Two-Phase Cross-Flow-Induced Vibration of Tube Arrays*

(Brief Review of Previous Studies and Summary of Design Methods)

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Many heat-exchange tubes in a shell-and-tube-type heat exchanger are oscillated by means of the flow of fluid through the heat exchanger. This flow is often cross flow and gas-liquid two-phase flow. Here, a brief review of studies on two-phase cross-flow-induced vibration is presented, and a summary of our work in this area provides a design methodology. Three types of vibration have been studied for tubes in two-phase cross-flow conditions: First, resonance of a tube due to vortex shedding is important primarily in single-phase flow, but also has been observed in homogeneous flow and even in two-phase flow. However, this vibration disappears in the slug flow or froth flow regions, which are important in numerous heat exchangers. Therefore, the vortex shedding phenomenon is not considered in this paper. Secondly, turbulent buffeting vibration is considered as the dominant phenomenon in the slug flow and froth flow regions. A method for estimating this type of random vibration is explained in this paper. Thirdly, an unstable form of vibration, so-called fluid elastic vibration, is also considered. The common method for estimating the instability boundary is similar to that used for single-phase flow; however, new insights on the method for two-phase flow are given in this paper.

Key Words: Fluid Vibration, Two-Phase Flow, Tube Array, Cross Flow, Random Vibration, Fluid Elastic Instability

1. Introduction

Heat-exchange tubes in many shell-and-tube-type heat exchangers are set in the cross-flow region of the heat exchanging fluid. Some of these fluids are steam-water mixtures. There have been many papers published⁽¹⁾ on cross-flow-induced vibration of tube arrays caused by a single-phase fluid. However, few papers⁽²⁾ have been published on vibration induced by a two-phase fluid, despite the fact that this type of vibration is an important subject in the design of many heat exchangers. This is because two-phase flow is a complex phenomenon and there are many parameters that affect it. In addition, two-phase flow testing which accurately models the actual phenomenon in heat exchangers is difficult to perform.

Almost all studies on two-phase cross-flow-induced vibration of heat-exchange tubes have been based on air-water two-phase flow tests under atmo-

spheric conditions, especially in the earlier stages of these studies⁽²⁾. However, steam-water two-phase flow tests at high-temperature and -pressure conditions should be performed; the flow patterns of two-phase flow may change due to variations in those conditions even if the flow volume is the same. A few test results have been published⁽³⁾ on steam-water two-phase flow, including results of our tests⁽⁴⁾.

The two-phase flow phenomenon is quite complex and it has not yet been clarified as a thermal-hydraulic subject. Purely theoretical study is likely to be impossible at this time due to the complexity, and only semi-empirical analysis and the experimental treatment are possible.

This paper presents a brief review of the papers published on two-phase cross-flow-induced vibration of tube arrays, and introduces our work, along with the commonly used methods, for establishing design tools for heat-exchange tubes using a semi-empirical treatment.

Nomenclature

$a(P)$: Coefficient ($N \cdot s/cm^3$)

$b(P)$: Coefficient relating theoretical flow velocity

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and slug speed
 c : Nondimensional fluid coefficient
 (Drag coefficient C_D or lift coefficient C_L)
 C_g, C_l : Nondimensional fluid force coefficient for gas flow and liquid flow, respectively
 $[C]$: Damping matrix of tube
 $d(\omega)$: Coefficient of correlation
 D : Diameter of tube
 D_e : Hydraulic equivalent diameter
 f : Natural frequency of tube
 F_{RMS} : Root-mean-square value of the exciting force
 $\{g\}$: Exciting force vector acting on tube ($=\{g_l\}$)
 G_g : Mass flux
 $G(\omega)$: One-sided power spectral density of the exciting force in a normalized form
 h : Damping ratio of tube
 h_j : Damping ratio of the j -th mode
 $H_j(\omega)$: Transfer function of the j -th mode
 j_g, j_l : Superficial gas velocity and liquid velocity, respectively
 $[K]$: Stiffness matrix of tube
 K_c : Critical coefficient for instability
 K_{cg}, K_{cl} : Critical coefficient based on Eq.(1) for gas and liquid single-phase flows, respectively
 l : Length of tube
 L_c : Correlation length of the exciting force
 m : Mass of tube per unit length
 m_h : Added mass of fluid
 m_o : Mass of tube per unit length in vacuum
 $[M]$: Mass matrix of tube
 M_j : Modal mass of the j -th mode
 P : Pressure
 P_s : Existence ratio of liquid slug
 q_j : Time-dependent function of the j -th mode ($\{y\}=[\phi]\{q_j\}$)
 $R(x, x', \omega)$: Statistical function of the exciting force
 $\bar{R}(\zeta, \tau)$: Correlation factor of time lag τ between two positions at distance ζ
 Re : Reynolds number
 $\bar{R}_{xx'}(\omega)$: Coherence function of the exciting force between positions x and x'
 S_g : Power spectral density of the random turbulence excitation (from Pettigrew and Gorman⁽²⁾)
 $S_x(\omega)$: Power spectral density of the exciting force at position x
 T : Tube pitch
 u_g, u_l : Theoretical flow velocity of gas and liquid, respectively
 U_c : Critical flow velocity
 U_{cg}, U_{cl} : Critical flow velocity for gas and liquid single-phase flows, respectively
 V_g, V_s : Speed of intermittently rising gas slug and

liquid slug, respectively
 $\{y\}$: Displacement vector of a tube
 $\bar{y}(x)$: Root-mean-square displacement of tube at point x
 α : Void fraction
 α_g, α_s : Void fraction of gas slug and liquid slug, respectively
 $\beta, \bar{\beta}$: Nondimensional factors
 γ : Nondimensional factor
 δ : Logarithmic damping of tube
 $\theta_{xx'}(\omega)$: Phase lag of the exciting force between positions x and x'
 Λ : Nondimensional length
 ρ : Density of fluid
 ρ_g, ρ_l : Density of gas, liquid, respectively
 $[\phi]$: Eigenvalue matrix
 $\phi_j(x)$: Mode component at position x of the j -th mode
 ϕ_l : Mode component of freedom l of the j -th mode
 ω_j : Circular natural frequency of the j -th mode

2. Brief Review of Previous Studies

Flow-induced vibration of tube arrays caused by a mixed liquid-gas flow (two-phase flow) is being increasingly investigated by some researchers. However, there have been few papers published on this subject.

The first paper published on two-phase flow-induced vibration of tube arrays is that by Pettigrew and Gorman⁽²⁾. They studied the response of a tube in a small tube array, as described in Table 1, which was subjected to air-water two-phase flow. They established a fundamental treatment of two-phase flow-induced vibration that was based on the method used for single-phase flow-induced vibration. They derived the following conclusions from their test results.

(1) Fluid elastic instability seems possible in two-phase flow as well as in liquid flow. The same instability criterion seemed to apply to both liquid and two-phase flow according to the following equation (where average flow velocity is denoted U , average density of the fluid ρ , and average damping δ in two-phase flow).

$$\frac{U_c}{fD} = K_c \left[\frac{m\delta}{\rho D^2} \right]^{1/2} \quad (1)$$

(2) Other than fluid elastic instability, vibration response to random flow turbulence was the dominant vibration excitation mechanism. This indicated that no resonant oscillation from vortex shedding was observed in their tests, and the tube's response to random flow turbulence was related to the exponent of flow velocity. The velocity exponent was roughly 1.0 for two-phase flow, despite the fact that it was between 1.5 and 2.6 for liquid flow.

Table 1 Representative published tests on two-phase flow

Researchers	Published *	Fluid	Tube Array	Void Fraction	Tube Length	Natural Frequency	Damping Ratio	Notes
Pettigrew & Gorman ⁽²⁾	1973	Air-Water	Triangular/Parallel Tri. Square/Rotated Square	10–20% ** (in quality)	50.8 mm	17Hz, 30Hz	2.5–2.7%	
Heilker & Vincent ⁽⁵⁾	1980	Air-Water	Triangular/Rotated Square	0.5 – 0.87	910 mm	56–62Hz	0.8–4.0%	
Hara & Ohtani ⁽⁶⁾	1981	Air-Water	Single Tube	0.02 – 0.61	60 mm	Rigid	—	
Remy ⁽⁷⁾	1982	Air-Water	Square	0.65 – 0.85	1000 mm	56.6Hz	0.6–1.75%	
Nakamura et al. ⁽⁸⁾	1982	Air-Water	Square/Rotated Square	0.2 – 0.94	190 mm	142Hz	(1.3–1.7%)***	Damping data is from stationary fluid.
Pettigrew et al. ⁽⁹⁾	1984	Air-Water	Triangular/Square	0.05 – 0.98	600 mm	26–32Hz	0.9–8.0%	
Axisa et al. ⁽³⁾	1984	Steam-Water	Square	0.52 – 0.98	1190 mm	74Hz	0.2–3.0%	
Nakamura et al. ⁽⁴⁾	1986	Steam-Water	Square	0.75 – 0.95	174 mm	15.2–16Hz	4.0–8.0%	
Hara ⁽¹⁰⁾	1987	Air-Water	Single/Row	0.01 – 0.5	58 mm	6.0–8.4Hz	2.9–15.6%	
Goyder ⁽¹¹⁾	1987	Air-Water	Triangular	0.5 – 0.8	360 mm	175Hz	—	
Gay et al. ⁽¹²⁾	1988	Freon-Water	Triangular	0.58 – 0.84	1000 mm	39.8Hz	(0.89–1.7%)	Damping data is from stationary fluid.
Nakamura & Fujita ⁽¹³⁾	1988	Air-Water	Square	0.02 – 0.95	600 mm	52Hz	(2.1%)	Damping data is from stationary fluid.
Funakawa et al. ⁽¹⁴⁾	1989	Air-Water	Square/Triangular	0.0 – 0.6	100 mm	12.8Hz	(3.3%)	Damping data is from stationary fluid.
Nakamura et al. ⁽¹⁵⁾	1990	Steam-Water	Square	0.33 – 0.91	174 mm	94–137Hz	—	
Axisa et al. ⁽¹⁶⁾	1990	Steam-Water	Square/Triangular Parallel/Triangular	0.52 – 0.99	1190 mm	72Hz	0.2–4.0%	

* published year means the year of the first publication, then they are some differences to the one of reference.

** calculated using different method.

*** () indicates approximate values.

(3) The vibration amplitude was generally larger in the flow direction than in the direction normal to the flow, and it was maximum at roughly 15% steam quality. Random excitation analysis has been proposed for two-phase flow based on the theory of wind-induced vibration proposed by Davenport⁽¹⁷⁾. The power spectral density of the random turbulence excitation has been inferred from the response of tubes as follows:

$$S_g = \bar{y}^2 \left(\frac{l}{2} \right) [4\pi^3 f^3 h m^2 l^2]. \quad (2)$$

In two-phase flow, upstream tubes vibrate most; however, no measurement of the correlation of the exciting forces along the tube axis (one of the important parameters in random vibration) has been conducted. They identified a new undeveloped area of the flow-induced vibration problem, but follow-up research did not appear until the report in 1981 by Heilker and Vincent⁽⁵⁾. They also showed that fluid elastic instability could be induced by two-phase flow.

Their measurement of damping was performed using the half-power point method. An important point in their paper is that the criterion for fluid elastic instability can be estimated using the same Eq. (1), but the results greatly depend on the estimation of the damping of a tube in a two-phase fluid.

Table 2 Examples of flow types

Flow Type	Average Void Fraction	Specification
Bubble	~ 0.3	Some bubbles are present in liquid flow and move with the same velocity.
Slug	0.3 ~ 0.5	Liquid slugs flow intermittently.
Froth	0.5 ~ 0.8	More violent intermittent flow.
Annular	0.8 ~ 0.9	Mainly gas flow, liquid adheres to the tube surface.
Mist	0.9 ~	Almost gas flow. Mist sometimes causes energy dissipation.

Two-phase flow characteristics greatly depend on the type of flow occurring; flow types are shown in Table 2. In a bubbly two-phase flow, vortex shedding has been observed by Hara and Ohtani⁽⁶⁾. However, they also found that vortex shedding disappears under high void fraction conditions.

In 1982, Remy⁽⁷⁾ showed results similar to those of Pettigrew and Gorman, but Remy's data showed that two-phase flow did not induce any extra damping.

In the same year, we published our first paper⁽⁸⁾ on this subject. Two-phase flow-induced turbulent random exciting forces in the bubbly flow and slug and froth flow regions were measured using a rigid tube as described in Table 1. The test results showed that there is no nondimensional treatment for the exciting force, because the maximum exciting forces are

almost constant in the slug flow region even with an increase of flow velocity. This was the starting point of our research on this subject (discussed later in this paper).

In 1984, Pettigrew et al.⁽⁹⁾ published new test results of an air-water flow test loop, where the hydrodynamic mass and damping from two-phase flow were treated. The results with respect to fluid elastic instability and random turbulence excitation were similar to those in their previous work. However, they did show an important result that the damping ratio could be expressed as a function of the void fraction.

The focus of our work is on the practical problem in which tube arrays in many heat exchangers are actually in steam-water flow under high pressure and temperature conditions, not in air-water flow. Steam-water two-phase flow tests have been conducted from this point of view. Axisa et al.⁽³⁾ published their results on steam-water two-phase flow tests with up to 2.5 MPa pressure and 210°C temperature in 1984. They showed that fluid elastic instability could easily be observed, that it could be treated using the Eq. (1), and that the two-phase flow damping proposed by Carlucci⁽¹⁸⁾ could be used. However, they gave no physical explanation for two-phase flow damping, and there was no investigation of the random turbulence exciting force.

With respect to fluid elastic instability, we published our steam-water two-phase flow test results⁽⁴⁾ in 1986 (rewritten in English⁽¹⁹⁾ in 1991, with added test results). The comparison of the test results with Eq. (1) was fair (as shown in another paper⁽²⁰⁾). However, we develop a new method in this paper to avoid the use of unknown two-phase flow damping; the new method is based on a physical interpretation of the two-phase phenomenon itself.

The steam-water flow test is technically difficult and costly to conduct, therefore a freon-water two-phase flow test was performed by Gay et al.⁽¹²⁾ They showed that instability can be observed in freon-water two-phase flow, and that it can be estimated by using Eq. (1).

Other papers have been published on fluid elastic instability in two-phase flow. Hara⁽¹⁰⁾ and Funakawa et al.⁽¹⁴⁾ investigated the fluid elastic mechanism through testing, and Goyder⁽¹¹⁾ attempted to explain the criterion with a theoretical interpretation. Pettigrew et al.⁽²¹⁾ have proposed a new criterion without the treatment of damping (shown later in this paper).

With respect to random turbulence excitation, two problems remain: the correlation along the tube axis of the random exciting forces, and the effect of temperature (or pressure). Goyder⁽²²⁾ estimated the

correlation length using theory he derived and his air-water two-phase flow test data. However, more detailed measurement of the exciting force correlation has been achieved in our air-water two-phase flow tests⁽¹³⁾. The exciting force in steam-water flow tests^{(15),(23)} has also been measured under high-temperature and -pressure conditions (up to 272°C and 5.8 MPa). These test data give us complete information for the calculation of the random response of a tube to intermittent liquid-gas two-phase flow. An important result of our tests is that random turbulence excitation greatly decreases with an increase in temperature and pressure (despite the fact that Axisa et al.⁽¹⁶⁾ did not indicate any difference between the exciting force in the air-water flow and that in steam-water flow).

According to the above brief review, the two most important vibration modes caused by two-phase flow are random excitation and fluid elastic instability. Vortex shedding can be ignored in practical application of many heat exchangers. Our proposed design method for treating these two mechanisms is discussed in the following section.

3. Random Excitation

3.1 General treatment

The equation of motion of a tube excited by an external load, such as fluid force, can be expressed as $[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = \{g\}$. (3)

By using the eigenvalue matrix $[\phi]$ of the above equation, which can be obtained when the damping term and the external load term are ignored, Eq. (3) can be arranged as a series of independent diagonal terms:

$$\ddot{q}_j + 2h_j\omega_j\dot{q}_j + \omega_j^2q_j = \frac{1}{M_j} \sum_{l=1}^n g_{l,j}\phi_l. \quad (4)$$

After taking a Fourier transformation of Eq. (4) and rearranging the terms according to Parseval's theorem, the following equation can be obtained. This equation gives the response of the tube as a root-mean-square value $\bar{y}(x)$:

$$\bar{y}(x)^2 = \sum_{j=1}^n \sum_{k=1}^n \phi_j(x)\phi_k(x) \times \int_0^\infty L_{jk}(\omega) H_j^*(\omega) H_k(\omega) d\omega, \quad (5)$$

$$\text{where } L_{jk}(\omega) = \frac{1}{M_j M_k \omega_j^2 \omega_k^2} \times \int_{\text{along tube axis}} R(x, x', \omega) \phi_j(x) \phi_k(x') dx dx',$$

$$H_j(\omega) = \left[\left(1 - \frac{\omega^2}{\omega_j^2} \right)^2 + 2ih_j \cdot \frac{\omega}{\omega_j} \right]^{-1}.$$

In this equation, $R(x, x', \omega)$ includes all information on the exciting force, and becomes a real value when the power spectral densities of the exciting forces acting at the different positions x and x' are assumed to be the same $S_x(\omega)$:

$$R(x, x', \omega) = \frac{1}{\pi} S_x(\omega) \cdot \tilde{R}_{xx'}(\omega)^2 \cdot \cos \theta_{xx'}(\omega). \quad (6)$$

It is important to compute the response of the tube using Eq. (5), where the right-hand terms of Eq. (6) are estimated for two-phase flow.

3.2 Power spectral density

Taylor et al.⁽²⁴⁾ summed up their air-water test results according to Eq. (2) and gave the power spectral density shown in Fig. 1. However, this is not sufficient for design estimation. Axisa et al.⁽¹⁶⁾ could not sufficiently verify the random turbulence excitation induced by steam-water two-phase flow, despite the fact that it may be reasonable in some flow regions such as that of homogenous bubbly flow. Axisa et al.⁽¹⁶⁾ give the following formula:

$$S_x(f=70 \text{ Hz}) \propto G_g^{1.5} \quad (7)$$

In the intermittent flow region (slug and froth flow), which can be considered as a typical flow pattern in some heat exchangers, the following method from our studies^{(15),(23)} may yield a better estimation. Here, power spectral density of the exciting force is normalized by the following equations:

$$(F_{RMS})^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega) d\omega, \quad S_x(\omega) = \frac{1}{2} (F_{RMS})^2 G(\omega),$$

$$\text{where } \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) d\omega = 1. \quad (8)$$

This indicates that the exciting force term has been given as a separate form by its magnitude F_{RMS} and its normalized frequency component $G(\omega)$.

The root-mean-square magnitude of the exciting force F_{RMS} can be given empirically as a function of slug speed V_s (the velocity of an intermittently rising liquid slug in the flow region of slug and of froth flow):

Fig. 1 Dimensionless power spectral density of air-water random turbulence excitation (from Taylor et al.⁽²⁴⁾)

$$F_{RMS} = a(P) \cdot c \cdot V_s. \quad (9)$$

Coefficient $a(P)$ may be a function of pressure or temperature. This coefficient function has not been clarified, but the following empirical formula has been derived from the test results shown in Fig. 2.

$$a(P=5.8 \text{ MPa}) \cdot c = \begin{cases} 0.28 & (\text{in drag direction}) \\ 0.18 & (\text{in lift direction}) \end{cases} \quad (10)$$

Parameter $a(P)$ is dimensional; however, coefficient c is nondimensional and should be regarded as the usual drag coefficient C_D or lift coefficient C_L . Since the coefficient c can be estimated as a function of Reynolds number Re and the geometry of the tube and tube array, we can estimate the effect on the tube array. Based on that, formula (10) has been derived for a square array with $P/D=1.42$.

The remaining problem is to estimate the slug speed V_s . This can be estimated using the following equations that are based on experimental studies^{(15),(23)}:

$$V_s = b(P) \cdot u_i, \quad u_i = \frac{j_i}{1 - \alpha}. \quad (11)$$

This relation can be derived from Fig. 3 and

$$b(P) = \begin{cases} 1.6 & (P \approx 5.8 \text{ MPa}) \\ 1.85 & (P = 0.5 \sim 3.0 \text{ MPa}) \\ 2.5 & (P \approx 0.1 \text{ MPa}). \end{cases} \quad (12)$$

The superficial flow velocity u_i can be estimated using standard thermal-hydraulic calculations, and the void fraction α can be estimated from known formulas. We use the following drift-flux model for the estimation of void fraction.

Fig. 2 Relationship between slug speed and exciting force (5.8 MPa) for (a) drag direction and (b) lift direction

$$\alpha \approx \frac{j_g}{1.2(j_g - j_l) + 0.1} \quad (13)$$

With respect to the normalized power spectral density $G(\omega)$, $G(\omega)$ can be estimated from the following equation, in the drag direction. This was derived from an assumption based on a physical interpretation where the intermittent exciting force is regarded as shot noise:

$$G(\omega) = \frac{4\nu}{\omega^2 + \nu^2} \quad (14)$$

Coefficient ν has been deduced from the test results shown in Fig. 4 and it may be a function of

Fig. 3 Relationship between average flow velocity and slug speed ($j_l = 0.3$ m/s)

Fig. 4 Frequency components of drag force ($j_l = 0.3$ m/s, $j_g = 3.0$ m/s) for (a) 0.5 MPa and (b) 5.8 MPa

Fig. 5 Relationship between pressure P and parameter ν ($j_l = 0.3$ m/s, $j_g = 3.0$ m/s)

pressure (or temperature), as shown in Fig. 5. For the lift direction, sufficient formulations which correspond to this phenomenon have not yet been proposed.

3.3 Correlation

With respect to the correlation of exciting forces along the tube axis, there are no test data for steam-water flow, so an estimation must be made from the results of air-water flow tests. Based on our measurements⁽¹³⁾, the following formula has been proposed as a coherence function of two-phase flow random turbulence excitation in the intermittent flow region:

$$\bar{R}_{xx'}(\omega) = e^{-d(\omega) \cdot \Lambda}, \quad (15)$$

where $\Lambda = |x - x'|/D$.

The nondimensional coefficient $d(\omega)$ has been determined from analysis of the correlation coefficient, as shown in Fig. 6. Then, $d(\omega) (=d(f))$ is given by the following formula:

$$d(f) = \begin{cases} 0.035 & (f = 0 \sim 10 \text{ Hz}) \\ 0.2 & (f = 10 \sim 30 \text{ Hz}) \\ 0.5 & (f = 30 \sim 50 \text{ Hz}) \\ 0.7 & (f = 50 \sim 100 \text{ Hz}). \end{cases} \quad (16)$$

If the correlation length L_c is defined by the following equation, its value would be about 300 mm. This indicates that the average size of a liquid slug should be 300 mm in air-water intermittent flow along the tube axis.

$$L_c = \int_{\text{along tube axis}} \bar{R}(\xi, \tau=0) d\xi \quad (17)$$

However, it is difficult to estimate the phase angle $\theta_{xx'}(\omega)$. Hence, the following assumption might be better for design estimation:

$$\cos \theta_{xx'}(\omega) = 1. \quad (18)$$

Summarizing the above formulae, the response of a tube to intermittent two-phase flow can be computed through a series of equations. Since the drag force is greater than the lift force in the intermittent flow

Fig. 6 Correlation functions for (a) 0~10 Hz, (b) 10~30 Hz, (c) 30~50 Hz and (d) 50~100 Hz

region, design calculations could be carried out using the drag force in each direction. In the homogenous flow region (such as of bubbly flow), data are insufficient for use in design. Therefore the data of Axisa et al.⁽²³⁾ might be used for analysis in homogeneous flow.

4. Fluid Elastic Instability

4.1 General treatment

Equation (1) has been shown to be useful in some studies, and the following assumptions have been adopted:

$$U = j_o + j_i, \rho = \rho_l(1 - \alpha) + \rho_g \alpha, m = m_o + \frac{\pi}{4} \rho D^2. \quad (19)$$

The last equation may be modified into the following formula of Pettigrew et al.⁽²⁴⁾:

$$m_h = \left(\frac{\pi \rho D^2}{4} \right) \left[\frac{\left(\frac{D_e}{D} \right)^2 + 1}{\left(\frac{D_e}{D} \right)^2 - 1} \right], \quad (20)$$

where $\frac{D_e}{D} = \begin{cases} \left(0.96 + 0.50 \frac{T}{D} \right) \frac{T}{D} & \text{(for triangular array)} \\ \left(1.07 + 0.56 \frac{T}{D} \right) \frac{T}{D} & \text{(for square array).} \end{cases}$

This equation gives reasonable correlation with their test data, as shown in Fig. 7, but some of our tests showed that m_h may actually be less than $\pi \rho D^2 / 4$. Therefore, some issues remain to be resolved.

The damping ratio is the largest unknown factor in the estimation using Eq. (1). Damping in this system is influenced by two mechanisms⁽²⁵⁾: structural damping and fluid damping. Structural damping includes damping due to the material, support-related damping such as gap clearance, and squeeze film damping. These damping terms should be estimated during design, but detailed discussion of this process is not presented here. Fluid damping includes several terms⁽¹⁸⁾: flow-related damping, viscous damping, and

Fig. 7 Hydrodynamic mass: comparison between theory and experiment (from Pettigrew et al.⁽²⁶⁾)

two-phase damping. In two-phase flow, damping caused by the two-phase condition may be the dominant mechanism of fluid damping. Two-phase flow damping is given as a function of the void fraction (shown in Fig. 8)⁽²⁶⁾. However, such data show a variety of values and have not yet been clarified with any physical explanation.

Using this data, the range of fluid elastic instability has been determined, and is shown in Fig. 9⁽²¹⁾. From the data in this figure, Pettigrew et al.⁽²¹⁾ proposed the following values for the critical coefficient K_c :

Fig. 8 Damping of tube bundles in two-phase crossflow (from Pettigrew et al.⁽²⁶⁾)

Fig. 9 Fluid elastic instability results presented in terms of mass parameter ($m/\rho D^2$) (from Pettigrew et al.⁽²¹⁾)

*Note: Curves were derived from additional data not shown here.

Fig. 10 Example criteria for homogeneous flow for (a) $\gamma=0.398$ ($P=0.1$ MPa) and (b) $\gamma=0.812$ ($P=5.8$ MPa)

Fig. 11 Existence ratio of a liquid slug for (a) effect of pressure and (b) liquid flow velocity

Fig. 12 Criteria for intermittent flow

$$K_c = \begin{cases} 3.3 & \text{(for non-square array)} \\ 4.0 & \text{(for square array)} \end{cases} \quad (21)$$

To avoid the estimation of damping, they also proposed the following criterion:

$$\frac{U_c}{fD} = 5 \left(\frac{m}{\rho D^2} \geq 7 \right), \quad \frac{U_c}{fD} = 0.7 \frac{m}{\rho D^2} \left(\frac{m}{\rho D^2} \leq 7 \right). \quad (22)$$

4.2 Physical treatment

Flow velocities of liquid and gas are theoretically given by the following equation in homogeneous flow regions (such as of bubbly flow):

$$u_l = \frac{j_l}{1-\alpha}, \quad u_g = \frac{j_g}{\alpha}. \quad (23)$$

From a comparison of the energy imparted to a tube by two-phase flow and that which dissipates while the tube oscillates, the following boundary con-

dition can be derived in a nondimensional form^{(4),(19)}:

$$\left(\frac{u_l}{U_{cl}} \right)^2 + \beta \left(\frac{u_g}{U_{cg}} \right)^2 = 1 + \beta, \quad (24)$$

where $\beta = \frac{\alpha}{1-\alpha} \cdot \gamma$, $\gamma = \frac{C'_g \rho_g U_{cg}^2}{C'_l \rho_l U_{cl}^2}$.

Nondimensional fluid coefficients C'_g and C'_l can be estimated as follows:

$$C'_g = \left[\frac{2\pi}{K_{cg}} \right]^2, \quad C'_l = \left[\frac{2\pi}{K_{cl}} \right]^2. \quad (25)$$

In these equations, all parameters are given using estimations from single-phase flow. These equations agree with actual test results, as shown in Fig. 10 (other related data cannot be shown here due to space limitations).

When two-phase flow is intermittent, the estimation of flow velocities and the existence ratio per cycle of tube motion changes. For flow in the intermittent region, the following slug speeds should be used.

$$V_s = b(P) \cdot u_l, \quad V_g = \frac{j_g + j_l - V_s P_s}{1 - P_s} \quad (26)$$

Here, P_s is the existence ratio of a liquid slug (averaged over time) and can be derived from the data reduction, as shown in Fig. 11. Then maintaining conservation of energy and from the above equation, the following relationship is proposed:

$$(1 - \alpha_s) \left(\frac{V_s}{U_{cl}} \right)^2 + \alpha_s \bar{\beta} \left(\frac{V_g}{U_{cg}} \right)^2 = 1 + \bar{\beta}, \quad (27)$$

where $\bar{\beta} = \frac{1 - P_s}{P_s} \cdot \gamma$.

However, this equation yields an absolute instability, i.e., the tube's oscillations become large with increased time. Alternately, the tube intermittently becomes unstable in the intermittent flow region, wherein the following criteria should be used:

$$\frac{V_s}{U_{cl}} = 1 \text{ and } \frac{V_g}{U_{cg}} = 1. \quad (28)$$

We call this condition intermittent instability. Almost all data are estimated as intermittent instability, as shown in the example in Fig. 12. Equations (24), (27), and (28) give a new physical insight into fluid elastic instability caused by two-phase flow. It is important to note that every parameter in these equations can be estimated using single-phase flow data, and that use of the unknown two-phase flow damping is not necessary.

5. Conclusions

Although the design methods for estimating two-phase flow-induced vibration of heat-exchange tubes are not fully explained in this paper, the following summarizes the essential conclusions from this work.

(1) Fluid elastic instability and random turbulent excitation should be the focus of analysis during the design stage.

(2) Design procedures must be established with

respect to both of these phenomena.

(3) Two-phase flow damping includes several unknown factors, hence other procedures based mainly on physical insights have been proposed for analyzing two-phase flow vibration.

(4) The test data on the two-phase flow under high-pressure and -temperature conditions are insufficient, therefore further study should be pursued.

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