An Experimental Study for Simulation of Pneumatic Conveying*

(Lift and Drag Applied to a Sphere in High- Reynolds-Number Linear Shear Flows)

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Effects of velocity gradient of linear shear flows at a high Reynolds number ($Re_s = 4\,000\text{--}35\,000$) on lift and drag applied to a spherical particle are investigated by measuring surface pressure distribution, using a pendant method, and by comparing computed and photographed trajectories of a sphere falling in upward shear flows. It is made clear that lift is applied to a sphere from the higher velocity side to the lower velocity side, that it increases as the velocity gradient becomes greater, and that drag is scarcely affected by the velocity gradient.

Key Words: Multiphase Flow, Shear Flow, Fluid Force, Particle, Lift, Drag, High Reynolds Number, Trajectory

1. Introduction

The present study was undertaken in order to answer demands for some basic data in the numerical simulation of gas-solid two-phase flows. The problem of lift application in shear flows has been long discussed by many researchers. Lift on a circle and lift on a sphere were investigated^{(1),(2)}. They showed by theoretical analysis, that lift is toward the higher velocity side in the creeping shear flows at low Reynolds numbers. On the other hand, Jordan and Fromm⁽³⁾ made clear by numerical analysis that the average lift force is toward the lower velocity side of the shear flow at a Reynolds number of 400. Lee and Durst⁽⁴⁾ investigated this problem in a developed pipe flow using laser-Doppler anemometer and showed that the

direction of lift depends on the sign of the relative velocity of fluid to particle. According to their study for smaller particles (100 μm and 200 μm) lift is directed toward the higher velocity side in the core part and toward the wall in the near-wall part of the pipe, and that for larger particles (400 μm and 800 μm) lift is toward the pipe center, that is, to the higher velocity side only.

At the beginning of our study⁽⁵⁾, lift applied to a circular cylinder was investigated in a two-dimensional duct flow with 1/7 power law velocity distribution by measuring the surface pressure distribution and using a pendant method. In the next step, linear shear flows were made to investigate the effects of velocity gradient on lift and drag applied to a circular cylinder. The previous paper⁽⁶⁾ showed that the value of the lift coefficient C_{Lc} increased with an increase of shear parameter, $K_c [= d_c/U_{cen}(\partial U/\partial y)]$, and that the drag coefficient, C_{Dc} , for linear shear flows take slightly larger values than those for the uniform flows.

The present study deals with lift and drag applied to a spherical particle in linear shear flows at much higher Reynolds numbers than those discussed by the above researchers. The fluid forces applied to a parti-

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cle are investigated by a pendant method and by measuring the pressure distribution on the sphere surface. The experiments are carried out using a vertical two-dimensional duct. The effects of the velocity gradients of the air flow and the Reynolds number on the lift and drag are discussed. After the experimentally obtained values of lift and drag coefficients are substituted into the equation of motion, the trajectories of a falling sphere in the duct flows are computed. The computed trajectories are compared with those which are photographed using a camera with a stroboscope.

Nomenclature

- α : inclination angle of pendant
 b : half width of duct section = 75 mm
 $\vec{c}$: velocity of sphere
 C_D : drag coefficient
 C_L : lift coefficient
 c_x, c_y : x, y component of $\vec{c}$
 d : diameter of sphere
 $\vec{F}_D, \vec{F}_L$: drag, lift applied to sphere
 G : nondimensional velocity gradient of duct flow
 $= 2b\lambda/U_d$
 g : acceleration of gravity
 $\vec{i}, \vec{j}, \vec{k}$: unit vectors in x, y, z directions
 K_s : shear parameter referring to sphere
 $= \lambda d_s/U_{cen}$
 m : mass of sphere
 p : pressure
 Re_d : Reynolds number referring to duct
 $= 2bU_d/\nu$
 Re_s : Reynolds number referring to sphere
 $= dU_{cen}/\nu$
 t : time
 T_u, T_v : turbulence intensity of x, y component of air velocity
 U : time mean velocity of air flow
 U_d : mean velocity of duct flow
 U_{cen} : velocity of air flow approaching to the center of particle
 w : relative velocity of air to sphere
 x, y, z : Cartesian coordinate system
 λ : velocity gradient $= \partial U/\partial y$
 ρ_a : density of air
 <Subscript>
 c : circular cylinder
 s : sphere
 p : particle

2. Experimental Apparatus and Procedure

2.1 Linear shear flows through vertical duct

Figure 1 shows the outline view of a test sphere and its rotation driver in linear shear flow through a

vertical type of duct. Air flows upwards in the x direction with a constant velocity gradient in the y direction and with uniform velocity in the z direction. The cross section of the duct is 150 mm $\times$ 150 mm wide. The linear shear flows are obtained by placement of unequally spaced parallel rods, whose spacing design is primarily based on the theory in Ref. (7). Four cases of velocity gradients, $G=0, 0.2, 0.4$, and 0.6 , are investigated. Their linearity and two-dimensionality are satisfactory for each value of G , as shown in the previous paper⁽⁶⁾. The distributions of turbulence intensity, T_u , in the x direction and Reynolds stress, $-\overline{uv}/U^2$, are shown in the paper.

2.2 Measurement of lift by pendant method

The measurement of the lift coefficient is obtained in the test section of the duct using a pendant method. Diameters of test spheres are $d_s=5$ mm and 10 mm. The inclination angle, α , of the pendant is measured on a video monitor. As the pendant swings with various amplitudes during the experiment, the mean value of inclination angle is used. The value of lift is calculated by the following equation:

$$F_L = (mg - F_b - F_D) \tan \alpha \quad (1)$$

where m is mass of the particle, g is acceleration of gravity, F_D is drag applied to the particle and F_b is buoyancy. The value of F_D is computed using the value of the drag coefficient, C_{Ds} , quoted from the Re_s - C_{Ds} curve in Ref. (8).

2.3 Measurement of lift and drag by surface pressure distribution

Surface pressure distributions of test spheres

Fig. 1 Outline of experimental apparatus

fixed in linear shear flows are measured in cases where $G=0, 0.2, 0.4$ and 0.6 , and $U_d=U_{cen}=12, 15, 17$ and 19 m/s, respectively (where U_{cen} is the velocity of air flow approaching the sphere center and U_d is mean velocity in the duct section). Test spheres are rotated around the x axis by an outside driver. Diameters of spheres are $d_s=25, 30$, and 35 mm. The shear parameters, K_s , are in the range of 0 to 0.1 . Denoting surface pressure on particles by p , and static pressure on the sidewall of the duct in a cross section containing the particle center by p_w , the pressure coefficient, C_p , is defined by the following equation:

$$C_p = (p - p_w) / (\rho_a U_{cen}^2 / 2) \quad (2)$$

where ρ_a is air density.

Drag coefficient C_{Ds} and lift coefficient C_{Ls} of a sphere are computed by the following numerical integration of the pressure coefficient C_p using Simpson's method:

$$C_{Ds} = (1/\pi) \int_0^\pi \int C_p \cdot \cos \theta \cdot \sin \theta \cdot d\phi d\theta \quad (3)$$

$$C_{Ls} = -(1/\pi) \int_0^\pi \int C_p \cdot \cos \phi \cdot \sin^2 \theta \cdot d\phi d\theta \quad (4)$$

2.4 Trajectory of a sphere falling in linear shear flows

In order to investigate the effect of lift application on the trajectory of sphere motion in linear shear flows, trajectories of a falling sphere are photographed using a camera with a stroboscope. The test sphere is a glass ball whose diameter is 5 mm and density is $4\,470$ kg/m³. Coordinates of the sphere's center at each time are measured on pictures, using a graphic digitizer with a personal computer. The velocity of the falling sphere is calculated using Wilkinson's method.

3. Computation of Trajectory of a Sphere Falling in Linear Shear Flows

Trajectories of a sphere falling in linear shear flows through the vertical duct are computed and are compared with experimental ones. For the forces applied to a sphere, gravity mg , lift, $\vec{F}_L$, and drag, $\vec{F}_D$, are considered. Magnus force and viscosity torque applied to a rotating sphere and collision between duct wall and sphere are neglected here. Velocity of a linear shear flow, U , is expressed by the following equation:

$$U = U_d \left(1 + \frac{Gy}{2b} \right) \quad (5)$$

where b ($=75$ mm) is the half-width of the duct, and $G=2b(\partial U/\partial y)/U_d$ is the nondimensional velocity gradient. The original point of the coordinate system is taken at the central point of the duct from which a test sphere starts to fall. Now the equation of motion is simply expressed as follows:

$$m \frac{d^2 \vec{r}}{dt^2} = -mg \vec{i} + \vec{F}_D + \vec{F}_L \quad (6)$$

where $\vec{r}$ is position vector. That is, $\vec{r} = x\vec{i} + y\vec{j}$, where $\vec{i}$ and $\vec{j}$ are unit vectors in the x and y direction, respectively. Drag $\vec{F}_D$ and lift $\vec{F}_L$ are given, respectively, by the following equations:

$$\vec{F}_D = \frac{1}{8} C_{Ds} \rho_a \pi d_s^2 |\vec{W}| \vec{W} \quad (7)$$

$$\vec{F}_L = -\frac{1}{8} C_{Ls} \rho_a \pi d_s^2 |\vec{W}|^2 \frac{\vec{K} \times \vec{W}}{|\vec{K} \times \vec{W}|} \quad (8)$$

where $\vec{W}$ is relative velocity of air flow to the sphere $\vec{W} = \vec{U} - \vec{c}$. The reason why the minus sign is used in Eq. (8) is that the value of C_{Ls} is expected to take a positive sign when the lift acts in the negative- y direction of the present coordinate system.

Velocity $\vec{c}$ of a sphere is given by the following equation:

$$\vec{c} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \vec{i} + \frac{dy}{dt} \vec{j} \quad (9)$$

Assuming here that $C_{Ds} \gg C_{Ls}$, $|c_y|/|\vec{W}| \ll 1$ and $\vec{K} \times \vec{W}/|\vec{K} \times \vec{W}| = \vec{j}$, Eq. (6) is resolved into the x and y components as follows:

$$\frac{d^2 x}{dt^2} = -g + \frac{3}{4} C_{Ds} \frac{\rho_a}{d_p \rho_p} |\vec{W}| (U - c_x) \quad (10)$$

$$\frac{d^2 y}{dt^2} = -\left[\frac{3}{4} C_{Ds} \frac{\rho_a}{d_p \rho_p} |\vec{W}| c_y + \frac{3}{4} C_{Ls} \frac{\rho_a}{d_p \rho_p} (U - c_x)^2 \right] \quad (11)$$

4. Experimental Results and Discussion

4.1 Lift coefficient measured by pendant method

Figure 2 is a photograph which shows that a pendant consisting of a sphere suspended by a thread is inclined from the vertical line to the lower velocity side due to the lift applied to it in a linear shear flow. As a result, the lift coefficient value of $C_{Ls}=0.02$ is obtained in the case of $G=0.6$ and $Re_{s,s}=4\,000$ to $7\,000$. Although the data of C_{Ls} versus G are scattered broadly, the value of C_{Ls} shows an upward trend as the value of G increases from zero to 0.6 .

4.2 Lift and drag measured by pressure distribution

Figure 3 shows surface-pressure distribution on a sphere in the direction of longitudinal angle ϕ with the parameter of latitudinal angle θ . Here the latitude of $\phi=0$ is located on the highest velocity side for a longitude θ , and the longitude of $\theta=0$ is located at the leading point of a sphere. As observed from this figure, the pressures on the higher velocity side are higher than those on the lower velocity side for the same angle of θ . Such pressure distributions cause lift applied to a particle to shift in the direction to the lower velocity side.

Figure 4 shows the drag coefficient C_{Ds} of the sphere as a function of Reynolds number, $Re_{s,s}$, with

two parameters, G and d . Since $K_s = Gd/2b$ by definition, each symbol for the same value of G and d has the same value of K_s . Marked differences cannot be found in this figure amongst the values of C_{Ds} in the case of the linear shear flows ($G \neq 0$) and in the case of uniform flow ($G=0$). According to the textbook⁽⁸⁾ the value of C_{Ds} is 0.40 at $Re_{s,0}=10\,000$, and increases

Fig. 2 Photograph of pendant in a linear shear flow ($G=0.6$)

Fig. 3 Surface pressure distribution on a sphere

slightly as Reynolds number becomes greater. It reaches 0.43 at $Re_{s,0}=30\,000$. The present data for $G=0$ agree well with the data of the textbook for uniform flows. For linear shear flows the values of C_{Ds} are found to be 0.46 ± 0.03 , slightly larger than the values for uniform flow.

Figure 5 shows that lift coefficient, C_{Ls} , increases with the increase of shear parameter, K_s , using the two parameters d and U_{cen} . In the case of uniform flow ($K_s=0$), the value of C_{Ls} is zero.

4.3 Trajectories of a sphere falling through vertical duct

Figures 6 (a) to (c) show photographs of a sphere falling in upward linear shear flows through the test duct. As observed from the photographs, the trajectories shift from the vertical line to the lower velocity side. It can be recognized that lift force acts on the sphere and that the shift in trajectories becomes greater with the increase of velocity gradient, G . In addition, some fluctuations of trajectories in the y direction are observed in the upper half. This is because the flow of air around the sphere and its

Fig. 4 Drag coefficient of sphere versus Reynolds number

Fig. 5 Lift coefficient of sphere versus shear parameter

holder has strong turbulence that affects the motion of the sphere.

Figure 7 shows the rearranged trajectories from the photographs and the computed trajectories for two values of C_{Ds} . When the value of $C_{Ds}=0.45$ is employed, the computed trajectories agree well with the experimental ones. The points A, B and C in the figure are the coordinates of the sphere at the same time. When $C_{Ds}=0.40$, the value of which is quoted from the textbook⁽⁸⁾, the sphere falls at a greater velocity than when $C_{Ds}=0.45$, because of the smaller drag.

The variations of experimental and computed coordinates and velocities of a falling sphere with a time lapse are shown in Figs. 8 (a) and (b). The

Fig. 6 Photographs of a sphere falling in linear shear flows

Fig. 7 Trajectories of a falling sphere

solid lines and dotted lines show the computed values. The values of the coordinates and the velocity components c_x and c_y computed using the values of $C_{Ls}=0.02$ and $C_{Ds}=0.45$, as described in the figure, also agree well with the experimental values at each time. The same phenomena were also observed for the other values of G . It is seen that the shift of trajectories from the vertical line becomes greater and the value of C_{Ls} also increases as the value of G becomes greater. This means that the stronger lift acts on the sphere as the velocity gradient becomes steeper.

In practice, the value of C_{Ds} is a function of the Reynolds number $Re_{s,s}$, and therefore should vary with the approaching relative velocity of air to the sphere. In the present experiment, the value of $Re_{s,s}$ varies slightly from 6 700 to 7 100. Since according to the curve of $Re_{s,s}-C_{Ds}$ in the textbook⁽⁸⁾ C_{Ds} takes a constant value in the region of $Re_{s,s}=3\,000-10\,000$, the value of C_{Ds} is taken for a constant in the computation.

(a) Coordinates

(b) Velocity components

Fig. 8 Coordinates and velocity components of a falling sphere ($G=0.6$)

5. Conclusions

The coefficients of lift and drag applied to a sphere in high Reynolds number linear shear flows through a duct were investigated and some data were proposed. The conclusions are summarized as follows:

(1) Lift is applied to a spherical particle in the direction from the higher velocity side of the approaching air flow to the lower velocity side.

(2) The value of the lift coefficient increases with increasing velocity gradient, and the value of drag coefficient is scarcely affected by the velocity gradient.

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References

- (1) Bretherton, F.P., Slow Viscous Motion Round a Cylinder in a Simple Shear, *J. Fluid Mech.*, Vol. 12, No. 4 (1962), p. 591.
- (2) Saffman, P.G., The Lift in a Small Sphere in a Slow Shear Flow, *J. Fluid Mech.*, Vol. 22, No. 2 (1965), p. 385. (Its Corrigendum is found in *J. Fluid Mech.*, Vol. 31, Part 4 (1968), p. 624.)
- (3) Jordan, S.K. and Fromm, J.E., Laminar Flow Past a Circular in a Shear Flow, *Phys. of Fluid*, Vol. 15, No. 16 (1972), p. 972.
- (4) Lee, S.L. and Durst, F., On the Motion of Particles in Turbulent Duct Flows, *International J. of Multiphase Flow*, Vol. 8, No. 2 (1982), p. 125.
- (5) Yamamoto, F. and Monya, H., An Experimental Study of Fluid Forces Applied on a Circular Cylinder in Air Flow with 1/7 Power Law Velocity Distribution through Horizontal and Vertical Ducts, *Memoirs of the Faculty of Engineering, Fukui University*, Vol. 36, No. 1 (1988), p. 139.
- (6) Yamamoto, F. and Monya, H., An Experimental Study for Simulation of Pneumatic Conveying (1st Report, Fluid Forces applied on a Circular Cylinder in Linear Turbulent Shear Flow), *Bull. JSME.*, (in Japanese), Vol. 55, No. 515 (1989), p. 1836.
- (7) Owen, P.R. and Zienkiewicz, H.K., The Production of Uniform Shear Flow in a Wind Tunnel, *J. Fluid Mech.*, Vol. 2, Part 6 (1957), p. 521.
- (8) Schlichting, H., *Boundary-Layer Theory*, 6th Edition, (1968), p. 17, McGraw Hill Book Co., New York.