Numerical Analysis of a Single Bubble by VOF Method*

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In order to examine the feasibility of direct simulation of bubbly flow, the applicability of the VOF (volume of fluid) method to analyses of a single rising bubble was examined in this study. Calculated bubble shapes and terminal velocities under wide ranges of Eötvös number and Morton number were compared with the experimental data summarized by Grace et al. Except for the cases in which bubble shapes were spherical-cap and skirted, the VOF method could predict them well by assigning only eight cells to the bubble diameter. Hence, it was confirmed that some modification of this method will enable us to simulate bubbly flow directly under a wide range of flow conditions. The relationship between bubble shape and velocity distribution was also examined within the ranges in which the VOF method is valid. It was found that the secondary vortex appearing in wobbling bubbles induces a velocity component normal to the bubble interface, and this velocity is one of the causes of the wobbling shape of the bubble.

Key Words: Multiphase Flow, Numerical Analysis, Bubble, Vortex, Volume of Fluid Method, Bubble Shape, Terminal Velocity

1. Introduction

Characteristics of gas-liquid two-phase bubbly flow in a pipe, such as pressure drops, area-averaged void fractions and heat transfer, depend on distributions of the local void fraction and local phase velocities in the cross-sectional area of the pipe. A number of experimental studies have therefore been conducted to clarify these distributions⁽¹⁾. Theoretical models based on the local time-averaged conservation equations have also been proposed to predict the distributions of the local quantities^{(2)–(5)}. Most of these models adopted the assumption that turbulent diffusion is equal to the linear combination of shear-induced turbulent diffusion in the liquid-phase and bubble-induced diffusion^{(2)–(4)}. However, it may be better to consider that the bubble-induced turbulence interacts nonlinearly with the shear-induced one.

If we can simulate the bubbly flow directly, that

is, if we can perform numerical analyses based on the local-instantaneous conservation equations for each phase, the nonlinear interaction can be estimated directly. One of the numerical methods, which enables us to make this simulation, may be the volume of fluid (VOF) method^{(6),(7)} since this method can predict the movement of the gas-liquid interface using the local-instantaneous Navier-Stokes equation and cell-averaged volumetric fractions of each phase. Here the cell means a computational control volume to which the local-instantaneous equations are applied. However, the VOF method has not yet been applied to bubbly flows. Hence, as a preliminary examination for the direct simulation of the bubbly flow, the applicability of the VOF method to analyses of a single bubble rising through stationary liquid was examined in this study. The relationship between bubble shape and the velocity distribution in the neighborhood of the bubble was also examined numerically within the ranges in which the validity of the VOF method was confirmed.

2. Brief Description of the VOF Method

An axisymmetric cylindrical coordinate system, i.e., (r, z) system, is used in this study. The mass and

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the momentum equations solved by the VOF method are given by

$$\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} + \frac{u}{r} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right] \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left[\frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial z^2} + \frac{1}{r} \frac{\partial v}{\partial r} \right] - g \quad (3)$$

where u and v denote the r and z components of the velocity, p the pressure, g the gravity constant, ν the kinematic viscosity, and t the time. The density ρ is defined by

$$\rho = (1 - F)\rho_G + F\rho_L, \quad (4)$$

where F is the volume of the liquid phase in a computational cell, and the subscripts G and L denote the gas and the liquid phase. Since F corresponds to the cell-averaged volumetric fraction of the liquid phase, the gas-liquid interface exists in a cell when its value of F lies between 0 and 1. The location of the interface in this cell is determined by using the values of F in eight cells surrounding the cell. The curvature of the interface is also calculated from these values, and is used to calculate the force due to surface tension. This force is taken into account when Eqs. (1)–(3) are solved to obtain the velocity and pressure fields in advanced time.

Since F moves with the fluid, F is governed by

$$\frac{\partial F}{\partial t} + \frac{1}{r} \frac{\partial r F u}{\partial r} + \frac{\partial F v}{\partial z} = 0. \quad (5)$$

This equation is used to advance the interface location.

3. Examination of the VOF Method

The applicability of the VOF method to the analyses of a single rising bubble was examined in order to check the feasibility of direct simulation of the bubbly flow. Grace et al.^{(8),(9)} summarized existing experimental data on the rising velocity and the shape of the single rising bubble in terms of three dimensionless numbers, that is, the Eötvös number Eo , the Morton number M and the bubble Reynolds number Re . They are defined by

$$Eo = g(\rho_L - \rho_G)d^2/\sigma, \quad (6)$$

$$M = g\mu_L^4(\rho_L - \rho_G)/(\rho_L^2\sigma^3), \quad (7)$$

$$Re = \rho_L V_t d/\mu_L, \quad (8)$$

where d denotes the equivalent bubble diameter, σ the surface tension, μ the viscosity and V_t the terminal velocity. Figure 1 shows the graphical correlation of the terminal velocity and the shape of the bubble quoted from Ref. (8). The bubble shapes were classified into six types, i.e., spherical, ellipsoidal,

wobbling, dimpled, skirted and spherical-cap bubbles in terms of Eo and M . The boundary between the spherical bubble and the ellipsoidal one was defined by using the minor to major axis ratio. That is, if the ratio was less than 0.9, the bubble was considered to be ellipsoidal. Other boundaries have not been defined clearly.

Figure 2 shows the computational mesh and boundary conditions used in the present analyses. The initial bubble shape was spherical. The size of the cells around the bubble was set fine. From the top boundary, the liquid was fed in a downward direction at a uniform velocity, V_{in} . The value of V_{in} was set nearly equal to that of the terminal velocity so that the bubble did not escape from the fine cell region. The

Fig. 1 Graphical correlation proposed by Grace(8): Numbers and bold lines denote the conditions examined

Fig. 2 Mesh geometry

number of cells assigned to the initial spherical bubble was determined by considering the allowable number of cells for one bubble when performing the direct simulation of three-dimensional bubbly flows using existing supercomputers. When an analysis of slow transient flow is carried out with the supercomputer, the total number of cells may be limited to the order of 10^5 . Hence, if we want to analyze the bubbly flow in a duct, and if we divide the flow area of the duct into 40×40 cells and its length into 100 cells, the total number of cells becomes 160 000. That is, the above division of the duct reaches the limitation. Then, if the volume-averaged void fraction is 5% and the number of bubbles in the duct is 30, the number of cells for one bubble becomes 267. In this case, the number of cells assigned to the bubble diameter, N , is approximately 8 since $\pi N^3/6=267$. Thus, only eight cells were assigned to the symmetric axis of the bubble, as shown in Fig. 2.

The conditions examined are summarized in Table 1. The bubble shapes represented in this table are ones which should be realized for each case. The number of each case is written in Fig. 1 at the position corresponding to the values of Eu and M for each

Table 1 Examined conditions

Case	Eu	$\log M$	Bubble Shape
1	0.5	-3.86	Spherical
2	0.5	-7.86	Spherical or Wobbling
3	10	-1.26	Ellipsoidal
4	10	-5.26	Ellipsoidal or Wobbling
5	10	-9.26	Wobbling
6	200	2.32	Dimpled
7	200	-1.68	Skirted
8	200	-5.68	Spherical-Cap

case. The rising velocity V_B in a stationary liquid was obtained by subtracting V_{in} from the interface velocity at the upper end of the bubble. When V_B does not change in time, it reaches the terminal velocity, V_t , in the stationary liquid.

The time histories of the calculated bubble shapes are summarized in Fig. 3. For cases 1, 3 and 6, that is, spherical, ellipsoidal and dimpled bubble, the bubble shapes reached a steady state within 30 seconds and the final shapes agreed with Grace's graphical correlation.

It is known that the minor to major axis ratio A of the ellipsoidal bubble can be estimated by the following empirical correlation⁽¹⁰⁾:

$$A = \begin{cases} 1 & (T \leq 1) \\ [0.81 + 0.206 \tanh \{2(0.8 - \log T)\}]^3 & (1 < T < 39.8) \\ 0.24 & (T \geq 39.8) \end{cases} \quad (9)$$

where T is the Tadaki number defined by

$$T = Re \cdot M^{0.23} \quad (10)$$

Substituting the fluid properties, the bubble diameter and the terminal velocity of case 3 into Eqs. (7)-(10) yields $A=0.82$. On the other hand, the value of A obtained from the calculated bubble shape was 0.78. The agreement of these two values is fair.

As for cases 2, 4 and 5, i.e., for wobbling bubbles, the bubble shapes continued to change within 30 sec. Since the present analyses were conducted using the axisymmetric coordinates, the bubbles retained symmetric shapes. However, if we perform three-dimensional analyses for these wobbling cases, there may be a possibility that the calculated bubbles take asymmetric shapes and move spirally. In cases 7 and 8, the skirted or spherical-cap bubbles were broken up because the distance between the upper- and the lower-interface of the bubble was small enough for one cell to include these two interfaces. We must therefore allot more cells to one bubble in order to simulate skirted and spherical-cap bubbles more accu-

Fig. 3 Time histories of bubble shapes

rately.

The calculated bubble rising velocities are shown against time in Fig. 4. Figure 4(a) corresponds to the wobbling bubble and Fig. 4(b) to the dimpled one. The rising velocity of the wobbling bubble fluctuated in time because its shape changed in time, as shown in Fig. 3. Except for the wobbling bubbles, the rising velocities converged to constant values, as shown in Fig. 4(b). These constant values correspond to the terminal rising velocities, V_t . The terminal velocity of the wobbling bubble was obtained by averaging the fluctuating velocity in time. Calculated V_t and experimental ones, which were obtained from Fig. 1, are summarized in Table 2. The time-averaged V_t could not be obtained for case 2 due to large fluctuation of the rising velocity. As shown in Table 2, the maximum difference between the experimental and the calculated V_t was 29%.

Consequently, it was confirmed that some

Table 2 Terminal velocity of rising bubbles

Case	1	2	3	4	5	6	7	8
V_t (Grace)	0.16	0.88	0.43	1.00	0.95	0.72	0.96	0.92
V_t (calc.)	0.12	-	0.45	0.71	0.75	0.71	0.68	1.16

(a) wobbling bubble, (b) dimpled bubble

Fig. 4 Rising velocity of a bubble

Fig. 5 Mesh geometry

modification of this method, especially for the cases of high Eötvös number and low Morton number which result in a small distance between interfaces, will enable us to simulate the motion of a single rising bubble under a wide range of flow conditions with a small number of cells. For the direct simulation of the bubbly flow, further examination of this method should be performed in the future concerning bubbles in laminar liquid flow, bubble-bubble interaction and bubble-wall interaction.

4. Velocity Distribution around Bubbles

Although a number of studies have been conducted to clarify the terminal velocity and the shape of the single rising bubble, there is little information on velocity distribution inside and outside of the bubble⁽¹¹⁾⁻⁽¹³⁾. Since the above analyses made it clear that the VOF method is a useful means of predicting the bubble motion, the relationship between the bubble shape and the velocity distribution was examined in the ranges denoted by bold lines in Fig. 1, in which the validity of the VOF method was confirmed. Calculations were carried out using 16 cells for the bubble diameter in order to obtain the velocity distributions in detail. The computational mesh and boundary conditions used in the following calculation are shown in Fig. 5. The number of cells in the axial direction was changed by considering the obtained size of the wake region.

In order to examine the effects of M and Eo on the velocity distribution systematically, the numerical analyses were carried out for the following two cases:

(case A) $Eo=10$, $10^{-6} \leq M \leq 10^1$,

(case B) $M=10^{-3}$, $1 \leq Eo \leq 70$.

Figures 6(a)-(d) show an example of the velocity distributions for case A. The velocities in these figures were plotted in the coordinates fixed at the upper end of the bubbles. The bubble shapes for $M \geq 10^{-3}$ were stable mainly due to the high viscosity, but wobbled when $M \leq 10^{-4}$. An estimated flow pattern for a stable deformed bubble quoted from Ref.(10) is shown in Fig. 6(e). In this flow pattern, the main and the secondary vortices locate within the bubble because the bubble is considered to be stable. However, in the wobbling bubble, the main and the secondary vortices locate across the gas-liquid interface as shown in Fig. 6(d). These vortices induce a velocity component normal to the interface. This velocity component causes the wobbling motion of the bubble. As an example, the time history of the bubble shape and the velocity distribution when $M=10^{-6}$ and $Eo=10$ is shown in Fig. 7. This figure clearly demonstrates that complex interaction among the main, the secondary and the toroidal vortices causes the periodical defor-

mation of the bubble.

Consequently, it was confirmed numerically that the velocity component normal to the interface induced by the interaction of the main, the secondary and the toroidal vortices is one of the causes of the wobbling motion of the bubble.

Typical results for case B, i.e., $M=10^{-3}$, are shown in Fig. 8. When $Eo < 10$, the toroidal vortex does not appear due to the high surface tension. However, since the main vortex occasionally grows larger than the bubble size, the bubble shape changes slightly in time. When $Eo \geq 20$, the toroidal and the

Fig. 6 Velocity distributions for $Eo=10$ (a)-(d) and an estimated flow pattern (e) for a stable deformed bubble⁽¹⁰⁾

Fig. 7 Time history of a wobbling bubble shape ($M=10^{-6}$, $Eo=10$)

Fig. 8 Velocity distributions for $M=10^{-3}$

Fig. 9 Bubble shape change ($M=10^{-3}$, $Eo=70$)

secondary vortices emerge as shown in Figs. 8 (c) and (d). Yabe and Kunii^{(11),(12)} and Coppus et al.⁽¹³⁾ investigated the flow pattern around spherical-cap bubbles with a tracer method. However, the existence of the secondary vortex was not confirmed in their experiments. This might be because the secondary vortex was weak and small enough to hide inside a bubble skirt, as shown in Fig. 8 (d). The fact that the secondary vortex is small can account for the fact that the spherical-cap bubble does not change its shape, as shown in Fig. 9.

5. Conclusions

In order to examine the feasibility of direct simulation of gas-liquid two-phase bubbly flow, the applicability of the VOF method to the analyses of a single rising bubble was examined in this study. Calculated bubble shapes and terminal velocities under a wide range of Eötvös and Morton numbers were compared with the experimental data summarized by Grace et al. Except for the cases when the bubble shapes are spherical-cap and skirted, the VOF method could predict them well by assigning only eight cells to the bubble diameter. Hence, it was confirmed that some modification of this method will enable us to simulate bubbly flows directly. Then, the relationship between the bubble shape and velocity distribution was examined within the ranges in which the validity of the VOF method was confirmed. It was found that the interaction of the main, the secondary and the toroidal vortices appearing in wobbling bubbles induces a velocity component normal to the bubble-liquid interface, and this velocity causes the wobbling motion of the bubble.

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