Capsule Velocity in Pipelines*

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Capsule velocity in hydraulic and pneumatic pipelines is analyzed in its equilibrium state based on a coaxial annular flow model. The capsule is cylindrical and is assumed to move coaxially, and Coulomb friction is considered when the capsule has wheels. Properties of carrier fluid are included in a nondimensional friction parameter which consists of kinematic viscosity of fluid, density ratio, Coulomb frictional coefficient, pipe diameter and gravity acceleration. Effects of length/diameter ratio, diameter ratio and friction parameter on the capsule velocity are calculated. The maximum capsule velocity is limited by the diameter ratio. Pressure and shear forces on the capsule and a pressure drop across the capsule are also discussed, and the possibility of drag reduction due to the capsule is suggested.

Key Words: Pipe Flow, Pipeline, Axisymmetric Flow, Hydraulic Capsule, Pneumatic Capsule, Capsule Velocity, Pressure Drop

1. Introduction

It is well known that the velocity of a hydraulic capsule can be higher than that of water⁽¹⁾. The hydraulic capsule tends to follow the water flow in a coaxial position even if the capsule is heavier than water⁽²⁾. Charles⁽²⁾ explains this phenomenon on the basis of a model of an infinite coaxial capsule; the capsule occupies the central region of pipe and has greater velocity than the average water velocity. He⁽²⁾ also predicts, on the basis of his model, that the pressure drop in the hydraulic pipelines can be reduced below the pressure drop due to water flow alone. The capsule velocity can be higher than that of water in single capsule flow as well⁽¹⁾. We simulate this capsule behavior numerically⁽³⁾. On the other hand, the velocity of a pneumatic capsule is usually lower than that of gas. In this paper we examine the

equilibrium capsule velocity in a fluid pipeline using the same model as before⁽³⁾, and discuss the factors affecting the capsule velocity other than the pressure drop.

2. Theory

We assume that the positioning of the capsule is maintained to be coaxial with the pipe axis by some means such as wheels.

The equation governing the capsule motion is

$$M \frac{d\dot{Z}}{dt} = F - f|M - \rho v_c|g \frac{\dot{Z}}{|\dot{Z}|}, \quad (1)$$

where M is the capsule mass, Z the axial coordinate of the capsule, t the time, F the fluid force on the capsule, f the coefficient of Coulomb friction if the capsule has wheels, ρ the density of fluid, v_c the volume of the capsule and g the acceleration of gravity. If we apply the equation of continuity for the fluid near the front and rear of the capsule, we have

$$\rho(V_F - \dot{Z})A = \rho(U_F - \dot{Z})(A - A_c) \quad (2)$$

$$\rho(V_R - \dot{Z})A = \rho(U_R - \dot{Z})(A - A_c), \quad (3)$$

where V is the average velocity of fluid in the pipe, U the average velocity of fluid in the space between the capsule and the pipe wall, A and A_c the cross-sectional areas of the pipe and the capsule, respectively.

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tional areas of the pipe and the capsule, and subscripts F and R mean near the front and rear of the capsule, respectively. If we assume that

$$U_F = U_R = U, \quad (4)$$

we have

$$V_F = V_R = V. \quad (5)$$

Then, Eq. (2) becomes

$$U - \dot{Z} = (V - \dot{Z}) / (1 - k^2), \quad (6)$$

where $k^2 = A_c/A$

If τ_c is the shear stress on the cylindrical surface of the capsule and p_1 and p_2 are the pressure at the rear and front of the capsule, the fluid force is given as

$$F = (p_1 - p_2)A_c + \tau_c \pi d l, \quad (7)$$

where d is the capsule diameter and l the capsule length. In the previous paper⁽³⁾ we included dynamic pressure in the pressure term, but it is difficult to justify the term and in this paper we omit it. We confirmed that this correction has a negligibly small influence on the previous calculation.

For fully developed laminar flow, τ_c is given as

$$\tau_c = \frac{kD}{8} \left(2 + \frac{1-k^2}{k^2 \ln k} \right) \frac{dp}{dx} + \frac{2\mu \dot{Z}}{kD \ln k}, \quad (8)$$

where dp/dx is the pressure gradient in the clearance, D is the pipe inner diameter, μ is the viscosity of fluid, and $U - \dot{Z}$ is

$$U - \dot{Z} = -\frac{D^2}{32\mu} \left(1 + k^2 + \frac{1-k^2}{\ln k} \right) \frac{dp}{dx} - \left(\frac{1}{1-k^2} + \frac{1}{2 \ln k} \right) \dot{Z}. \quad (9)$$

From Eqs. (6) and (9) we have

$$\frac{dp}{dx} = -\frac{V - \delta^2 \dot{Z}}{\epsilon^2 l}, \quad (10)$$

where

$$\epsilon^2 = \frac{D^2(1-k^2)}{32\mu l} \left(1 + k^2 + \frac{1-k^2}{\ln k} \right) \quad (11)$$

and

$$\delta^2 = -\frac{1-k^2}{2 \ln k}. \quad (12)$$

If we assume that clearance flow is fully developed, dp/dx is given by

$$\frac{dp}{dx} = -\frac{\Delta p_f}{l}, \quad (13)$$

where Δp_f is the pressure drop in the clearance.

The pressure difference across the capsule can be expressed as a function of the velocities of fluid and the capsule. The pressure changes due to the change of flow area from A to $A - A_c$ or vice versa, and drops due to the accompanying area changes. We estimate the pressure change and drop from those given by flow around the coaxial capsule at rest in a pipe flow of a relative velocity of $V - \dot{Z}$ and approximate them from the pipe flow data. Thus, the pressure change due to the area change is given by

$$\Delta p_B = \left\{ \frac{1}{(1-k^2)^2} - 1 \right\} \frac{1}{2} \rho (V - \dot{Z})^2. \quad (14)$$

The pressure drop due to sudden contraction is

$$\Delta p_d = \left\{ \frac{k^2}{2(1-k^2)^2} \right\} \frac{1}{2} \rho (V - \dot{Z})^2, \quad (15)$$

where the loss coefficient is approximated by $k^2/2$, and the pressure drop due to sudden expansion is given as

$$\Delta p_e = \left(\frac{k^2}{1-k^2} \right)^2 \frac{1}{2} \rho (V - \dot{Z})^2. \quad (16)$$

2.1 Capsule velocity

The fluid flow through the clearance is classified by the sign of $V - \dot{Z}$ and of dp/dx . The sign of dp/dx is the same as that of $\delta^2 \dot{Z} - V$. Thus, the possible cases are easily established as (1) $\dot{Z} \leq V$, (2) $V < \dot{Z} \leq V/\delta^2$ and (3) $\dot{Z} > V/\delta^2$. Figure 1 schematically shows the fluid velocity profiles in the clearance for $\dot{Z} > V$ in a moving coordinate system of $\dot{Z}$, and it is seen that in case (3) both the shear and pressure forces become negative, which is only possible when the capsule is decelerated. Thus, in the equilibrium state of capsule motion, the capsule velocity is always lower than V/δ^2 . In the following we examine the capsule velocity in the equilibrium state.

2.1.1 The case of $\dot{Z} \leq V$ In this case $p_1 - p_2$ and pressure drop across the capsule Δp are given by

$$p_1 - p_2 = \Delta p_f + \Delta p_d + \Delta p_B \quad (17)$$

and

$$\Delta p = \Delta p_f + \Delta p_e + \Delta p_d. \quad (18)$$

If we substitute Eqs. (8), (10), (13), (14), (15) and (17) for Eq. (1), we get

$$\begin{aligned} s \frac{d\dot{Z}^*}{dt^*} = & K_1 \dot{Z}^{*2} - (2K_1 + K_2 - K_3) \dot{Z}^* \\ & + (K_1 + K_2 - K_4), \end{aligned} \quad (19)$$

where $\dot{Z}^*$ and t^* are nondimensionalized on V and l , and the coefficients K_1 to K_4 are all positive and are

$$K_1 = \frac{k^2(5-2k^2)}{2(1-k^2)^2} \quad (20)$$

$$K_2 = \frac{-1}{(1+k^2) \ln k + 1 - k^2} \frac{16b}{kRe} \quad (21)$$

$$K_3 = \frac{1-k^2}{2} K_2 \quad (22)$$

and

$$K_4 = \frac{abk}{Re^2}, \quad (23)$$

where Re is the pipe Reynolds number VD/ν , b is the length/diameter ratio l/d and a is friction parameter defined by

Fig. 1 Velocity profiles in the clearance.

$$a = f|s-1|\frac{gD^3}{\nu^2} \quad (24)$$

where s is the density ratio of the capsule to the fluid, that is, $s = \sigma/\rho$, σ being the density of the capsule $M/\nu c$, and ν is the kinematic viscosity of the fluid. Since we consider the equilibrium state, $d\dot{Z}^*/dt^*$ is zero. A possible solution for this case is given by

$$\dot{Z}^* = 1 + \frac{K_2 - K_3 - \sqrt{(K_2 - K_3)^2 - 4K_1(K_3 - K_4)}}{2K_1} \quad (25)$$

Since $K_2 > K_3$, $\dot{Z}^*$ is less than unity when K_3 is less than K_4 . If K_4 is equal to zero, there is no solution which satisfies $\dot{Z}^* < 1$.

2.1.2 The case of $V < \dot{Z} \leq V/\delta^2$ Figure 2 schematically shows a pressure distribution across the capsule for case (2). In this case,

$$p_1 - p_2 = \Delta p_f - (\Delta p_d + \Delta p_B) \quad (26)$$

and

$$\Delta p = \Delta p_f - (\Delta p_e + \Delta p_d). \quad (27)$$

The equation of capsule motion is given as

$$s \frac{d\dot{Z}^*}{dt^*} = -K_1 \dot{Z}^{*2} + (2K_1 - K_2 + K_3) \dot{Z}^* - (K_1 - K_2 + K_4). \quad (28)$$

A possible solution to the equilibrium state is given as

$$\dot{Z}^* = 1 - \frac{K_2 - K_3 - \sqrt{(K_2 - K_3)^2 + 4K_1(K_3 - K_4)}}{2K_1}. \quad (29)$$

Thus, if K_3 is larger than K_4 , $\dot{Z}^*$ becomes larger than unity.

Therefore, we can summarize as

If $K_3 \leq K_4$, then $\dot{Z} \leq V$.

If $K_3 > K_4$, then $V < \dot{Z} \leq V/\delta^2$.

When $K_3 = K_4$, $\dot{Z}$ is equal to V and the Reynolds number is

$$Re_0 = -ak^2 \left(\frac{1+k^2}{1-k^2} \ln k + 1 \right). \quad (30)$$

2.2 Fluid force and pressure drop

If we define a nondimensional pressure force on the capsule p^* as $(p_1 - p_2)A_c/\rho V^2 l^2$, p^* is given as

$$p^* = \frac{\pi}{4b_2} \left\{ \frac{1 - \delta^2 \dot{Z}^*}{\rho V \varepsilon^2} \mp K_1 (1 - \dot{Z}^*)^2 \right\}. \quad (31)$$

The nondimensional shear force is similarly given as

Fig. 2 Pressure distribution across the capsule

$$S^* = \frac{\pi}{b} \left\{ -\frac{1}{8b} \left(2 + \frac{1-k^2}{k^2 \ln k} \right) \frac{1}{\rho V \varepsilon^2} (1 - \delta^2 \dot{Z}^*) + \frac{2\dot{Z}^*}{kRe \ln k} \right\}, \quad (32)$$

where S^* is $\tau_c \pi d l / \rho \nu^2 l^2$. In Eqs. (31) and (32) the plus sign indicates the case of $V > \dot{Z}$. The nondimensional Coulomb frictional force is

$$C^* = \frac{\pi a k}{4bRe^2}. \quad (33)$$

If we make dimensionless the pressure drop across the capsule by the frictional pressure drop for the capsule length $\lambda(l/D)(\rho V^2/2)$, the nondimensional pressure drop Δp^* is given by

$$\Delta p^* = \frac{1}{\lambda b k} \left\{ \frac{64 b k (1 - \delta^2 \dot{Z}^*)}{Re(1-k^2) \left(1 + k^2 + \frac{1-k^2}{\ln k} \right)} \mp \frac{k^2(1+2k^2)}{2(1-k^2)^2} (1 - \dot{Z}^*)^2 \right\}, \quad (34)$$

where λ is the pipe friction coefficient for the smooth wall.

3. Results and Discussion

Equations (25) and (29) show that the capsule velocity depends on the diameter ratio k , Reynolds number Re , length/diameter ratio b and friction parameter a . Figure 3 shows the diagram used to derive a from a given pipe size, carrier fluid and frictional coefficient when σ is 800 kg/m³. It is evident that the parameter a of a pneumatic capsule is larger than that of a hydraulic capsule for given D , f and σ , since s and ν are generally smaller for the hydraulic capsule.

Figure 4 shows the nondimensional capsule velocity versus Reynolds number with a as a parameter for $k=0.9$ and $b=5$. The capsule velocity asymptotically approaches the fluid velocity in the limit of large Reynolds number. If the capsule is always coaxial without wheels or the capsule has the same density as that of fluid, a , as well as K_4 , is zero. In this case, the capsule velocity is always higher than that of fluid.

Fig. 3 Friction parameter

Fig. 4 Capsule velocity and friction parameter

Fig. 5 Pressure and shear forces versus Reynolds number

Fig. 6 Capsule velocity and length/diameter ratio

Fig. 7 Capsule velocity and diameter ratio

Figure 5 shows the nondimensional pressure and shear forces for $k=0.9$ and $b=5$. When a is zero, the pressure force always balances the shear force. When Re decreases, both forces become positive for a non-zero value of a , which means that both forces drive the capsule against the frictional force. However, it was found that when Re becomes very large, the shear force alone drives the capsule, while the pressure force acts as resistance for small values of a . It is also evident that for large values of a , the pressure force governs the capsule motion.

3.1 Capsule of small a

As mentioned above, for given D , f and σ , the hydraulic capsule is characterized by relatively small values of a as compared to the pneumatic capsule. In the following, we examine the characteristics of a capsule with $a=2 \times 10^5$, which can be regarded as representative of the hydraulic capsule.

Fig. 8 Shear force and length/diameter ratio

Fig. 9 Shear force and diameter ratio

Fig. 10 Capsule velocity and length/diameter ratio

Fig. 12 Pressure force and length/diameter ratio

Fig. 13 Pressure force and diameter ratio

Fig. 11 Capsule velocity and diameter ratio

Fig. 14 Pressure drop across the capsule

Figures 6 and 7 show the capsule velocity versus Reynolds number, the diameter ratio being 0.9 with b as a parameter in the former and the length/diameter ratio being 5 with k as a parameter in the latter. It is easily shown using Eq.(29) that Z^* increases with increasing b and approaches unity when b becomes small. In fact, Ellis⁽¹⁾ experimentally shows that the capsule velocity increases with increasing b for an equal-density capsule, that is, $s=1$, without wheels.

The capsule velocity also increases with decreasing k for large Re .

Figures 8 and 9 show the shear force versus Reynolds number. In the former, b is varied for $k=0.9$ and in the latter k is varied for $b=5$. When k becomes small, the shear force increases. This means that the shear force becomes a major driving force of the capsule when obstruction of the capsule to the fluid is decreased.

3.2 Capsule of large a

If the above results for $a=2\times 10^5$ are for the hydraulic capsule, the results for $a=2\times 10^8$ can be regarded as characteristic of the pneumatic capsule for the same combination of D , f and σ . The following are the results for such a case. Figures 10 and 11 show $\dot{Z}^*$ versus Re . In the former the parameter is b when k is 0.9 and in the latter the parameter is k when b is 5. It is shown that the capsule velocity is generally smaller than that of fluid for normal operating Reynolds number and that the capsule velocity increases with increasing k , the influence of b being relatively weak. Figures 12 and 13 show the pressure force versus Reynolds number. It was found that the pressure force is a primary driving force for the capsule to overcome the larger value of C^*

3.3 Pressure drop across the capsule

Figure 14 shows the pressure drop across the capsule. The nondimensional pressure drop decreases with increasing Re , and it is shown to be positive always. When Δp^* is unity, the pressure drop across the capsule is the same as the pressure drop due to the pipe friction for the capsule length. Therefore, if Δp^* is less than unity, the pressure drop is reduced due to the capsule. Thus, for the hydraulic capsule it is probable that the overall pressure drop becomes less than that of the water flow alone for the same flow

rate of water.

4. Conclusions

Capsule motion in the equilibrium state is examined on the basis of a coaxial annular flow model. It is shown that when there is no solid friction the capsule velocity is always higher than that of fluid velocity and that the pressure drop in the capsule pipeline can become smaller than that of the fluid flow alone.

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