of DS is to solve the dynamical equations, for instance, Navier-Stokes equation (NS-eq.) or other equation derived from NS-eq. numerically in space and time. On the other hand, in the KS, the flows do not necessarily satisfy the dynamical equations. However these random flow fields are generated whose one or two-points statistics are determined from experimental measurements or other reliable computer simulations. In the DS, there are two typical methods: direct numerical simulation (DNS)^{(2)~(4)} and large eddy simulation (LES)^{(5),(6)}. On the other hand, the representatives of the KS are discrete vortex model^{(7)~(9)} and random Fourier mode method^{(10)~(12)}. At present, the most extensively studied methods are DNS and LES. But in order to ensure sufficient spatial and temporal resolution requirements and numerical stability, both methods require a huge memory capacity and computational time, so that the simulation inevitably becomes very expensive. On the other hand, the KS makes it possible to simulate certain aspects of high-Reynolds-number turbulent flow fields at a relatively low cost. For instance, the discrete vortex model has very often been applied to separation flow^{(7),(8)} and an axisymmetric jet⁽⁹⁾. In the random Fourier mode method^{(10)~(12)}, the turbulent field is described by the finite summation of the statistically independent random Fourier modes. Fung et al.⁽¹²⁾ simulated the homogeneous isotropic turbulence which specified the three-dimensional Eulerian spectrum, and investigated the statistical characteristics of this “turbulence”. They found that the statistics of the simulated turbulence are in good agreement with the past experimental results.

Next, we turn to the Lagrangian particle tracking method. The typical Lagrangian particle tracking method is a random flight (RF) model^{(13),(14)} which is based on the ITO type of stochastic differential equation⁽¹³⁾. Although this model gives an accurate description of the particle motion in homogeneous and isotropic turbulence, in the case of inhomogeneous turbulence, it is necessary to estimate some parameters (e.g., Lagrangian integral time scale which is a function of position), so that an empirical model for each flow must be developed^{(13)~(16)}. Here it should be noted that the random flight model gives only ensemble statistics involving many realizations of particle trajectories. Also, since in this model the particle trajectory is approximated by the sequence of straight

lines, reliable statistics are assumed to be obtained when the turbulence scale is small compared to the flow scale⁽¹³⁾. However, there are some merits in that the model is simple and the computation is inexpensive.

In this study, the combined model of the random Fourier mode and RF was adopted in consideration of the merits and demerits of the various methods above, in particular, the applicability to a three-dimensional high-Reynolds-number turbulence, understandability of physical meaning, capacity of memory and cost. The random Fourier mode method is used to simulate the strongly inhomogeneous turbulence of large scale in front of the cylinder, and the small-scale motion of the particle, advected by the large-scale motion, is simulated by the RF model with a memory effect. In using the random Fourier mode method for the large-scale turbulence around the cylinder, we should take into account the change of the Eulerian velocity spectrum due to the cylinder which is modelled by the rapid distortion theory (RDT). Here it should be noted that according to Hunt⁽¹⁷⁾ the complete asymptotic expression to describe the change of Fourier components of small eddies is very complicated, and this causes prolongation of the computation time. On the other hand, although the RF model is not suitable for the simulation of particle motion in large-scale inhomogeneous turbulence which produces correlated curving trajectories, the motions smaller than the cylinder diameter are approximately straight and can be modelled relatively accurately by the RF model if the parameters (Lagrangian time scale, Lagrangian velocity scale and the time increment of calculation) are properly chosen. The present combined model can be regarded as a practical method in which the demerits of both models are compensated and only the merits are utilized for effective use. Also, it should enable the individual realization of the concentration field to be modelled on a large scale, and the statistics of the concentration field to be modelled on a small scale.

We should point out that the idea of the present combined model is similar to LES in that the velocity field is divided into two parts, i.e., the large- and small-scale motions. In the case of LES, fluid motions larger than the grid size are exactly computed as the numerical solution of the filtered dynamical equation, whereas the velocity field smaller than the grid size is simply modelled by the eddy viscosity assumption in which the memory effect is not considered. There are two main points of difference between LES and the present model: Firstly, in the present model the large-scale motion is simulated not dynamically but kinematically, and secondly the small-scale random

motion of the present model has a definite memory effect. The latter is caused by the fact that the critical spatial scale, by which the velocity is divided into large- and small-scale motions, is determined on the basis of the geometric scale of the body, and so the memory effect of the turbulence comparable to the critical scale can not be neglected.

This kind of simulation of particle motion has not been reported to date, so at the preparatory stage before going on to simulate practical problem of the particle trajectory around the cylinder, it is important to understand the fundamental Lagrangian characteristics of particle motion simulated by this model. In this report, first, the diffusion problem of fluid particles in homogeneous isotropic turbulence which travel with a uniform mean velocity has been examined using the present combined model. In particular, the following two problems are considered here, mainly using physical arguments to develop an appropriate algorithm.

(1) Do the particle motions of large and small scale have simply additive effects on the total Lagrangian statistics (e.g., the mean square displacement, velocity autocorrelation function) of the particle moving in the resultant velocity of both motions, or do they have some interaction effects on total Lagrangian statistics? (see section 3)

(2) How should the appropriate Lagrangian time scale of small-scale motion be determined? (see section 4)

With regard to problem (1), we should refer to Saffman's work⁽¹⁸⁾. Saffman studied the effect of the molecular diffusivity of a substance in turbulent diffusion, and found that the interaction between the molecular diffusion and the turbulent motion reduces the dispersion from the value it would have if the process of molecular and turbulent diffusion were independent and additive. A most important point of difference between Saffman's and our cases is that the small-scale random motion in our model has a definite memory effect, whereas the molecular diffusion in Saffman's problem does not hold effective memory. As a natural course, problem (1) was motivated by the question of whether or not the present model leads to the same results as Saffman.

Here it will be emphasized that in this paper we report only some Lagrangian statistics of fluid particles in homogeneous turbulence by the combined model, so this paper has preliminary nature to go into the simulation of the concentration field around the circular cylinder, which is our final purpose. However, we believe that a good understanding of the model is useful to interpret the future simulation results of the diffusion problem.

2. Model and Calculation Method

2.1 Random Fourier mode method

Since this method is explained in detail in Refs. (1) and (2), here it is given only in summary.

In general, the homogeneous turbulent velocity field $\mathbf{u}(\mathbf{x}, t)$ can be expressed in terms of Fourier transform $S(\mathbf{k}, \omega)$ as

$$\mathbf{u}(\mathbf{x}, t) = \iiint_{-\infty}^{+\infty} S(\mathbf{k}, \omega) \exp\{i(\mathbf{k} \cdot \mathbf{x} - \omega t)\} d\mathbf{k} d\omega, \quad (1)$$

where $\mathbf{k}$ is the wavenumber vector and ω is the angular frequency. By discretizing the above integral and choosing only a finite number of modes N , we can express $\mathbf{u}$ as a finite Fourier series,

$$\mathbf{u}(\mathbf{x}, t) = \sum_{n=1}^N (\mathbf{a}_n \times \hat{\mathbf{k}}_n) \cos\{\mathbf{k}_n \cdot (\mathbf{x} - \mathbf{x}_n) - \omega_n t\} + (\mathbf{b}_n \times \hat{\mathbf{k}}_n) \sin\{\mathbf{k}_n \cdot (\mathbf{x} - \mathbf{x}_n) - \omega_n t\}. \quad (2)$$

The subscript n means the quantity with regard to the n -th Fourier mode. $\mathbf{x}_n$ is the displacement of the eddy of wavenumber $\mathbf{k}_n$. The values of $\mathbf{a}_n$, $\mathbf{b}_n$ and $\mathbf{k}_n$ are independent of $\mathbf{x}$; hence such a velocity field automatically satisfies the continuity condition, $\nabla \cdot \mathbf{u} = 0$. The wavenumber vectors $\mathbf{k}_n$ are randomly distributed on a sphere of wavenumber $|\mathbf{k}_n|$ in each realization of the flow field. $\hat{\mathbf{k}}_n$ is the unit vector which is defined as $\hat{\mathbf{k}}_n = \mathbf{k}_n / k_n$, where $k_n = |\mathbf{k}_n|$. The vectors $\mathbf{a}_n$ and $\mathbf{b}_n$ are chosen randomly with a three-dimensional Gaussian distribution with zero mean and variance determined as

$$\langle |\mathbf{u}_n|^2 \rangle = \frac{2}{3} \langle |\mathbf{a}_n|^2 \rangle = \frac{2}{3} \langle |\mathbf{b}_n|^2 \rangle = 2 \int_{k_n}^{k_{n+1}} E(k) dk, \quad (3)$$

where $k = |\mathbf{k}|$, $E(k)$ is a prescribed three dimensional energy spectrum and $\langle \rangle$ means the ensemble average of many realizations of turbulent fields generated by Eq. (2). In our simulation, the Von Karman energy spectrum has been used:

$$E(k) = \frac{g_1 k^4}{(g_2 + k^2)^{17/6}}, \quad (4)$$

where $g_1 = \pi \Gamma^2\left(\frac{5}{6}\right) / \Gamma^2\left(\frac{1}{3}\right)$ and $g_2 = g_1^{5/6} / \pi = 1.196^{(17)}$. $E(k)$ is normalized to give

$$\int_0^\infty E(k) dk = \frac{1}{2} \langle |\mathbf{u}|^2 \rangle = \frac{3}{2}, \quad (5)$$

where $\langle |\mathbf{u}|^2 \rangle = \langle |u_1|^2 \rangle + \langle |u_2|^2 \rangle + \langle |u_3|^2 \rangle = 3 \langle |u_1|^2 \rangle = 3$ (subscripts 1, 2, 3 mean individual components in a Cartesian coordinate system; 1 is the main streamwise direction, 2 is the horizontal direction and 3 is the vertical direction). The Eulerian integral length scale L^E derived from the energy spectrum is given by

$$L^E = \frac{3\pi}{4} \int_0^\infty k^{-1} E(k) dk / \int_0^\infty E(k) dk = 1. \quad (6)$$

The displacement of the eddy of wavenumber $\mathbf{k}_n$, $\mathbf{x}_n$ is

modelled simply as being advected from the upstream by the mean flow ($\mathbf{U}=(U_0, 0, 0)$), and consequently

$$\mathbf{x}_n = \int_0^t \mathbf{U} dt = (U_0 t, 0, 0). \quad (7)$$

The dependency of the Fourier component $S(\mathbf{k}, \omega)$ on the frequency ω_n of the n -th mode is not clear. But we consider the critical wavenumber k_c which separates small and large scales, and we adopted different expressions for ω_n between the low wavenumber range ($k < k_c$) and the high wavenumber range ($k_c \leq k$) —(the modes of large wavenumbers will be used in estimating the Lagrangian integral time scale of the small-scale motion of particles in section 4). With regard to the large scales of low wavenumbers ($k < k_c$) we choose the following model⁽¹⁹⁾.

$$\omega_n = k_n \cdot v_n, \quad v_n^2 = \frac{2}{3} \int_{k_n}^{k_{n+1}} E(k) dk \quad (8)$$

This model simply states that the unsteady frequency of the n -th mode is of the same order of magnitude as the turnover frequency of eddies of wavenumber k_n .

On the other hand, the small scales at very high Reynolds number are in statistical equilibrium. Therefore, the mean and standard deviations of the frequency distribution are defined by the inertial range scaling laws and must be proportional to $\varepsilon^{1/3} k^{2/3}$. Hence, here we simply choose a single frequency which is equal to the mean value of the distribution for the high wavenumber region ($k_c \leq k$). Therefore

$$\omega_n = \lambda \varepsilon^{1/3} k_n^{2/3}, \quad (9)$$

where λ is a constant of order unity, and ε is the mean rate of dissipation per unit mass, which has also the value on the order of unity (see Ref. (12)). In the present simulation, we simply choose $\lambda \varepsilon^{1/3} = 1$.

In the normalized energy spectrum, the critical wavenumber has been chosen as $k_c = 5.0$ and then the wavenumbers are distributed as suggested by Fung et al.⁽¹²⁾ i.e.,

$$\begin{cases} 0 < k_n < 5.0, & k_n = 5.0 \times \frac{n}{9}, & n = 1, 2, \dots, 8; \\ 5.0 \leq k_n < 50.0, & k_n = 5.0 \times (50.0/5.0)^{\frac{n-9}{41}}, & n = 9, 10, \dots, 50. \end{cases}$$

In our simulation, the total number of modes N is chosen to be 50, the maximum wavenumber k_N is 50.0 and the number of modes in the low wavenumber range N_c is 9.

In order to examine the behavior of the fluid particle around the circular cylinder, we need to make another division between large and small scales compared with the radius of the cylinder. We select this wavenumber to be k'_c , where $k'_c = 1/a$. In order to use the rapid distortion theory⁽¹⁷⁾, the values of a and U_0 have to be constructed by

$$a/L^E \lesssim U_0/u'_0$$

where $u'_0 = \langle u_{i0}^2 \rangle^{1/2}$, therefore we choose $a = 0.2$ and $U_0 =$

10.0, and thus the critical wavenumber $k'_c = 1/a = 5.0$ coincides with the value of k_c in the spectrum. The distribution of the energy derived by $k_c = k'_c = 5.0$ is 59.9% for the small wavenumber range ($0 < k < 5.0$) and 40.1% for the high wavenumber range ($5.0 \leq k < 50.0$).

2.2 Random flight model

The small-scale motion ($k > k'_c$) of the fluid particle compared with the cylinder diameter is simulated by the Random flight model with the memory effect. The contribution to the particle velocity of the small-scale eddies $\mathbf{v}_s$ is assumed to obey the following ITO type of stochastic differential equation⁽¹³⁾ (which is known as the Langevin equation),

$$d\mathbf{v}_s = -\frac{\mathbf{v}_s}{T_s^L} dt + \sigma_{v_s} \sqrt{\frac{2}{T_s^L}} d\boldsymbol{\xi}_t, \quad (10)$$

where T_s^L is the Lagrangian time scale of the small-scale motion, σ_{v_s} is the total root mean square velocity of the high wavenumber eddies ($k > k'_c$) which is given by

$$\sigma_{v_s}^2 = \frac{2}{3} \int_{k'_c}^{\infty} E(k) dk, \quad (11)$$

and $d\boldsymbol{\xi}_t$ is the random velocity increments of the Wiener process with zero mean and variance of dt .

2.3 Simulation condition

Each fluid particle is tracked by using an extended Runge-Kutta scheme modified by Drummond et al.⁽¹¹⁾, and statistical quantities are obtained by taking the ensemble average of many realizations of the particle trajectories. Each particle trajectory is executed in different realizations of the large-scale velocity field and the small-scale random motion. The time increment of calculation Δt is 0.01 under the normal simulation, but in the case of simulating the initial diffusion, we chose $\Delta t = 0.001$. The results in this section were computed from 6 000 realizations of the particle trajectories.

3. Lagrangian Properties of the Fluid Particles in Homogeneous Isotropic Turbulent Flow by the Combined Model

3.1 Theoretical examination

3.1.1 Mean square displacement and Lagrangian autocorrelation function The Lagrangian velocity of a fluid particle at time t for a particle that was at $\mathbf{x}_0$ at time $t=0$, $\mathbf{V}(\mathbf{x}_0, t)$, is described as follows in the present model:

$$\mathbf{V}(\mathbf{x}_0, t) = \mathbf{U}(\mathbf{x}(\mathbf{x}_0, t)) + \mathbf{u}_t(\mathbf{x}(\mathbf{x}_0, t), t) + \mathbf{v}_s(t), \quad (12)$$

where $\mathbf{x}(\mathbf{x}_0, t)$ is a position vector of a particle at time t which was at $\mathbf{x}_0$ at time $t=0$; $\mathbf{U}(\mathbf{x}(\mathbf{x}_0, t))$ is the Eulerian mean velocity at $\mathbf{x}(\mathbf{x}_0, t)$ at time t (here $\mathbf{U} = (U_0, 0, 0)$); $\mathbf{u}_t(\mathbf{x}(\mathbf{x}_0, t))$ is the Eulerian velocity fluctuation advected by the large scale eddies at $\mathbf{x}(\mathbf{x}_0,$

t) at time t ; $v_s(t)$ is the Lagrangian velocity fluctuation of the small-scale motion at time t . $V(\mathbf{x}_0, t)$ can be divided into two parts as follows:

$$V(\mathbf{x}_0, t) = \langle \overline{V(\mathbf{x}_0, t)} \rangle + \mathbf{v}(\mathbf{x}_0, t), \quad (13)$$

where $\overline{}$ denotes an ensemble average over the realizations of small-scale motion of particles which are simulated by the RF model, and $\langle \rangle$ an ensemble average over realizations of large-scale turbulent flow which are generated by the random Fourier mode method. $\mathbf{v}(\mathbf{x}_0, t)$ is the total Lagrangian velocity fluctuation of the particle.

From Eqs. (12) and (13), we obtain

$$\langle V(\mathbf{x}_0, t) \rangle = U(\mathbf{x}(\mathbf{x}_0, t)) = (U_0, 0, 0) \quad (14\cdot a)$$

$$\text{and } \mathbf{v}(\mathbf{x}_0, t) = \mathbf{u}_l(\mathbf{x}(\mathbf{x}_0, t), t) + \mathbf{v}_s(t). \quad (14\cdot b)$$

The total displacement of the particle which was at the position $\mathbf{x}_0$ at time $t=0$, $\mathbf{X}(\mathbf{x}_0, t)$, is given by integrating Eq. (12) over the diffusion time t ,

$$\mathbf{X}(\mathbf{x}_0, t) = \int_0^t \mathbf{V}(\mathbf{x}_0, t') dt'. \quad (15)$$

Now, we consider only the y -component of $\mathbf{X}$, i.e., X_2 . From Eqs. (13), (14·a, b) and (15), X_2 is described by the summation of two parts as follows:

$$X_2 = X_{l2} + X_{s2}, \quad (16)$$

where X_{l2} and X_{s2} are the contributions to the y -direction displacement of particles by the large-scale turbulent fluctuation u_{l2} , and by small-scale random velocity v_{s2} respectively. X_{l2} and X_{s2} are given by

$$X_{l2} = \int_0^t u_{l2}(\mathbf{x}(\mathbf{x}_0, t'), t') dt', \quad X_{s2} = \int_0^t v_{s2}(t') dt'.$$

The mean square value of X_2 can be obtained from Eq. (16).

$$\langle X_2^2 \rangle = \langle X_{l2}^2 \rangle + \langle X_{s2}^2 \rangle + 2\langle X_{l2}X_{s2} \rangle \quad (17)$$

On the other hand, $\langle X_2^2 \rangle$ can be expressed in terms of the Lagrangian velocity autocorrelation function, i.e.,

$$\begin{aligned} \langle X_2^2 \rangle &= \int_0^t dt' \int_0^{t'} dt'' \langle v_2(\mathbf{x}_0, t') v_2(\mathbf{x}_0, t'') \rangle \\ &= 2 \int_0^t d\tau (t-\tau) B_{22}^L(\tau), \end{aligned} \quad (18)$$

where $B_{22}^L(\tau) = \langle v_2(\mathbf{x}_0, t') v_2(\mathbf{x}_0, t'+\tau) \rangle$, $t'' - t' = \tau$. Using Eq. (14·b), $B_{22}^L(\tau)$ can be described as

$$B_{22}^L(\tau) = B_{l2l2}^L(\tau) + B_{s2s2}^L(\tau) + B_{l2s2}^L(\tau) + B_{s2l2}^L(\tau), \quad (19)$$

where

$$\begin{aligned} B_{l2l2}^L(\tau) &= \langle u_{l2}(\mathbf{x}(\mathbf{x}_0, t'), t') u_{l2}(\mathbf{x}(\mathbf{x}_0, t'+\tau), t'+\tau) \rangle, \\ &= \langle u_{l2}(t') u_{l2}(t'+\tau) \rangle, \end{aligned} \quad (20\cdot a)$$

$$B_{s2s2}^L(\tau) = \langle v_{s2}(t') v_{s2}(t'+\tau) \rangle = \overline{v_{s2}(t') v_{s2}(t'+\tau)}, \quad (20\cdot b)$$

$$B_{l2s2}^L(\tau) = \langle u_{l2}(\mathbf{x}(\mathbf{x}_0, t'), t') v_{s2}(t'+\tau) \rangle, \quad (20\cdot c)$$

$$B_{s2l2}^L(\tau) = \langle v_{s2}(t') u_{l2}(\mathbf{x}(\mathbf{x}_0, t'+\tau), t'+\tau) \rangle. \quad (20\cdot d)$$

$\langle X_{l2}^2 \rangle$ and $\langle X_{s2}^2 \rangle$ are expressed by

$$\langle X_{l2}^2 \rangle = 2 \int_0^t d\tau (t-\tau) B_{l2l2}^L(\tau), \quad (21\cdot a)$$

$$\langle X_{s2}^2 \rangle = 2 \int_0^t d\tau (t-\tau) B_{s2s2}^L(\tau). \quad (21\cdot b)$$

From Eqs. (17), (18), (19), (20·a~d) and (21·a, b), we obtain

$$\langle X_{l2}X_{s2} \rangle = \int_0^t d\tau (t-\tau) \{ B_{l2s2}^L(\tau) + B_{s2l2}^L(\tau) \}. \quad (21\cdot c)$$

B_{l2s2}^L , B_{s2l2}^L (Eq. (20·c, d)) and $\langle X_{l2}X_{s2} \rangle$ (Eq. (21·c)) are the interaction terms between the large- and small-scale motion. Here, considering that $\mathbf{u}_l$ and $\mathbf{v}_s$ are simulated independently of each other, and in particular, that $\mathbf{v}_s$ is a function of only the time t , not dependent on the spatial position, we can expect that $B_{l2s2}^L(\tau) \equiv 0$. On the other hand, $B_{s2l2}^L(\tau)$ is not equal to 0 in general because $u_{l2}(\mathbf{x}(\mathbf{x}_0, t+\tau), t+\tau)$ depends on $v_{s2}(t)$. However, noting that $B_{s2l2}^L(\tau=0) = 0$, $B_{s2l2}^L(\tau) |_{\tau \gg T_l^L, T_s^L} \approx 0$ (T_l^L is the Lagrangian integral time scale of the large-scale motion, which will be defined by Eq. (32) later), it can be anticipated that $B_{s2l2}^L(\tau)$ is small compared with $B_{l2l2}^L(\tau)$, $B_{s2s2}^L(\tau)$ (except for the region of $\tau > T_l^L$, T_s^L in which B_{l2l2}^L , B_{s2s2}^L comes to be of the same order as B_{s2l2}^L). Consequently, in the region where the diffusion time t is relatively short ($t \lesssim T_l^L$, T_s^L), the contribution of $B_{s2l2}^L(\tau)$ to $B_{22}^L(\tau)$ can be expected to be small, and this leads to the following approximation of $B_{22}^L(\tau)$.

$$B_{22}^L(\tau) \approx B_{l2l2}^L(\tau) + B_{s2s2}^L(\tau) \quad (22)$$

In the integrated expressions, we can expect that $\langle X_{l2}X_{s2} \rangle$ is also small compared with $\langle X_{l2}^2 \rangle + \langle X_{s2}^2 \rangle$, so

$$\langle X_2^2 \rangle \approx \langle X_{l2}^2 \rangle + \langle X_{s2}^2 \rangle. \quad (23)$$

Equation (23) implies for the mean square displacement, the interaction effect between the large- and small-scale motions is small, so the total mean square displacement $\langle X_2^2 \rangle$ can be described approximately by the summation of $\langle X_{l2}^2 \rangle$ and $\langle X_{s2}^2 \rangle$.

3.1.2 Expressions of $B_{l2l2}^L(\tau)$ and $B_{s2s2}^L(\tau)$

$B_{s2s2}^L(\tau)$ can be expressed as the autocorrelation function of the solution process of the stochastic differential equation (10), as follows⁽¹⁴⁾:

$$B_{s2s2}^L(\tau) = \sigma_{v_s}^2 \exp(-\tau/T_s^L). \quad (24)$$

Next, we will calculate $B_{l2l2}^L(\tau)$. The complex expression of u_{l2} is given by

$$\begin{aligned} u_{l2}(\mathbf{x}(\mathbf{x}_0, t), t) &= \sum_{n=-N_c}^{N_c} C_{2n} \exp \left[i \left\{ \mathbf{k}_n \cdot \left(\mathbf{x}(\mathbf{x}_0, t) \right. \right. \right. \\ &\quad \left. \left. \left. - \int_0^t \mathbf{U}(\mathbf{x}(\mathbf{x}_0, t')) dt' \right) - \omega_n t \right\} \right], \end{aligned} \quad (25)$$

where

$$\begin{aligned} C_{2n} &= 0.5 \{ [(\mathbf{a}_n \times \hat{\mathbf{k}}_n)_2]^2 + [(\mathbf{b}_n \times \hat{\mathbf{k}}_n)_2]^2 \}^{1/2} \cdot e^{-i\theta}, \\ \theta &= \tan^{-1} \{ (\mathbf{b}_n \times \hat{\mathbf{k}}_n)_2 / (\mathbf{a}_n \times \hat{\mathbf{k}}_n)_2 \}, \\ \sum_{n=1}^{N_c} 2\langle |C_{2n}|^2 \rangle &= \sum_{n=1}^{N_c} \langle \overline{u_{l2n}^2} \rangle = \langle \overline{u_{l2}^2} \rangle. \end{aligned}$$

Considering that $\mathbf{U}$ is uniform and the probability density function of $\mathbf{u}_l$ is Gaussian, the following

expression of $B_{i2i2}^L(\tau)$ can be derived from Eqs. (20) and (25),

$$B_{i2i2}^L(\tau) = \sum_{n=1}^{N_c} \langle \overline{u_{i2n}^2} \rangle e^{-1/2 k_n^2 \sigma_{X2}^2} \cos \omega_n \tau, \quad (26)$$

where

$$\sigma_{X2}^2 = \langle (\Delta X_2)^2 \rangle, \\ \Delta X_2 = \int_{t'}^{t'+\tau} \{u_{i2}(\mathbf{x}(\mathbf{x}_0, t''), t'') + v_{s2}(t'')\} dt''.$$

3.1.3 Asymptotic expressions of mean square displacement and Lagrangian integral time scale

From Eqs. (24) and (21•b), $\langle \overline{X_{s2}^2} \rangle$ can be easily obtained as

$$\langle \overline{X_{s2}^2} \rangle = 2\sigma_{vs}^2 (T_s^L)^2 \{ (t/T_s^L) - 1 + \exp(-t/T_s^L) \}. \quad (27)$$

The asymptotic expressions for $t \ll T_s^L$ and $t \gg T_s^L$ are given as

$$\langle \overline{X_{s2}^2} \rangle \approx \sigma_{vs}^2 t^2 \quad (t \ll T_s^L), \quad (28\cdot a)$$

$$\approx 2\sigma_{vs}^2 T_s^L t \quad (t \gg T_s^L). \quad (28\cdot b)$$

On the other hand, substituting Eq. (26) into Eq. (21•a) and considering Eq. (23), we obtain the following expression of $\langle \overline{X_{i2}^2} \rangle$.

$$\langle \overline{X_{i2}^2} \rangle \approx 2 \int_0^t d\tau (t-\tau) \cdot \left[2 \sum_{n=1}^{N_c} \langle |C_{2n}|^2 \rangle \cdot e^{-1/2 k_n^2 (\langle \overline{X_{i2}^2} \rangle + \langle \overline{X_{s2}^2} \rangle)} \right] \cdot \cos \omega_n \tau \quad (29)$$

In the above equation, in order to estimate $\langle \overline{X_{i2}^2} \rangle$, we need $\langle \overline{X_{i2}^2} \rangle$ itself, so it is very difficult to obtain the analytical expression of $\langle \overline{X_{i2}^2} \rangle$. However for the purpose of giving a guideline for the later numerical simulation, here only asymptotic behavior of $\langle \overline{X_{i2}^2} \rangle$ for $t \ll T_i^L$ and $t \gg T_i^L$ will be examined here.

Firstly, we will consider the case of $t \ll T_i^L$. If $\tau \rightarrow 0$ in Eq. (26), we obtain $B_{i2i2}^L(\tau)|_{\tau \rightarrow 0} \approx 2 \sum_{n=1}^{N_c} \langle |C_{2n}|^2 \rangle = \langle \overline{u_{i2}^2} \rangle = \text{const.}$ Therefore, from Eq. (21 a), $\langle \overline{X_{i2}^2} \rangle$ reduces to

$$\langle \overline{X_{i2}^2} \rangle|_{\tau \ll T_i^L} \approx 2 \int_0^t d\tau (t-\tau) \langle \overline{u_{i2}^2} \rangle \\ = \langle \overline{u_{i2}^2} \rangle t^2. \quad (30)$$

Substituting Eqs. (30) and (28•a) into Eq. (29), we obtain the initial asymptotic expression, which is described explicitly to $O(t^4)$,

$$\langle \overline{X_{i2}^2} \rangle \approx \langle \overline{u_{i2}^2} \rangle t^2 - \frac{1}{6} t^4 \sum_{n=1}^{N_c} \langle |C_{2n}|^2 \rangle \cdot \{ k_n^2 (\sigma_{vs}^2 + \langle \overline{u_{i2}^2} \rangle) + \omega_n^2 \} + O(t^6). \quad (31)$$

This means that in the initial stage of the diffusion, the effect of the small-scale motion on $\langle \overline{X_{i2}^2} \rangle$ appears of order $O(t^4)$.

Next, we will consider long-time diffusion. In this case, we find that $B_{i2i2}^L(\tau)$ tends to 0 from Eq. (20). Now, introducing the reference time t^* at which $B_{i2i2}^L(t^*) \approx 0$, and the integral time scale T_i^L defined by

$$T_i^L = \int_0^{t^*} d\tau B_{i2i2}^L(\tau) / \langle \overline{u_{i2}^2} \rangle, \quad (32)$$

we find that from Eq. (21•a), the asymptotic expression of $\langle \overline{X_{i2}^2} \rangle$ at $t \gg T_i^L$ is given by

$$\langle \overline{X_{i2}^2} \rangle \approx 2 \langle \overline{u_{i2}^2} \rangle T_i^L t, \quad (33)$$

i.e. like a classical diffusion process by G. I. Taylor⁽²⁰⁾.

From the above consideration, we can summarize the asymptotic expression of $\langle \overline{X_{i2}^2} \rangle$ as follows.

$$\langle \overline{X_{i2}^2} \rangle \approx \langle \overline{u_{i2}^2} \rangle + \sigma_{vs}^2 \} t^2 - \frac{1}{6} t^4 \sum_{n=1}^{N_c} \langle |C_{2n}|^2 \rangle \cdot \{ k_n^2 (\sigma_{vs}^2 + \langle \overline{u_{i2}^2} \rangle) + \omega_n^2 \} + O(t^6), \\ (t \ll T_s^L, T_i^L) \quad (34\cdot a)$$

$$\langle \overline{X_{i2}^2} \rangle \approx 2 \langle \overline{u_{i2}^2} \rangle T_i^L + \sigma_{vs}^2 T_s^L \} t = 2 \langle \overline{v_{i2}^2} \rangle T_i^L t, \\ (t \gg T_s^L, T_i^L) \quad (34\cdot b)$$

T_i^L in Eq. (34•b) indicates the integral time scale of the particle which moves at the resultant turbulent velocity $v_{i2} = u_{i2} + v_{s2}$. Here we can easily find the following relation between T_i^L , T_s^L and T^L :

$$x_e = \langle \overline{u_{i2}^2} \rangle T_i^L + \sigma_{vs}^2 T_s^L = \langle \overline{v_{i2}^2} \rangle T^L, \quad (35)$$

where x_e is the turbulent diffusivity.

3.1.4. Decorrelation of $B_{i2i2}^L(\tau)$ by the small-scale random motion From Eqs. (24) and (26), we find that $B_{s2s2}^L(\tau)$ is independent of u_i , but $B_{i2i2}^L(\tau)$ depends on v_s . In this section, the dependence of $B_{i2i2}^L(\tau)$ and $\langle \overline{X_{i2}^2} \rangle$ on v_s will be examined.

Now we define $B_{i2i2}^L(\tau)_{\text{pure}}$ and $\sigma_{X2, \text{pure}}^2$, respectively, as the Lagrangian velocity autocorrelation function of the large-scale velocity and the mean square displacement in the time interval τ due to the large-scale motion in the case where there is no small-scale motion. From Eq. (26), if $\sigma_{X2}^2 > \sigma_{X2, \text{pure}}^2$, which corresponds approximately to $\langle (\Delta X_{i2})^2 \rangle + \langle (\Delta X_{s2})^2 \rangle > \langle (\Delta X_{i2})^2 \rangle_{\text{pure}}$, in consideration of Eq. (23), we obtain the following inequality between $B_{i2i2}^L(\tau)$ and $B_{i2i2}^L(\tau)_{\text{pure}}$.

$$B_{i2i2}^L(\tau) < B_{i2i2}^L(\tau)_{\text{pure}} \quad (36)$$

The above inequality means that the value of the large-scale Lagrangian autocorrelation function is reduced by the occurrence of the displacement of particles due to the small-scale random motion ΔX_{s2} . This decorrelation effect suggests the following inequality in terms of the mean square displacement due to the large-scale motion,

$$\langle \overline{X_{i2}^2} \rangle < \langle \overline{X_{i2}^2} \rangle_{\text{pure}}, \quad (37)$$

where $\langle \overline{X_{i2}^2} \rangle_{\text{pure}}$ is the mean square displacement only due to the large-scale motion not accompanied by the small-scale motion. Now, we estimate the above decorrelation of $B_{i2i2}^L(\tau)$ and the reduction of $\langle \overline{X_{i2}^2} \rangle$. From Eqs. (23) and (26), the decorrelation $\Delta B_{i2i2}^L(\tau)$ can be calculated as

$$\Delta B_{i2i2}^L(\tau) = B_{i2i2}^L(\tau)_{\text{pure}} - B_{i2i2}^L(\tau) \\ \approx \sum_{n=1}^{N_c} \langle \overline{u_{i2n}^2} \rangle \{ e^{-(1/2) k_n^2 \langle (\Delta X_{i2})^2 \rangle_{\text{pure}}} - e^{-(1/2) k_n^2 (\langle (\Delta X_{i2})^2 \rangle + \langle (\Delta X_{s2})^2 \rangle)} \} \cos \omega_n \tau,$$

where $\langle (\Delta X_{i2})^2 \rangle \approx \langle \overline{u_{i2}^2} \rangle \tau^2$, $\langle (\Delta X_{i2})^2 \rangle_{\text{pure}} \approx \langle \overline{u_{i2}^2} \rangle_{\text{pure}} \tau^2 = \langle \overline{u_{i2}^2} \rangle \tau^2$. Considering $\langle (\Delta X_{s2})^2 \rangle \approx \sigma_{vs}^2 \tau^2$, we obtain the following expression of $\Delta B_{i2i2}^L(\tau)$,

$$\Delta B_{i2i2}^L(\tau) \approx \left(\frac{1}{6} \right) \sum_{n=1}^{N_c} \langle \overline{u_{i2n}^2} \rangle k_n^2 \sigma_{vs}^2 \tau^2, \quad (38)$$

where $\langle \overline{u_{in}^2} \rangle = \langle \overline{u_{1n}^2} \rangle + \langle \overline{u_{2n}^2} \rangle + \langle \overline{u_{3n}^2} \rangle = 3\langle \overline{u_{2n}^2} \rangle$. Therefore, the reduction of the large scale mean square displacement $\Delta \langle \overline{X_{l2}^2} \rangle$ which corresponds to $\Delta B_{l2l2}^l(\tau)$ can be expressed as

$$\begin{aligned} \Delta \langle \overline{X_{l2}^2} \rangle &= \langle \overline{X_{l2}^2} \rangle_{\text{pure}} - \langle \overline{X_{l2}^2} \rangle \\ &= 2 \int_0^t d\tau (t-\tau) \Delta B_{l2l2}^l(\tau) \\ &\simeq \left(\frac{1}{36} \right) \sum_{n=1}^{N_c} \langle \overline{u_{ln}^2} \rangle k_n^2 \sigma_{v_s}^2 t^4. \end{aligned} \quad (39)$$

From the above equations, we find that $\Delta B_{l2l2}^l(\tau)$ and $\Delta \langle \overline{X_{l2}^2} \rangle$ are proportional to τ^2 and t^4 , respectively, at the initial stage of the diffusion.

It is worthwhile investigating the relationship between the present model and Saffman's theory⁽¹⁹⁾. In the following, we will derive the asymptotic expressions of the decorrelation $\Delta B_{l2l2}^l(\tau)$ and the reduction of the large scale mean square displacement $\Delta \langle \overline{X_{l2}^2} \rangle$ at the initial diffusion time when applying the pure random walk model without the memory effect for the small-scale motion of the fluid particle in the present model. In this case, the mean square displacement by the small-scale motion can exactly expressed as $\langle (\Delta X_{s2})^2 \rangle = 2\chi_s \tau^{(14)}$, where χ_s is the diffusion coefficient due to the small-scale random walk. From the same procedure as deriving the previous expressions (38) and (39), we can readily obtain the following expressions of $\Delta B_{l2l2}(\tau)$ and $\Delta \langle \overline{X_{l2}^2} \rangle$,

$$\begin{aligned} \Delta B_{l2l2}(\tau) &\simeq (1/3) \sum_{n=1}^{N_c} \langle |u_{ln}|^2 \rangle k_n^2 \chi_s \tau, \\ \Delta \langle \overline{X_{l2}^2} \rangle &\simeq (1/9) \chi_s t^3 \sum_{n=1}^{N_c} \langle |u_{ln}|^2 \rangle k_n^2. \end{aligned}$$

Now, if the intensity of the reference vorticity ω_l by the large-scale motion is estimated by

$$\omega_l^2 = \sum_{n=1}^{N_c} \langle |u_{ln}|^2 \rangle k_n^2,$$

Fig. 1 Time changes of various mean square displacements $\langle \overline{X_{l2}^2} \rangle$, $\langle \overline{X_{s2}^2} \rangle$, $\langle \overline{X_{l2}^2} \rangle_{\text{pure}}$ and the cross-correlation of the large- and small-scale displacements

then the above expressions of $\Delta B_{l2l2}(\tau)$ and $\Delta \langle \overline{X_{l2}^2} \rangle$ are consistent with Eqs. (3.8) and (3.9) in Saffman's paper⁽¹⁸⁾. This means that the present combined model is not physically contradictory to Saffman's interaction theory⁽¹⁸⁾.

3.2 Numerical verification

Figure 1 shows the various mean square displacements, $\langle \overline{X_{l2}^2} \rangle$, $\langle \overline{X_{s2}^2} \rangle$, $\langle \overline{X_{l2}^2} \rangle_{\text{pure}}$ and the cross-correlation of the large- and small-scale displacements $\langle \overline{X_{l2} X_{s2}} \rangle$ as a function of time, where we tentatively chose $T_s^L = 0.5$ (all the results of simulation given in this section are those for the case of $T_s^L = 0.5$). From the figure, the value of $\langle \overline{X_{l2} X_{s2}} \rangle$ is shown to be almost zero over the wide range of diffusion time, $0 \leq t \leq 10.0$. This means the relationship given by Eq. (23), i.e., $\langle \overline{X_{l2}^2} \rangle \simeq \langle \overline{X_{l2}^2} \rangle + \langle \overline{X_{s2}^2} \rangle$, is realized quite accurately. The inequality relationship (37), i.e., $\langle \overline{X_{l2}^2} \rangle < \langle \overline{X_{l2}^2} \rangle_{\text{pure}}$ is also clearly seen from the figure.

Figure 2 shows the time change of $\Delta \langle \overline{X_{l2}^2} \rangle$ at the initial stage of the diffusion. It is found that at the range $t \leq 0.1$, relationship (39), i.e., $\Delta \langle \overline{X_{l2}^2} \rangle \propto t^4$, is realized.

Figure 3 gives the distributions of various Lagrangian auto-correlation coefficients of the particle velocity defined by

$$R_{22}^l(\tau) = B_{22}^l(\tau) / \{ \langle \overline{v_2^2}(\mathbf{x}_0, t') \rangle \langle \overline{v_2^2}(\mathbf{x}_0, t' + \tau) \rangle \}^{1/2},$$

$$R_{l2l2}^l(\tau) = \frac{B_{l2l2}^l(\tau)}{\{ \langle \overline{u_{l2}^2}(\mathbf{x}(\mathbf{x}_0, t'), t') \rangle \cdot \langle \overline{u_{l2}^2}(\mathbf{x}(\mathbf{x}_0, t' + \tau), t' + \tau) \rangle \}^{1/2}},$$

$$R_{s2s2}^l(\tau) = B_{s2s2}^l(\tau) / \{ \langle \overline{v_{s2}^2}(t') \rangle \langle \overline{v_{s2}^2}(t' + \tau) \rangle \}^{1/2},$$

$$R_{l2l2}^l(\tau)_{\text{pure}} = \frac{B_{l2l2}^l(\tau)_{\text{pure}}}{\{ \langle \overline{u_{l2}^2}(\mathbf{x}(\mathbf{x}_0, t'), t') \rangle_{\text{pure}} \cdot \langle \overline{u_{l2}^2}(\mathbf{x}(\mathbf{x}_0, t' + \tau), t' + \tau) \rangle_{\text{pure}} \}^{1/2}}.$$

From the figure, we can clearly see the following

Fig. 2 Time variation of $\Delta \langle \overline{X_{l2}^2} \rangle$

inequality, $R_{i2i2}^L(\tau) < R_{i2i2}^L(\tau)_{\text{pure}}$ at $\tau \leq 1.0$, which corresponds to the inequality (36). The Lagrangian time scales T^L , T_i^L and T_s^L , were calculated to be $T^L = 0.593$, $T_i^L = 0.632$ and $T_s^L = 0.571$, which were estimated by integrating $R_{22}^L(\tau)$, $R_{i2i2}^L(\tau)$ and $R_{s2s2}^L(\tau)$ in the figure from $\tau=0$ to $\tau=10.0$. Subsequently, from Eq. (35), we can estimate the eddy diffusivity χ_e as

$$\chi_e = \langle \overline{v_{i2}^2} \rangle T_i^L + \sigma_{v_s}^2 T_s^L = 0.599 \times 0.632 + 0.401 \times 0.571 = 0.608.$$

On the other hand, from the right-hand side of Eq. (35), χ_e is also estimated to be $\chi_e = \langle \overline{v^2} \rangle T^L = 1.00 \times 0.593 = 0.593$. By comparing these two values of χ_e , we find that Eq. (35) holds true within the error of 3% in this simulation.

In order to verify the relation (22), $R_{22}^L(\tau)$ is now divided into the following four terms, and their distributions are compared in Fig. 4.

$$R_{22}^L(\tau) = \frac{B_{22}^L(\tau)}{\sigma_{v_2}^2} = \frac{B_{i2i2}^L}{\sigma_{v_2}^2} + \frac{B_{s2s2}^L}{\sigma_{v_2}^2} + \frac{B_{i2s2}^L}{\sigma_{v_2}^2} + \frac{B_{s2i2}^L}{\sigma_{v_2}^2}, \quad (40)$$

(I) (II) (III) (IV)

where $\sigma_{v_2}^2 = \{ \langle \overline{v_2^2(x_0, t')} \rangle \langle \overline{v_2^2(x_0, t' + \tau)} \rangle \}^{1/2}$. From the figure, it is found that the (III) and (IV) terms are very small compared with the (I) and (II) terms at $\tau \leq T_s^L$, T_i^L , although they include the inevitable statistical error of the simulation. This means that the relation (22) is accurate.

Although it is not shown in the figure, we investigated the time change of the reduction of the Lagrangian autocorrelation coefficient for the large-scale motion $\Delta R_{i2i2}^L(\tau) = R_{i2i2}^L(\tau)_{\text{pure}} - R_{i2i2}^L(\tau)$. Consequently, it was confirmed that at the short-time diffusion range $\tau \leq 0.1$, the relationship of $\Delta R_{i2i2}^L(\tau) \propto \tau^2$ (which corresponds to Eq. (38)) is realized well.

Fig. 3 Comparison between various Lagrangian autocorrelation coefficients of the particle velocity, R_{22}^L , R_{i2i2}^L , R_{s2s2}^L and R_{i2i2}^L, pure

4. Selection of Lagrangian integral time scale for small-scale random motion T_s^L

The main problem when using the combined model of the random Fourier mode and RF is how to determine the value of T_s^L , which is not known, even for the homogeneous isotropic turbulence. Suggested here is a simple estimation on the basis of the dimensional argument which assumes that the main factor of determining T_s^L is only the energy spectrum $E(k)$. Next, we tried to verify the propriety of such an estimation of T_s^L by investigating the Lagrangian properties of the fluid particle in the velocity field generated only through the summation of random Fourier modes over a whole range of wavenumbers ($k = 0 \sim 50.0$).

Now, we define the Lagrangian length scale and velocity scale of the small-scale motion as L_s^L and $v_s(L_s^L)$ respectively. Then T_s^L can be expressed as $T_s^L = L_s^L / v_s(L_s^L)$ and this T_s^L can be described logically as the product of the following three terms:

$$T_s^L = \frac{L_s^L}{v_s(L_s^L)} = \frac{L_s^L}{L^L} \cdot \frac{L^L}{v_t} \cdot \frac{v_t}{v_s(L_s^L)},$$

where L^L and v_t are the Lagrangian length scale and velocity scale of the fluid particle moving at a resultant velocity v , respectively. Since $T^L = L^L / v_t$, we obtain

$$\frac{T_s^L}{T^L} = \frac{L_s^L}{L^L} \cdot \left(\frac{E^L}{E_s^L} \right)^{1/2}, \quad (41)$$

where E^L and E_s^L indicate the kinematic energy of the Lagrangian velocities v and v_s . In the homogeneous isotropic turbulence, E^L and E_s^L are determined as

$$E^L = \frac{1}{3} \int_0^\infty E(k) dk, \quad E_s^L = \frac{1}{3} \int_{k_c}^\infty E(k) dk. \quad (42 \cdot a, b)$$

Now, assuming that L_s^L and L^L are determined only by the distribution of $E(k)$ and the modulus of the wavenumbers k , for example, the following expressions of L_s^L and L^L can be derived from the dimen-

Fig. 4 Contributions of each component of $R_{22}^L(\tau)$

sional arguments⁽²¹⁾:

$$L_s^L = C \int_{k_c}^{\infty} \frac{1}{k} E(k) dk / \int_{k_c}^{\infty} E(k) dk, \quad (43 \cdot a)$$

$$L^L = C \int_0^{\infty} \frac{1}{k} E(k) dk / \int_0^{\infty} E(k) dk, \quad (43 \cdot b)$$

where C is a constant whose value can be considered to depend on the characteristics of the turbulence. Since $k_c=5.0$ and $E(k)$ is expressed as Eq. (4) in our problem, from Eqs. (41), (42·a, b) and (43·a, b), T_s^L/T^L can be estimated to be $T_s^L/T^L \approx 0.295$. Subsequently, we obtain $T^L/T^L \approx 1.47$ from Eq. (35).

The theoretical propriety of this estimation is not clear at present. However, this estimation seems to be of some value since it gives some criteria to determine T_s^L .

Next, in order to examine the propriety of the above estimation, by using the turbulent field generated by the summation of a finite number of Fourier modes over the entire range of wavenumbers considered here ($k=0 \sim 50.0$), T_s^L/T^L was calculated numerically and compared with the values estimated above. In the following are given the results of examination on the contribution of large- ($k < 5.0$) and small- ($k \geq 5.0$) scale motions to the dispersion of the particle, by means of this pure KS.

Figure 5 shows the Lagrangian velocity autocorrelation coefficients. The solid line is the autocorrelation coefficient for the total Lagrangian velocity $R_{22}^L(\tau)$, the broken line is that for the large-scale velocity, and the dotted line that for the small-scale velocity. It is found that $R_{22}^L(\tau)$, $R_{1212}^L(\tau)$ and $R_{s2s2}^L(\tau)$ clearly show separate curves. This means that the large- and small-scale motions have a different Lagrangian time scale from that due to the total velocity.

Fig. 5 Comparison between Lagrangian autocorrelation coefficients of the total velocity ($R_{22}^L(\tau)$), large-scale velocity ($R_{1212}^L(\tau)$) and small-scale velocity ($R_{s2s2}^L(\tau)$) by the pure random Fourier mode method.

ity.

Figure 6 shows the semitheoretical profiles of R_{22}^L , R_{1212}^L and R_{s2s2}^L , which were calculated from Eq. (26), where the simulated change of the total mean square displacement $\langle \bar{X}_2^2 \rangle$ was substituted into $\sigma_{dX_2}^2$. $R_{22}^L(\tau)$, $R_{1212}^L(\tau)$ and $R_{s2s2}^L(\tau)$ were calculated as the summation of modes from $n=1$ to 50, from 1 to N_c and from N_c to 50, respectively. Comparing Fig. 5 and Fig. 6, it is found that the corresponding profiles in the two figures are in good agreement. From the figures, we can estimate the values of T^L , T^L and T_s^L by integrating the profiles of $R_{22}^L(\tau)$, $R_{1212}^L(\tau)$ and $R_{s2s2}^L(\tau)$ in Fig. 6 over $\tau=0 \sim 10.0$. The calculated values are $T^L=0.494$, $T^L=0.750$ and $T_s^L=0.112$, respectively. Hence we can estimate T_s^L/T^L and T^L/T^L as 0.227 and 1.52, respectively. It is found that these values are similar to the values estimated previously, i.e., $T_s^L/T^L=0.295$ and $T^L/T^L=1.47$. Here, it should be noted that in our simulation, many important properties of turbulence, for example, the random sweeping of the large-scale eddies and the advection of small eddies by large-scale motion, which were modelled by Fung et al.⁽¹²⁾, are not taken into account. However, we would like to emphasize that our results on the fundamental properties of particle dispersion, for instance, the mean square displacement, the eddy diffusivity and the Lagrangian velocity autocorrelation, are not significantly different from the results of Fung et al.⁽¹²⁾. Therefore, we were able to estimate reasonable values of time scales by our method mentioned above.

Furthermore, in Fig. 5, the theoretical autocorrelation curve, i.e., $\exp(-\tau/T_s^L)$ of the RF model with the time scale $T_s^L=0.11$, is drawn as the chainline for comparison with the curve of R_{s2s2}^L by the pure random Fourier mode method. We find that two curves are in good agreement, which supports the idea of using the

Fig. 6 Semitheoretical profiles of R_{22}^L , R_{1212}^L and R_{s2s2}^L by the pure random Fourier mode method.

RF model as a simple substitution of the small-scale motions.

From the results thus far obtained, we can conclude that the appropriate value of T_s^L is 0.11 in our combined model.

5. Conclusions

In order to clarify the diffusion properties of the fluid particles in a turbulent flow approaching a circular cylinder with a uniform mean velocity, a new simulation model in which the random Fourier mode method and RF model are combined has been developed. As the first and preliminary stage of the study, it is important to understand that fundamental properties of the particle diffusion simulated by this combined model. In this paper, the Lagrangian statistics of fluid particles in the homogeneous isotropic turbulence convected by the uniform mean velocity in the case without a cylinder have been examined theoretically and numerically, in particular, from the viewpoint of interference between the large- and small-scale motions. Furthermore, the method for estimating the Lagrangian integral time scale of the small-scale motion T_s^L has also been considered. The main conclusions obtained in this paper are as follows.

(1) The cross-correlation of the displacement by the large-scale motion X_{12} and that by the small scale motion X_{s2} , i. e., $\langle \overline{X_{12}X_{s2}} \rangle$ is shown to be very small compared to each individual mean square displacement by the large- and small-scale motions, i.e., $\langle \overline{X_{12}^2} \rangle$ and $\langle \overline{X_{s2}^2} \rangle$. This means that the total mean square displacement $\langle \overline{X_2^2} \rangle$ can be described by the summation of $\langle \overline{X_{12}^2} \rangle$ and $\langle \overline{X_{s2}^2} \rangle$ fairly accurately. In correspondence with the above results, it is also found that in the short diffusion time ($\tau \lesssim T_s^L, T_s^l$), the Lagrangian cross correlation functions of the large- and small-scale velocities, i.e., B_{12s2}^L and B_{s212}^L , are almost negligible compared with each autocorrelation function of large- and small-scale velocities, i.e., B_{1212}^L and B_{s2s2}^L .

(2) The decorrelation of the large-scale Lagrangian velocity due to the effect of the small-scale random motion has been predicted by the theoretical analysis and confirmed by the numerical simulation.

(3) The value of T_s^L has been calculated by the pure random Fourier mode method, and compared with the value of T_s^L estimated from the simple dimensional arguments. Consequently, the appropriate value of T_s^L in this model is about 0.11.

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