Definition of Turbulence in In-Cylinder Flow Fields*

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The definition of turbulence of in-cylinder flow fields has been an important issue for researchers working on in-cylinder gas motion. This study proposes a definition of the "turbulence component" in both the calculation and measurement. Also, a new method to make a precise comparison between the calculation and measurement is presented theoretically. This method implies changing the coefficient, C_D , which is included in expressions for turbulence energy, its dissipation rate and turbulence length scale. Employing this method demonstrates the possibility of reducing the ambiguity of the cut-off frequency method to determine turbulence experimentally and also of improving the reliability of the calculation code.

Key Words: Internal Combustion Engine, Numerical Analysis, Turbulent Flow, Turbulence Model

1. Introduction

In the analysis of the in-cylinder gas motion of internal combustion engines, there arises the question of how to detect "turbulence" from the cycle-to-cycle fluctuating velocity. If the definition of turbulence is not consistent, different results of turbulent characteristics are derived from velocity data. This resulted in various inconsistencies in the experimental evaluation of the mathematical models. There have been several methods to define the turbulence component⁽¹⁾, each method based on the concept that a certain cut-off frequency is decided above which is the turbulence component and under which is the mean velocity component in each cycle. Then, the problem is how to

decide the cut-off frequency. On the other hand, the physical definition of turbulence is unclear in the calculation. The initial conditions of calculations or referenced data for comparisons which are given by experiments using the cut-off frequency method have some ambiguity. In this report, the problem is considered in detail from both the experimental and theoretical viewpoints. First, the physical meaning of turbulence in both the calculation and measurement is clarified and then, a proposal to determine "turbulence" is made. In addition, the validity of the proposal is discussed by making comparisons between the calculation and measurement.

2. Definition of Turbulence in In-Cylinder Flow Field

2.1 Turbulent characteristics determined in experiments

The authors have proposed a criterion in a previous paper by which to determine the cut-off frequency, f_c , which separates the turbulence from the mean flow in nearly isotropic turbulent in-cylinder fields⁽²⁾. The conditions for the criterion are that i) power spectra of both the ensemble-averaged mean

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velocity, $\bar{U}$, and moving-averaged mean velocity, $\tilde{U}$ (velocity component included under f_c in the frequency range of each cycle), should be similar to each other, and ii) the rms value, $\bar{u}'$, of turbulent component, u' , (velocity component included over f_c in frequency range of each cycle) should have a power spectrum profile that has one peak of power at a certain frequency, f_0 , and the power, E , decreases almost logarithmically in the frequency range either over or under the frequency, f_0 . This power spectrum profile, which can be observed in a nearly isotropic flow field where the cycle-to-cycle variation does not occur⁽³⁾, is illustrated in Fig. 1. The ordinate and abscissa stand for power, E (energy per unit mass and frequency), and frequency, f (or wave number, κ), respectively. The frequency where E reaches a maximum corresponds to the frequency of the energy containing eddy, f_0 (or the wave number, κ_0). If the cut-off frequency of f_c is increased, f_0 (or κ_0) shifts to a higher frequency (or wave number), and then, the turbulent-energy-containing scale, L_0 , corresponding to f_0 (κ_0) decreases. As a result, the turbulent energy, k , that is, the integrated value of the power over the whole frequency range decreases.

Next, we discuss ε , that is, the dissipation rate of the turbulent energy, k . In the high Reynolds number flow field, the turbulent power, E , can be expressed by both ε and κ as $E = \alpha \varepsilon^{2/3} \kappa^{-5/3}$ in the region known as Kolmogorov's $-5/3$ power domain. As Kolmogorov's domain is generally observed in an engine cylinder flow field, we can determine ε from the power spectrum pattern in a high wave number region which can be measured experimentally. The value of ε obtained in this way basically does not depend on the cut-off frequency at all, differing from k and L_0 mentioned in the above. Regarding the value of α , 1.5 and 0.55 are used, respectively, in the three-dimensional energy spectrum and the one-dimensional energy spectrum usually obtained from the time-series data⁽⁴⁾.

Ultimately, k and L_0 , which depend on the cut-off frequency, f_c , and ε which does not depend on the cut-off frequency are obtained experimentally. However, it is difficult to decide f_c by the established method. This issue will be mentioned in the next section.

Fig. 1 Model of turbulence power spectrum

2.2 Theoretically determined turbulent characteristics

Two-equation turbulence models such as the k - ε model can provide the eddy viscosity, ν_t , the diffusivity term included in the N - S equations, on the assumption of isotropic turbulence as in the following:

$$\nu_t = C_D k^2 / \varepsilon, \quad (1)$$

where the coefficient, C_D , is generally taken to be 0.09 for flows with high Reynolds number. Another correlation among L_0 , ε and k shown in Eq. (2) is commonly used. Combining Eqs. (1) and (2), ν_t can be expressed as Eq. (3).

$$L_0 = C_D^{3/4} k^{3/2} / \varepsilon \quad (2)$$

$$\nu_t = C_D^{1/4} k^{1/2} L_0 \quad (3)$$

Now, Eq. (2) is derived from the power spectrum profile of turbulence as follows. Considering the power spectrum sketched in Fig. 1, the integrated value over the entire wave number equals k , as expressed in Eq. (4).

$$k = \int_0^{\kappa_0} A \kappa^m d\kappa + \int_{\kappa_0}^{\infty} \alpha \varepsilon^{2/3} \kappa^{-5/3} d\kappa \\ = \frac{A}{m+1} \kappa_0^{m+1} + \frac{3\alpha \varepsilon^{2/3}}{2} \kappa_0^{-2/3} \quad (4)$$

At a wave number of κ_0 , we obtain the relationship $A \kappa_0^m = \alpha \varepsilon^{2/3} \kappa_0^{-5/3}$, where m is the power-coefficient of the profile under κ_0 region. Then, the turbulent energy, k , can be expressed by $k = \alpha [1/(m+1) + 3/2] \varepsilon^{2/3} \kappa_0^{-2/3}$. Assuming that the reciprocal of κ_0 corresponds to L_0 , Eq. (5) is given as

$$L_0 = k^{3/2} / \left[\left\{ \alpha \left(\frac{1}{m+1} + \frac{3}{2} \right) \right\}^{3/2} \cdot \varepsilon \right] \\ = C' k^{3/2} / \varepsilon. \quad (5)$$

This expression has the same form as the algebraic Eq. (2). As a result, $C' = C_D^{3/4}$.

Rotta⁽⁵⁾ and Makita and Iida⁽⁶⁾ derived the value of the coefficient, C' , using Heisenberg's semi-empirical expression of the power spectrum for an isotropic turbulence. Figure 2 shows the result obtained by Rotta that indicates a relationship between the values of C' and the turbulence Reynolds number, $Re_t (= k^{1/2} L_0 / \nu)$. In this figure, C' is shown in the range of

Fig. 2 Variation of coefficient C' with turbulence Reynolds number⁽⁵⁾

Re_t up to 50. When we extrapolate this curve to a higher Re_t , C' becomes 0.164, which is determined from the $C_D=0.09$ usually used in the k - ϵ model, at $Re_t=135$. From this result, C' is a constant of 0.164 in the Re_t region higher than 135.

Here, an estimation of Re_t in the actual in-cylinder flow field is made. According to the authors' results⁽⁷⁾ obtained from experiments performed on an axi-symmetrical transparent cylinder engine, whose specifications are listed in Table 1, at 320 rpm motoring operation with a cut-off frequency of 25 Hz, the rms value of $\bar{u}'$ and lateral integral length scale of turbulence were 75% of the mean piston speed and 5% of the cylinder bore diameter, respectively, at the intake-valve-closing time. Assuming L_0 is equal to the lateral integral length scale of turbulence, L_θ (Appendix), Re_t becomes 94 at the same timing. Then, C' and C_D are 0.273 and 0.177, respectively. This value of $C_D=0.177$ is around twice the usual value of 0.09. This is because the value of Re_t in the in-cylinder flow is lower than 135 and then, C' takes a large value at the low Re_t range.

At a higher engine speed such as 1 000 rpm, C' becomes constant, 0.164, as Re_t is higher than 135.

3. Discussion of the Proposed Definition of Turbulence in In-Cylinder Flow

In the previous section, it was mentioned that turbulence obtained in both the calculation and measurement can be determined from the same power

Table 1 Specifications of model engine

Displacement volume (cc)	402
Bore x Stroke (mm)	80 x 80
Top-clearance length (mm)	20.0
Compression ratio	5.0
Intake valve timing Open (deg)	0° TDC
Close	30° ABDC
Exhaust valve timing Open	30° BBDC
Close	0° TDC
Con-rod length (mm)	200
Single valve diameter (mm)	32
Maximum valve lift (mm)	8.4

Table 2 Effect of cut-off frequency on turbulent characteristics

f c(Hz)	Lg(mm)	k(m ² /s ²)	Ret	C _D	ε (m ² /s ³)	ν _t (m ² /s)
25	2.1	0.233	89	0.189	15.4	6.7x10 ⁻⁴
				0.09	8.83	5.7x10 ⁻⁴
50	1.8	0.163	64	0.247	12.9	5.1x10 ⁻⁴
				0.09	6.05	4.0x10 ⁻⁴
Estimated value of ε from Fig.3					14.9	

spectrum profile, and the relationship of k , L_0 and ϵ was deduced. In this section, investigations are made on some topics which have been significant problems in making a precise comparison between the calculation and measurement.

3.1 Effect of cut-off frequency on initial conditions

Because the cut-off frequency has some optionality, the initial condition for ν_t changes qualitatively depending on the cut-off frequency, f_c . The effect of f_c on ν_t was investigated first.

Table 2 indicates turbulent characteristics obtained from two different cut-off frequencies at 90 degrees BTDC. Shown here is an example at $r=15$ mm, $z=10$ mm, where r and z designate positions from the cylinder center and cylinder head surface, respectively. Values of L_θ , k , Re_t and C_D were measured directly, while ϵ and ν_t were derived by Eqs. (2) and (3), respectively, using measured data. Values of L_θ were obtained by the spatial correlation method employing the two-point simultaneous LDV measurement⁽⁶⁾. As for ϵ and ν_t , both results in the cases of fixed C_D and C_D varied by Re_t are also presented.

It is found that the difference in ν_t caused by the cut-off frequency is reduced for the case of changing C_D , and the value of ν_t increases about 20% compared to that in the case of fixed C_D . This explanation can be attributed to the fact that a high value of f_0 results in a reduction of ν_t due to the decrease in both k and L_0 in the case of fixed C_D , whereas it does not cause reduction of ν_t in the case of varied C_D in spite of the decrease in both values of k and L_0 , because $C_D^{1/4}$ in Eq. (3) increases at the same time.

On the other hand, a significant difference in ϵ is shown in the case of varied C_D ; almost twice as high a value as that in the case of fixed C_D . For comparison, another estimation of ϵ was made using an analysis of the turbulence dissipation process in the high wave number region mentioned in the previous section.

Fig. 3 Power spectrum of instantaneous velocity

Figure 3 shows the power spectrum of the instantaneous velocity from 94 degrees to 84 degrees BTDC at the same measured point as in Table 2. This shows that the turbulence decay is on the $-5/3$ power line. Because the relationship of the energy versus wave number is required to calculate ε , the horizontal line of f is converted into κ by dividing f by the convection velocity. In this conversion of f into κ , because the narrow region in the high wave number, where turbulence fluctuation is adequately smaller than that of the mean velocity, is focussed, the use of the mean velocity, $\bar{U}$, for the convection velocity seems to be justified. Then, Eq. (6) is adopted.

$$\left. \begin{aligned} \kappa &= 2\pi f / \bar{U} \\ E(\kappa) &= E(f) / 2\pi \bar{U} \end{aligned} \right\} \quad (6)$$

ε can be expressed in the $-5/3$ power region as Eq. (7).

$$\varepsilon = [E(\kappa) / (\alpha \kappa^{-5/3})]^{3/2} \quad (7)$$

For example, we can read $E(f_1) = 10^{-5} (\text{m}^2/\text{s})$ at $f_1 = 1010 \text{ Hz}$ in Fig. 3. Then, when $\bar{U} = 1.06 (\text{m/s})$, $\kappa_1 = 5990 (1/\text{m})$, and $E(\kappa_1) = 1.69 \times 10^{-6} (\text{m}^3/\text{s}^2)$ are given. The value of ε calculated from these κ_1 and $E(f_1)$ using Eq. (7) agrees approximately with that obtained in the case of C_D varied by Re_t , as shown in Table 2.

The above result is one example, but a similar tendency was observed at several other measured points. Accordingly, changing C_D by Re_t is found to

(a) Turbulence velocity

(b) Integrated length scale of turbulence

Fig. 4 Effect of cut-off frequency on temporal variations of turbulent characteristics

reduce the quantitative difference of the initial condition. Also, the value of ε obtained by means of the power spectrum showed almost the same value calculated by Eq. (2) using k and L .

3.2 Problems in comparison between calculation and measurement

In order to obtain a theoretical profile of the turbulent power spectrum like that of Fig. 1, the cut-off frequency must be changed spatially and temporally. However, it is actually quite difficult to satisfy that condition such that the cut-off frequency is made constant in the experiment. Then, the difference caused by the selection of cut-off frequency is mentioned first.

Figure 4 shows variations of the measured turbulent characteristics in both the tangential and axial directions at $(r, z) = (5, 16) \text{ mm}$. Figure 4(a) indicates the turbulent velocity, $\bar{u}'$, while Fig. 4(b) shows the lateral integral length scale of turbulence, L_g , obtained using two different cut-off frequencies of $f_c = 25$ and 50 Hz . Regarding the turbulent velocity, little difference in quality or quantity is found, while a large difference in L_g is noted. The explanation is thought to be that since L_g is the eddy size corresponding to the turbulent-energy-containing eddy, the turbulent component around this range is strongly affected by the cut-off frequency, and that since $\bar{u}'$ or k is an integrated value over the whole frequency range, the turbulent velocity is not greatly affected by the cut-off frequency. This tendency should be noted before making comparisons between the calculation and measurement.

Figure 5 shows temporal variations of both the mean velocity and turbulent characteristics at $(r, z) = (5, 12) \text{ mm}$. As the turbulent characteristics, values of ν_t , ε , L_g , Re_t , C_D and $\bar{u}'$ are plotted. The calculated value of $\bar{u}'$ is given from that of k as $(2k/3)^{1/2}$. Marks of 'O' and 'x' represent measured values in the tangential and axial directions, respectively. In Fig. 5(a), both the calculated and measured results in the case of $f_c = 25 \text{ Hz}$ using C_D varied by Re_t are shown. Experimental values of ε and ν_t are obtained by Eqs. (2) and (1), respectively. Figure 5(b) shows both the calculated and measured results in the case of $f_c = 25 \text{ Hz}$ with fixed C_D of 0.09 and 0.18. Note that experimental results, however, are plotted only in the case with the fixed C_D at 0.18. The value of 0.18 as C_D is selected to quantitatively give the same initial value of ν_t as in the case of (a). Calculated results with fixed C_D of 0.09 and 0.18 are plotted in broken and solid lines, respectively. Figure 5(c) shows both the calculated and measured results in the case of $f_c = 50 \text{ Hz}$ with fixed C_D and varied C_D , but note that the experimental results are plotted only in the case with

Fig. 5 Comparison of temporal variations of mean velocity and turbulent characteristics between calculation and measurement

varied C_D . The calculated results with fixed and varied C_D are plotted in broken and solid lines, respectively.

First, Fig. 5(a) in the case of $f_c = 25$ Hz shows good agreement of the mean velocity between the calculation and measurement. Quantitatively good agreement of ν_t and L_g is found during the compression stroke. However, the calculated values become large compared to the measured data in the expansion stroke. This is probably due to the fact that the value of f_c is too large during the expansion stroke and then L_g has a small value. Good agreement is found between $\bar{u}'$ and ε during both the compression and expansion strokes.

Next, for Fig. 5(b) in the case of $f_c = 25$ Hz, the calculated mean velocity shows as good an agreement as Fig. 5(a), while the calculated values of ν_t , L_g , $\bar{u}'$ and ε show humps during the compression stroke which are not found in the experiments. Comparing the calculated result of $C_D = 0.09$ (broken line) to that of $C_D = 0.18$ (in solid line), the latter shows good agreement with experiments. Furthermore, comparing the calculated results of Fig. 5(a) of varied C_D to Fig. 5(b) in the solid line of fixed C_D at 0.18, they have almost the same ability to predict the mean velocity, while the former shows better agreement of turbulent characteristics with the measured values during the compression stroke.

Finally, for Fig. 5(c) in the case of $f_c = 50$ Hz, the calculated tangential mean velocity with the C_D varied by Re_t in the solid line shows as good a prediction as Fig. 5(a) or Fig. 5(b), but that with the fixed C_D of 0.09 in the broken line does not show good

prediction. Other characteristics predicted with the varied C_D show a similar tendency as Fig. 5(a), while the calculation with the fixed C_D shows poor agreement of the mean and turbulent velocities in the experiments of the effect of the cut-off frequency, in contrast to the result shown in Fig. 5(b) as the solid line. Thus, comparisons between the calculation and measurement successfully show the validity of the new proposal to change C_D by Re_t .

4. Conclusions

A new proposal on the definition of turbulence, which is a significant problem in analysis of the in-cylinder gas motion, was presented. The evaluation of this proposal by comparisons between the calculation and measurement enabled the following conclusions.

(1) In an algebraic equation of $\varepsilon = C_D^{3/4} k^{3/2} / L_0$, it was shown theoretically that changing the value of C_D as a function of the turbulence Reynolds number enables us to obtain the same power spectrum profile of turbulence in both the calculation and measurement.

(2) In the experiments, since the cut-off frequency has some ambiguity in determining turbulence, there is the problem that different initial conditions are given by different cut-off frequencies. With respect to this problem, the new proposal shown in this paper offers the possibility of reducing the effect of the cut-off frequency on the calculated result.

(3) Problems in making a detailed comparison between the calculation and measurement were carefully considered. Turbulent characteristics were calculated at some measured points in cases using both

the newly proposed method and the conventional one. Then, when only the newly proposed method is adopted, good prediction of turbulence is possible compared to the measured data. This result has made it possible to give more credibility to the $k-\epsilon$ model by which it was thought that the mean velocity was predictable but prediction of the turbulent characteristics uncertain.

Appendix

According to the previous experimental study⁽⁶⁾, the turbulent-energy-containing eddy size of L_0 has almost as great a value as the longitudinal integral length scale of turbulence, L_f . In this study, turbulence was determined as the component over a cut-off frequency to remove the cyclic variation. In this case, the turbulent power spectrum takes a maximum value at the frequency corresponding to the turbulent-energy-containing eddy. The profile of the autocorrelation function which is obtained by the inverse-FFT from the power spectrum becomes the same as that of the spatial correlation function to measure the lateral integral scale of turbulence which was obtained experimentally⁽⁷⁾. Therefore, L_g was made equal to L_0 in this study.

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