Study on Heat Transfer of Heat Exchangers in the Stirling Engine*

(Heat Transfer in a Heated Tube under the Periodically Reversing Flow Condition)

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Heat transfer characteristics in heated tubes under periodically reversing flow conditions have been experimentally investigated, using a test apparatus that simulates heat exchangers for an actual Stirling engine. It is shown that the heat transfer characteristics under these conditions are greatly affected by the piston phase difference that generates the reversing flow of working fluid, and this phenomenon is peculiar to heat transfer under periodically reversing flow, which is different from the conventional convective heat transfer under steady flow. The experimental correlation for the heat transfer coefficient under these conditions is induced through the use of the working gas velocity evaluated from the Schmidt cycle model, which is one of the ideal Stirling cycles concerning the influence of the piston phase difference.

Key Words: Stirling Engine, Heat Exchanger, Heat Transfer, Forced Convection, Unsteady Flow

1. Introduction

The Stirling engine is a kind of external combustion engine in which the heat transfer from heat sources to working gas is carried out through heat exchangers. The heater, the cooler, which exchanges heat between the working gas and heating medium or coolant, and the regenerator located between the two heat exchangers, are necessary component of the engine. One of the remarkable characteristics of these heat exchangers in the Stirling engine is the working gas flow condition, under which flow direction changes periodically (periodically reversing flow, hereinafter). Especially in the heater and the cooler, heat exchange between the periodically reversing flow and the steady flow is actualized, which is regarded as an exceptional characteristic of the heat exchanger in the Stirling

engine.

Heat exchange under high-temperature and high-pressure conditions above 1000 K and 10 MPa is required in the heater, and hence the working-gas-side heat transfer is an important factor that not only influences the performance of the heat exchanger but also directly affects the heater tube temperature determined by the heat exchanging condition, and so must govern the endurance of the heater working under the above conditions. Also in the cooler chilled by water, because the heat transfer characteristics at both sides of a tube are nearly comparable, the working-gas-side heat transfer affects the cooler performance considerably, and is an important parameter influencing the thermal efficiency of the engine as well as that of the heater.

As for the thermal design of the Stirling engine, it is important to accurately predict time-averaged heat transfer rate, which means the time-averaged instantaneous heat transfer rate corresponding to the periodical flow change of working gas. Therefore, the time-averaged heat transfer characteristics under the periodically reversing flow condition of working gas will be dealt with in this paper.

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Most of the previous studies on the heat transfer of the heater and the cooler in the Stirling engine have been concerned with heat transfer in the heating medium of the heater or the analysis of heat transfer characteristics in an experimental engine⁽¹⁾⁻⁽⁵⁾. Though some research taking account of the effect of periodically reversing flow, which is peculiar to the heat exchanger of the Stirling engine, has been conducted⁽⁶⁾⁻⁽⁸⁾, there remain many unsolved problems, including the establishment of the empirical correlations for the heat exchanger design.

The authors have already clarified heat transfer characteristics in the periodically reversing flow on the basis of the experimental investigation⁽⁹⁾, and showed that the heat transfer coefficient for the gas flow in tubes of the heat exchangers in the Stirling engine varied with the phase difference in reciprocating motions of pistons generating the periodically reversing flow. In this paper, we describe the results of our experiment conducted to correlate the piston phase difference generating the periodically reversing flow and the heat transfer within tubes, and also the method predicting the heat transfer characteristics considering the above effect, because the periodically reversing flow is an important factor in heat exchanger design of the Stirling engine.

Nomenclature

- α_i : thermal coefficient of thermal conductivity of tube material, K
- C' : correction factor for heat transfer coefficient
- d_o : outer tube diameter, m
- d_i : inner tube diameter, m
- d_p : piston diameter, m
- G_i, G_t : mass of working gas, kg
- g_i : dimensionless mass of working gas
- L_h : tube length, m
- N_r : revolutions per minute of the engine, rpm
- Nu : Nusselt number
- P : working gas pressure, MPa
- p : dimensionless working gas pressure
- Q_e : heating rate, W
- q_{v_i} : heat generation, W/m³
- R : gas constant, m/K
- Re : Reynolds number (based on mean piston velocity)
- Re_{sc} : Reynolds number (based on Schmidt cycle model)
- S_i : piston stroke, m
- T_b : working gas temperature (experimental value), K
- T_i : working gas temperature (i -th working gas space), K
- T_o : outlet tube wall temperature, K

- T_w : inner tube wall temperature, K
- U : working gas velocity (based on mean piston velocity), m/s
- V_i : volume of i -th working gas space, m³
- V_1^0 : stroke volume of No. 1 cylinder, m³
- V_2^0 : stroke volume of No. 2 cylinder, m³
- v_i : dimensionless volume of i -th working gas space
- α : heat transfer coefficient, W/m²K
- λ : thermal conductivity, W/mK
- μ : viscosity of working gas, Pa·s
- ν : kinematic viscosity of working gas, m²/s
- τ_i : dimensionless temperature of working gas
- ϕ : phase difference of pistons, rad
- θ : rotating angle, rad

2. Experimental Apparatus

The experimental apparatus consisted of a test section transferring heat to working gas, a reciprocating device generating the periodical flow in a tube, and auxiliary units. Figures 1 and 2 show the flow sheet of the experimental apparatus and the details of the test section, respectively. The ends of the test section were connected by two cylinders built into the reciprocating device, which was adapted from a diesel engine and generated the periodically reversing flow in the tube by the reciprocating motion of pistons. The heat transferring part was electrically heated.

The fluid temperatures within the tube were

Fig. 1 Flow chart of test apparatus for heat transfer of periodically reversing flow

Fig. 2 Configuration of test section

measured at both ends and their midpoint. The tube wall temperature was measured by thermocouples attached to the outer tube wall.

The outer diameter of the test section tube was 10.5 mm, and the tube wall thickness was 1.7 mm. The pistons generating the periodical flow within the tube were 150 mm long in stroke, and their phase difference angle and diameter were variable. Phase difference values of 90, 120 and 180 deg and piston diameters of 20 and 30 mm were selected in the experiment. The kinds of working gas used were nitrogen and helium. The main objects and instruments of measurement are shown in Table 1 and Fig. 3.

3. Experimental Results

The time-averaged heat transfer coefficient under the periodically reversing flow condition was defined by using the temperature difference between the time-averaged values of the measured working gas temperature and those of the measured inner tube wall temperature, and can be represented by the following equation.

$$\alpha = \frac{Q_e}{\pi d_i L_h} \cdot \frac{1}{T_w - T_b} \quad (1)$$

The time-averaged value of tube inner-wall temperature was obtained from the measured tube outer-wall temperature through the use of the following equations. As for working gas temperature and tube outer-wall temperature, the time-averaged value

without fluctuation was measured by the measuring method shown in Table 1.

$$T_w = T_o - \Delta T_o \left\{ 1 + \frac{\alpha_h \cdot \Delta T_o}{2(1 + \alpha_h \cdot T_o)} \right\} \quad (2)$$

$$\Delta T_o = \frac{q_v}{8\lambda_o} \left\{ d_o^2 \ln \left(\frac{d_o}{d_i} \right) - \frac{d_o^2 - d_i^2}{2} \right\} \quad (3)$$

Figure 4 presents the experimental data on the heat transfer coefficient of working gas flowing periodically within the tube together with the data presented in the previous report. In this figure, the abscissa shows measured tube wall temperatures and the ordinate indicates the ratio of the measured value of heat transfer coefficient and the value predicted by the heat transfer correlation⁽¹⁰⁾ for turbulent steady flow in tubes given by the following equation.

$$Nu = 0.021 Re^{0.8} Pr^{0.4} \left(\frac{T_w}{T_b} \right)^{-0.5} \quad (4)$$

In the calculation of heat transfer coefficient by Eq. (4), the Reynolds number in the equation was evaluated by using the working gas velocity that corresponds to the mean piston speed in the tube at the experimental apparatus, as shown in the following equations, and it does not vary with the phase difference.

$$Re = \frac{U d_i}{\nu} \quad (5)$$

$$U = \left(\frac{d_p}{d_i} \right)^2 \frac{2 S_t N_r}{60} \quad (6)$$

As clearly shown in Fig. 4, if the heat transfer coefficient in the periodically reversing flow condition is compared with that in the turbulent steady flow given by Eq. (4), it greatly differs with the phase difference of pistons generating the periodical flow and reaches a maximum at the phase difference of 180 degrees where the gas velocity assumes the same value as is given by Eq. (6). As described in the previous report, the heat transfer correlation for the periodical flow within the tube requires a correction

Table 1 Measuring items and points

Measured Item	Symbol	Measuring Instruments
Pressure of Working Gas	P00	Bourdon Tube
	P01 ~ P07	Strain Gauge
Temperature of Working Gas	T11 ~ T17	1.0 mm Thermocouple
Heater Tube Wall Temperature	T21 ~ T28	0.3 mm Thermocouple
Revolution	RPM	Tachometer
Heat Input	W10	Wattmeter
Flow Rate of Cooling Water	F40, F45	Flowmeter
Temperature of Cooling Water	T41 ~ T47	1.6 mm Thermocouple

Fig. 3 Schematics and symbols of measured points in test section

Fig. 4 Influence of tube wall temperature on heat transfer under the periodically reversing flow

factor varying with the phase difference for Eq. (4).

4. Analysis according to Schmidt Cycle Model

We analyze here the effect of the phase difference on the heat transfer characteristics in the periodical flow in conjunction with the Schmidt cycle model, which is a simplified model of the Stirling cycle. In the Schmidt cycle model, it is assumed that the pressure varies in a consistent manner throughout the entire working gas space and the working gas temperatures in the each space comprising the cycle do not change with time. In this analysis, the working gas space was divided into seven spaces corresponding to the experimental apparatus, and working gas temperatures for each divided space were assumed, as shown in Fig. 5 on the basis of the experimental result. The working gas temperature, except for that of the heat transfer section, was kept nearly constant, independent of experimental conditions, as described in the previous report⁽⁹⁾.

Assuming the working gas to be an ideal gas, the following equations can be derived from the equation of state.

$$P = \frac{G_i R T_i}{V_i} = \frac{G_T \cdot R}{\sum_{i=1}^7 (V_i / T_i)} \quad (i=1 \sim 7) \quad (7)$$

$$G_T = \sum_{i=1}^7 G_i \quad (8)$$

$$V_i = \frac{V_1^0}{2} (1 - \cos \theta) \quad (9)$$

$$V_7 = \frac{V_7^0}{2} \{1 - \cos(\theta - \phi)\} \quad (10)$$

The volume (V_i), temperature (T_i) and mass (G_i)

Fig. 5 Schmidt cycle model

in each working space and pressure (P) are nondimensionalized by using the following equations.

$$v_i = V_i / V_1^0 \quad (i=1 \sim 7) \quad (11)$$

$$\tau_i = T_i / T_1 \quad (i=1 \sim 7) \quad (12)$$

$$p = \frac{P V_1^0}{G_T \cdot R T_1} \quad (13)$$

$$g_i = G_i / G_T \quad (i=1 \sim 7) \quad (14)$$

In accordance with the above nondimensionalization, Eq. (7) is transformed to the following equations.

$$p = \frac{1}{\sum_{i=1}^7 (v_i / \tau_i)} \quad (15)$$

$$g_i = p v_i / \tau_i \quad (16)$$

From the change of mass existing in each working space, gas flow rates between working spaces are presented as follows.

$$\left(\frac{dg}{d\theta} \right)_{1-2} = - \frac{dg_1}{d\theta} \quad (17)$$

$$\left(\frac{dg}{d\theta} \right)_{2-3} = \left(\frac{dg}{d\theta} \right)_{1-2} - \frac{dg_2}{d\theta} = - \frac{dg_1}{d\theta} - \frac{dg_2}{d\theta} \quad (18)$$

$$\left(\frac{dg}{d\theta} \right)_{3-4} = \left(\frac{dg}{d\theta} \right)_{2-3} - \frac{dg_3}{d\theta} = - \frac{dg_1}{d\theta} - \frac{dg_2}{d\theta} - \frac{dg_3}{d\theta} \quad (19)$$

$$\left(\frac{dg}{d\theta} \right)_{4-5} = \left(\frac{dg}{d\theta} \right)_{3-4} - \frac{dg_4}{d\theta} = - \frac{dg_1}{d\theta} - \frac{dg_2}{d\theta} - \frac{dg_3}{d\theta} - \frac{dg_4}{d\theta} \quad (20)$$

$$\begin{aligned} \left(\frac{dg}{d\theta} \right)_{5-6} &= \left(\frac{dg}{d\theta} \right)_{4-5} - \frac{dg_5}{d\theta} \\ &= - \frac{dg_1}{d\theta} - \frac{dg_2}{d\theta} - \frac{dg_3}{d\theta} - \frac{dg_4}{d\theta} - \frac{dg_5}{d\theta} \end{aligned} \quad (21)$$

$$\left(\frac{dg}{d\theta} \right)_{6-7} = \frac{dg_7}{d\theta} \quad (22)$$

$$\left(\frac{dg}{d\theta} \right)_{i \rightarrow i+1} = \sum_{n=1}^i \left(- \frac{dg_n}{d\theta} \right) \quad (23)$$

From Eqs. (15) and (16), changes of mass and pressure in each working gas space are given by the following equations.

$$\frac{dg_i}{d\theta} = \left(v_i \frac{dp}{d\theta} + p \frac{dv_i}{d\theta} \right) \cdot \frac{i}{\tau_i} \quad (24)$$

$$\frac{dp}{d\theta} = \frac{- \sum_{i=1}^7 \frac{dv_i}{d\theta} \cdot \frac{1}{\tau_i}}{\left[\sum_{i=1}^7 (v_i / \tau_i) \right]^2} \quad (25)$$

on the basis of the above equations, the pressure change and working gas flow rates between each working gas space at any piston position during one cycle can be obtained.

The calculated pressure and flow change of working gas based on the analytical model as shown above are presented in Fig. 6. The lower figures in Fig. 6 indicate the flow changes between test section ④ and the No. 1 cooler ③ and that between ④ and the No. 2 cooler ⑤. In the case of the phase difference of 180 degrees, the two flows coincide with each other, but not in the other cases. Also in the latter case the difference between the flow rates into and from each working gas space corresponds to the pressure fluctua-

tion. This fluctuation does not occur in the case of the phase difference of 180 degrees.

If the gas flow rate into test section ④ from its neighboring coolers is assumed to be represented by that from cooler ③, the Reynolds number in the tube can be expressed by the following equation.

$$Re_{sc} = \left(\frac{dg}{d\theta} \right)_{3 \rightarrow 4} \left(\frac{PV_1^0}{pRT_1} \cdot 6N_r \right) \left(\frac{4}{\pi \mu d_i} \right) \\ = \left(\frac{dg}{d\theta} \right)_{3 \rightarrow 4} \left(\frac{\bar{P}V_1^0}{pRT_1} \cdot 6N_r \right) \left(\frac{4}{\pi \mu d_i} \right) \quad (26)$$

Also the Reynolds number based on the time-averaged gas velocity (average of the absolute value) is given by the following equation.

$$\overline{Re}_{sc} = \left| \frac{dg}{d\theta} \right|_{3 \rightarrow 4} \cdot \left(\frac{\bar{P}V_1^0}{pRT_1} \cdot 6N_r \right) \left(\frac{4}{\pi \mu d_i} \right) \quad (27)$$

The Reynolds number based on the piston mean speed is given by the following equation.

$$Re = \left(\frac{\bar{P}V_1^0}{RT_1} \cdot \frac{N_r}{30} \right) \left(\frac{4}{\pi \mu d_i} \right) \quad (28)$$

Therefore, the ratio $(\overline{Re}_{sc}/Re)$ of Reynolds number by Schmidt cycle model to that by piston mean speed becomes as follows.

$$\frac{\overline{Re}_{sc}}{Re} = \left| \frac{dg}{d\theta} \right|_{3 \rightarrow 4} \cdot \frac{180}{p} \quad (29)$$

$$\frac{\overline{Re}_{sc}^{0.8}}{Re^{0.8}} = \left| \frac{dg}{d\theta} \right|_{3 \rightarrow 4}^{0.8} \cdot \left(\frac{180}{p} \right)^{0.8} \quad (30)$$

The calculated time-averaged values of working gas pressure, flow rates and Reynolds number for the conditions given in Fig. 4 are shown in Table 2. From this table it can be seen that the phase difference of 90 degrees or 120 degrees gives lower flow rates than the angle of 180 degrees. Regarding the heat transfer coefficient shown in Fig. 4, the value for the phase difference of 90 degrees or 120 degrees was evaluated lower than that for the angle of 180 degrees. This can be regarded as reflecting the difference in the actual

working gas velocities flowing within the tube, as shown in Table 2. The ratios $(\overline{Re}_{sc}/Re)$ and $(\overline{Re}_{sc}/Re)^{0.8}$ given by Eqs. (29) and (30) are presented against the phase difference in Fig. 7. Though $\overline{Re}_{sc}$

Table 2 Calculated results of time-averaged values using Schmidt cycle model

CASE	1	2	3	4	5
d_p (mm)	20	20	20	20	20
ϕ (deg)	90	90	90	120	180
τ_4 —	1.50	1.75	2.00	1.50	1.50
(T_4) (K)	523	613	698	523	523
$\bar{p}$	0.3439	0.3510	0.3564	0.3390	0.3342
$\left \frac{dg}{d\theta} \right _{3 \rightarrow 4}$	1.36×10^{-3}	1.39×10^{-3}	1.41×10^{-3}	1.64×10^{-3}	1.86×10^{-3}
$\left \frac{dg}{d\theta} \right _{3 \rightarrow 4}^{0.8}$	4.97×10^{-3}	5.05×10^{-3}	5.10×10^{-3}	5.81×10^{-3}	6.37×10^{-3}
$\overline{Re}_{sc}/Re$	0.7137	0.7122	0.7111	0.8686	1.00
$\overline{Re}_{sc}^{0.8}/Re^{0.8}$	0.7435	0.7423	0.7413	0.8707	0.9751

Fig. 7 Influence of phase difference on Reynolds number

Fig. 6 Calculated results of pressure and flow rate of working gas using Schmidt cycle model ($d_p=20$ mm, $\tau_4=1.50$)

Fig. 8 Nondimensional heat transfer correlation using Reynolds number based on Schmidt cycle model

coincides with Re in the case of the angle of 180 degrees, $\overline{Re}_{sc}$ becomes smaller than Re at other angles.

Figure 8 shows the result of re-evaluation of the same data shown in Fig. 4 by evaluating the Reynolds number with Eq. (27). As shown in this figure, the heat transfer coefficient within tubes under the periodically reversing flow condition can be presented independent of the phase difference by evaluating the gas velocity with the Schmidt cycle model as follows.

$$Nu = 0.021 \overline{Re}_{sc}^{0.8} Pr^{0.4} \left(\frac{T_w}{T_b} \right)^{-0.5} \cdot C' \quad (31)$$

$$C' = 0.923 + 0.750 \left(\frac{T_w}{1000} \right) \quad (32)$$

where C' is a correction term. In the calculation of the heat transfer coefficient, physical properties in each equation should be evaluated at the time-averaged value of working gas.

5. Conclusions

The heat transfer in heat exchangers for the Stirling engine was dealt with in this study. The effect of the phase difference of pistons on heat transfer was analyzed on the basis of the Schmidt cycle model, which is one of the simplified models for the Stirling cycle. The result shows that the phase difference affects the gas velocity within tubes. A prediction method for heat transfer coefficient in which the gas velocity is evaluated by the Schmidt cycle model was presented.

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