Influence of Internal Phenomena on Gas Bubble Motion*

(Effects of Thermal Diffusion, Phase Change on the Gas-Liquid Interface and Mass Diffusion between Vapor and Noncondensable Gas in the Collapsing Phase)

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The motion of a single bubble is simulated directly using the full conservation equations for mass, momentum and energy when the surrounding pressure increases stepwise. The numerical results for several cases reveal that considerable distributions of temperature and concentration of vapor are formed inside the bubble, and heat and mass transfer inside the bubble have a great influence on the bubble motion. Heat transfer on the bubble wall, which is mainly governed by the temperature gradient, causes the thermal damping of the bubble motion. This damping behavior is influenced by the initial bubble radius. Mass transfer on the bubble wall, which is influenced by diffusion phenomena between vapor and noncondensable gas, also greatly effects the bubble motion.

Key Words: Cavitation, Bubble Dynamics, Direct Simulation, Nonequilibrium, Heat Transfer Phase Change, Diffusion between Vapor and Noncondensable Gas

1. Introduction

A cavitation bubble is formed from a small gas bubble; cavitation is the process in which the bubble, consisting of vapor and noncondensable gas, expands and collapses according to the surrounding pressure which decreases and increases rapidly. The internal phenomena, such as thermal diffusion, diffusion between vapor and noncondensable gas, mist formation due to homogeneous condensation and heat and mass transfer through the bubble wall, significantly influence the cavitation process.

The motion of cavitation bubble has been investigated by many researchers. Some real effects such as compressibility of liquid⁽¹⁾ and deformation of the bubble from the spherical shape near a solid wall⁽²⁾ have been studied. Nonlinear oscillation of a bubble^{(3),(4)}, chaotic behavior⁽⁵⁾ and behavior of

multibubbles^{(6),(7)} have also been analyzed. There are some analyses in which the internal conditions of the bubble are considered. Nigmatulin et al.⁽⁸⁾ performed a numerical analysis in which the temperature distribution was considered. Fujikawa and Akamatsu⁽⁹⁾ discussed that the effects of nonequilibrium in the process of condensation at the bubble wall, assuming the boundary layer of temperature is thin. One of the present authors proposed a model⁽¹⁰⁾ in which the phase change at the bubble wall is considered under the condition that temperature inside the bubble is uniform, and analyzed the effect of diffusion between noncondensable gas and vapor. He reported the effect of mist⁽¹¹⁾ which generates inside the bubble. Bubble motion⁽¹²⁾ has also been simulated by direct integration of the conservation equations of mass, momentum and energy in both gas and liquid phases under the conditions that the bubble includes only noncondensable gas and phase change is ignored. The results revealed that a temperature distribution is formed inside the bubble, and when the gas inside the bubble has high heat conductivity, like hydrogen, the motion is affected by the heat transfer at the gas-liquid interface. Kamath and Prosperetti⁽¹³⁾ solved the energy

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equation using by the spectrum method and analyzed the temperature distribution inside the bubble with good accuracy. However, there are few analyses which take into account such factors as heat transfer through the interface, phase change at the bubble wall, and diffusion due to the phase change.

In this paper, for the case in which the surrounding pressure increases stepwise, the bubble motion is calculated numerically using the full equations for mass, momentum and energy conservation inside and outside the bubble, and the effects of internal phenomena such as phase change at the bubble wall, mass transfer of noncondensable gas through the interface, and diffusion between vapor and noncondensable gas are analyzed.

The numerical results reveal that bubble motion is affected by heat transfer inside and outside the bubble. In particular, the size of the initial radius greatly affects the decay of bubble motion. It is also shown that a boundary layer of vapor concentration is formed inside the bubble, and that the diffusion affects bubble motion. The initial size of the bubble, which affects the developing speed of the boundary layers, also influences bubble motion.

2. Nomenclature

C	: speed of sound or specific heat
D	: diffusion coefficient
L	: latent heat
R	: bubble radius
$\mathcal{R}$	: gas constant
T	: temperature
c	: concentration
e	: specific internal energy
p	: pressure
r	: coordinate of radial direction
t	: time
u	: radial velocity
α	: mass accommodation factor
λ	: thermal conductivity
μ	: viscosity
ϕ	: velocity potential
ρ	: density
σ	: surface tension
subscripts and superscripts	
0	: initial condition
B	: boundary of calculation
g	: gas phase
gl	: gas in liquid
l	: liquid phase
m	: mixture condition
n	: noncondensable gas
s	: saturated condition
v	: vapor

w	: bubble wall
∞	: infinity
*	: nondimension
$\dot{}$	: time differential calculus

3. Numerical Calculation

Bubble motion is calculated by means of direct integration of the full equations of mass, momentum and energy conservation both inside and outside the bubble. In solving bubble motion, the following assumptions are employed.

(1) The gas inside the bubble and the surrounding liquid move in spherical symmetry.

(2) The gases inside the bubble obey the perfect gas law.

(3) Noncondensable gas obeys Henry's law at the bubble wall. The concentration of noncondensable gas at the gas-liquid interface has a rate of solution proportional to the partial pressure of the gas.

(4) Mist generated in the rebounding phase is ignored, because the time scale of mist growth is longer than that of bubble motion, and mist has little effect on bubble motion.

3.1 Normalization

All physical quantities are made dimensionless through the following normalization :

$$\begin{aligned}
 t &= \tau t^*, \quad R = R_0 R^*, \quad r = R_0 r^*, \quad U = R_0 / \tau, \\
 \rho_{mg} &= \rho_{mg0} \rho_{mg}^*, \quad \rho_n = \rho_{mg0} \rho_n^*, \quad \rho_v = \rho_{mg0} \rho_v^*, \\
 \rho_{ml} &= \rho_{ml0} \rho_{ml}^*, \quad \rho_l = \rho_{ml0} \rho_l^*, \quad \rho_{gl} = \rho_{ml0} \rho_{gl}^*, \\
 u_{mg} &= U u_{mg}^*, \quad u_n = U u_n^*, \quad u_v = U u_v^*, \\
 u_{ml} &= U u_{ml}^*, \quad u_l = U u_l^*, \quad u_{gl} = U u_{gl}^*, \\
 p_{mg} &= (p_{n0} + p_{v0}) p_{mg}^*, \quad p_{ml} = p_{ml0} p_{ml}^*, \\
 \rho_n e_n &= (\rho_{n0} e_{n0} + \rho_{v0} e_{v0}) \rho_n^* e_n^*, \\
 \rho_v e_v &= (\rho_{n0} e_{n0} + \rho_{v0} e_{v0}) \rho_v^* e_v^*, \\
 T &= T_0 T^*, \quad \dot{M} = \rho_{mg0} U \dot{M}^*, \quad \dot{G} = \rho_{mg0} U \dot{G}^*
 \end{aligned}$$

where $\tau = R_0 \sqrt{\rho_{l\infty} / p_{l0}}$ and is of the same order of magnitude as the time scale of bubble motion. $\dot{M}^*$ is mass flux of vapor at the interface due to evaporation and $\dot{G}^*$ is the mass flux of noncondensable gas through the interface.

3.2 Governing equations and boundary conditions

The governing equations and boundary conditions at the interface for bubble motion are described in detail by Hsieh⁽¹⁴⁾. The equations are nondimensionalized using the above normalization. The governing equations and boundary conditions are written as follows. In the following equations, subscript $i=g$ denotes gas phase and subscript $i=l$ denotes liquid. Also, subscript $j=v$ denotes vapor in the gas phase and $j=gl$ denotes noncondensable gas in the liquid phase.

3.2.1 Governing equations

- The conservation equation of mass for gas mixture and the mixture of liquid and noncondensable gas is

$$\frac{\partial \rho_{mi}^*}{\partial t^*} + \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} (r^{*2} \rho_{mi}^* u_{mi}^*) = 0. \quad (1)$$

- The conservation equation of mass for vapor in gas phase and noncondensable gas in liquid is

$$\begin{aligned} \frac{\partial \rho_j^*}{\partial t^*} + \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} (r^{*2} \rho_j^* u_{mi}^*) \\ = \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} \left(r^{*2} \rho_{mi}^* D_{mi}^* \frac{\partial c_{mi}}{\partial r^*} \right), \end{aligned} \quad (2)$$

where $\rho_{mg}^* = \rho_n^* + \rho_v^*$, $\rho_{mg}^* u_{mg}^* = \rho_n^* u_n^* + \rho_v^* u_v^*$, $\rho_{mi}^* = \rho_l^* + \rho_{gi}^*$, $\rho_{mi}^* u_{mi}^* = \rho_l^* u_l^* + \rho_{gi}^* u_{gi}^*$, $c_v^* = \rho_v^* / \rho_{mg}^*$, $c_{gl}^* = \rho_{gi}^* / \rho_{mi}^*$, $D_{mi}^* = D_{mi} / (R_0 U)$.

- The conservation equation of momentum for both gas and liquid phases is

$$\begin{aligned} \frac{\partial}{\partial t^*} (\rho_{mi}^* u_{mi}^*) + \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} (r^{*2} \rho_{mi}^* u_{mi}^* u_{mi}^*) \\ = - \frac{p_{mi0}}{\rho_{mi0} U} \frac{\partial p_{mg}}{\partial r} \\ + \frac{4}{3} \frac{\mu_{mi}^*}{r^{*2}} \left(\frac{\partial u_{mi}^*}{\partial r^*} - \frac{u_{mi}^*}{r^*} \right) \\ + \frac{4}{3} \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} \left\{ r^{*2} \mu_{mi}^* \left(\frac{\partial u_{mi}^*}{\partial r^*} - \frac{u_{mi}^*}{r^*} \right) \right\}, \end{aligned} \quad (3)$$

where $\mu_{mi}^* = \mu_{mi} / (\rho_{mi0} R_0 U)$.

- The conservation equation of energy for a gas mixture is

$$\begin{aligned} \frac{\partial}{\partial t^*} (\rho_n^* e_n^* + \rho_v^* e_v^*) + \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} \{ r^{*2} (\rho_n^* e_n^* u_n^* \\ + \rho_v^* e_v^* u_v^*) \} = - \frac{\rho_{n0} \mathcal{R}_n + \rho_{v0} \mathcal{R}_v}{\rho_{n0} C_n + \rho_{v0} C_v} \frac{P_{mg}^*}{r^{*2}} \frac{\partial}{\partial r^*} (r^{*2} u_{mg}^*) \\ + \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} \left(r^{*2} \lambda_{mg}^* \frac{\partial T^*}{\partial r^*} \right) \\ + \frac{4}{3} \frac{\mu_{mg} U}{(\rho_{n0} e_{n0} + \rho_{v0} e_{v0}) R_0} \left(\frac{\partial u_{mg}^*}{\partial r^*} - \frac{u_{mg}^*}{r^*} \right)^2, \end{aligned} \quad (4)$$

where $\lambda_{mg}^* = \lambda_{mg} / \{ (\rho_{n0} C_n + \rho_{v0} C_v) R_0 U \}$.

- The conservation equation of energy for liquid is

$$\begin{aligned} \frac{\partial}{\partial t^*} (\rho_l^* T_l^*) + \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} (r^{*2} \rho_l^* e_l^* u_l^*) \\ = - \frac{p_{l0}}{\rho_{l0} C_l T_0} \frac{p_l^*}{r^{*2}} \frac{\partial}{\partial r^*} (r^{*2} u_l^*) \\ + \frac{1}{r^{*2}} \frac{\partial}{\partial r^*} \left(r^{*2} \lambda_l^* \frac{\partial T^*}{\partial r^*} \right) \\ + \frac{4}{3} \frac{\mu_l U}{(\rho_{l0} C_l T_0 R_0)} \left(\frac{\partial u_l^*}{\partial r^*} - \frac{u_l^*}{r^*} \right)^2, \end{aligned} \quad (5)$$

where $\lambda_l^* = \lambda_l / (\rho_{l0} C_l R_0 U)$.

- The equations of state for gas and liquid are

$$p = \rho \mathcal{R} T, \quad \frac{p+B}{p_\infty+B} = \left(\frac{\rho}{\rho_\infty} \right)^n, \quad (6)$$

where $B = 304.9 \text{ MPa}$, $n = 7.15$ in the case where the liquid is water.

3.2.2 Boundary conditions at the gas-liquid interface

- Boundary conditions for mass conservation

At the gas-liquid interface, the mass flux due to phase change $\dot{M}^*$ is estimated by the following equation:

$$\dot{M}^* = \alpha p_{mg0} (p_s^* - p_v^*) / \left(\rho_{mg0} U \sqrt{2\pi \mathcal{R}_v T_0 T^*} \right), \quad (7)$$

where $\alpha = 0.4^{(15)}$. The concentrations and gradients of vapor and noncondensable gas in liquid at the interface are related to $\dot{M}^*$ and $\dot{G}^*$ by the following equations.

$$\dot{M}^* = \frac{D_{mg}^* \rho_{mgw}^* \left(\frac{\partial c_v}{\partial r^*} \right)_w}{1 - c_{vw}} + \frac{c_{vw}}{1 - c_{vw}} \frac{\rho_{m10}}{\rho_{mg0}} \dot{G}^* \quad (8)$$

$$\dot{G}^* = \frac{D_{gl}^* \rho_{m1w}^* \left(\frac{\partial c_{gl}}{\partial r^*} \right)_w}{1 - c_{glw}} + \frac{c_{glw}}{1 - c_{glw}} \frac{\rho_{mg0}}{\rho_{m10}} \dot{M}^* \quad (9)$$

- The boundary condition for momentum conservation is

$$\begin{aligned} - p_{lw}^* + \frac{4}{3} \frac{\mu_l U}{p_{m10} R_0} \left(\frac{\partial u_l^*}{\partial r^*} - \frac{u_l^*}{r^*} \right)_w + \frac{\rho_{mg0}}{\rho_{m10}} \{ \rho_{mg}^* (u_{mg}^* \\ - \dot{R}) (u_{mg}^* - u_l^*) \}_w \\ = - \frac{p_{mg0}}{p_{m10}} p_{mgw}^* - \frac{\sigma}{R_0 p_{m10}} \frac{2}{R^*} \\ - \frac{4}{3} \frac{\mu_{mg} U}{p_{m10} R_0} \left(\frac{\partial u_{mg}^*}{\partial r^*} - \frac{u_{mg}^*}{r^*} \right)_w. \end{aligned} \quad (10)$$

- The boundary condition for energy conservation is

$$\begin{aligned} \left(\frac{T_0 \lambda_l}{\rho_{mg0} R_0 U^3} \frac{\partial T^*}{\partial r^*} \right)_w - \left(\frac{T_0 \lambda_{mg}}{\rho_{mg0} R_0 U^3} \frac{\partial T^*}{\partial r^*} \right)_w \\ + \left(\frac{5}{2} \frac{k T_0}{U^2} \frac{m_v - m_n}{m_v m_n} T_g^* D_{mg}^* \rho_{mg}^* \frac{\partial c_v}{\partial r^*} \right)_w \\ = \dot{M}^* \left\{ L^* + \frac{4}{3} \frac{\mu_l^*}{\rho_{lw}^*} \left(\frac{\partial u_l^*}{\partial r^*} - \frac{u_l^*}{r^*} \right)_w \right. \\ \left. - \frac{4}{3} \frac{\mu_{mg}^*}{\rho_{mgw}^*} \left(\frac{\partial u_{mg}^*}{\partial r^*} - \frac{u_{mg}^*}{r^*} \right)_w \right\} \\ + \frac{1}{2} \dot{M}^* \{ (\dot{R}^* - u_{lw}^*)^2 - (\dot{R}^* - u_{mgw}^*)^2 \}. \end{aligned} \quad (11)$$

where m is mass of one molecule, k is the Boltzmann constant and $L^* = L/U^2$. Temperature and concentration of noncondensable gas in liquid at the wall are

$$T_{lw}^* = T_{gw}^*, \quad \text{and} \quad c_{glw}^* = H_n p_{nw}^*, \quad (12)$$

where H_n is Henry's constant.

3.2.3 Boundary conditions at interface between near field and far field

In this calculation, the liquid phase is divided into two regions. Numerical solution is used in the near field of the bubble and analytical solution is used in the far field of the bubble. The pressures and velocities in the two fields are connected at the interface so that the pressure wave from the bubble is not reflected there. Longer calculations can be performed by this method. The boundary conditions of the pressure and the liquid velocity are derived as follows.

The velocity potential of the wave equation, which is described in terms of spherical coordinates, can be shown as

$$\phi = \frac{F(t + r/C_\infty) + G(t - r/C_\infty)}{r}. \quad (13)$$

Now, only the pressure wave from the bubble exists in this calculation and there is no pressure wave which comes from the far field into the near field. Therefore,

the velocity potential can be written as

$$\phi = \frac{G(t - r/C_\infty)}{r}. \quad (14)$$

Also, velocity and pressure are described as follows using velocity potential.

$$u = \frac{\partial \phi}{\partial r}, \quad \frac{p - p_\infty}{\rho_\infty} = -\frac{\partial \phi}{\partial t} \quad (15)$$

With Eq. (14) and Eq. (15), the velocity potential can be eliminated, and the equation which represents the relationship between the velocity and the pressure at the interface is written as

$$\frac{\partial u_B}{\partial t} = \frac{1}{\rho_\infty C_\infty} \frac{\partial p_B}{\partial t} + \frac{p_B - p_\infty}{\rho_\infty r_B}. \quad (16)$$

This equation gives the boundary conditions of velocity and pressure at the interface. The temperature and the concentration of noncondensable gas are

$$T_B^* = \text{const. and } C_{gIB} = \text{const.} \quad (17)$$

3.3 Numerical procedure

The following transformation of variables is used in order to maintain numerical accuracy.

$$r^* = f(y, t^*), \quad (18)$$

where

$$f(y, t) = \begin{cases} R^* y & (0 \leq y < y_A) \\ C_1 y^2 + C_2 y + C_3 & (y_A \leq y \leq y_B) \end{cases}$$

The boundary conditions are $f(y_A, t^*) = y_A R^*$ and $f'(y_A, t^*) = R^*$ at $y = y_A$, and also $f(y_B, t^*) = R_B^*$ at $y = y_B$. In this calculation y_A , y_B and R_B^* are 10, 15 and 100, respectively.

For the differential method, the forward-time and centered-space scheme is applied. The CIP (cubic interpolated pseudo-particle) method⁽¹⁶⁾ and staggered mesh are applied. The reason for using the CIP method is that it is not only accurate but also simple, and it is convenient for solving many equations such as the present calculation.

The numerical procedure is as follows:

(1) A new radius is calculated using the previous radius and bubble wall speed.

(2) A new grid is set in the calculation domain.

(3) The concentration of noncondensable gas in liquid at the interface, which is determined from Eq. (12) and Eq. (17), is set to be the boundary condition of the conservation equation of noncondensable gas in liquid. The new concentrations of noncondensable gas are determined from Eq. (2) using the above boundary conditions.

(4) Equation (7) is substituted into Eq. (8) and Eq. (9) as $\dot{M}^*$. Then Eq. (9) is substituted into Eq. (8) as $\dot{G}^*$. The concentration and mass flux of vapor at the interface is calculated using the new concentration of noncondensable gas from process (3) and the previous temperature. The conservation equations for energy and vapor inside the bubble are solved using this mass flux, and the new temperature and concen-

tration of vapor are calculated iteratively until the temperature converges.

(5) The new density is calculated using the conservation equations for gas mixture and liquid and the boundary condition shown in Eq. (10). The new pressure is estimated using the temperature calculated in process (4), the concentration of vapor and the equation of state.

(6) The conservation equation for momentum is solved using the pressure from process (5) and new radial velocities inside and outside the bubble and the bubble wall speed are calculated.

(7) Return to process (1).

In this calculation bubble motion is simulated for surrounding pressure increasing stepwise from the equilibrium state in low pressure. The bubble radius, liquid pressure, temperature in the equilibrium state and liquid pressure after the pressure increase are given as the initial conditions. In the equilibrium state, the pressure and the temperature in the gas and liquid phases are uniform. The initial liquid pressure is 10 kPa and the temperature is 20°C except when otherwise noted. In this condition the gas pressure is higher than liquid one due to surface tension. The liquid pressure after pressure increase is 100 kPa. Heat conductivity, viscosity and the diffusion coefficient depend on the pressure and temperature, hence their changes must be considered in this calculation⁽¹⁷⁾.

4. Results and Discussion

The behavior of a bubble filled with vapor and noncondensable gas, which exists in stationary infinite water, is simulated for surrounding pressure increasing stepwise.

4.1 Uncertainty analysis of numerical solution

The numerical solution is not completely accurate because the basic equations are solved by means of differential method. Thus it is important to estimate the precision of the numerical solution.

The present numerical solution for bubble motion in compressible and inviscid liquid is compared with the solution calculated by means of fourth-order Runge-Cutta method using the first-order approximate equation⁽¹⁸⁾ with respect to the compressibility. The result reveals that the present method has enough accuracy.

The precision of the numerical solution depends on the grid size. The effect of the grid size is estimated for the following conditions: the grid sizes inside and outside the bubble are (A) 0.02 and 0.04, (B) 0.01 and 0.02 and (C) 0.005 and 0.01. Time steps in all cases are determined for the Courant number of 0.2. Figure 1 shows the time histories of the radii and the pressures at the bubble center for the above three cases, where

only the heat conduction inside and outside the bubble is considered. The initial radius is 100 μm long. It is shown that the three results of (A), (B) and (C) almost coincide. In particular, there is little difference between (B) and (C). In this calculation, the grid size of case (B) is selected.

The next problem is the time step. When many equations are solved at the same time, as in this calculation, the time scale of each phenomenon must be investigated. The time step in the calculation is determined by the speed of sound; the cases where the Courant number in liquid is 0.4, 0.2 and 0.1 are examined. The results almost coincide. It is confirmed that the time step can be determined from the speed of

sound, and the Courant number in this calculation is selected as 0.2.

4.2 Effect of thermal diffusion on bubble motion

The effect of thermal diffusion on bubble motion is investigated in the case of a gas bubble which contains only noncondensable gas. In this case, the governing equations and the boundary conditions are simplified, and the nondimensional heat conductivity is $\lambda_{mg}/(\rho_{n0}C_nR_0U)$. One of the authors⁽¹²⁾ made calculations for the case of nitrogen gas, whose heat conductivity is relatively low, and for that of hydrogen gas, whose heat conductivity is relatively high, and discussed the difference in temperature distribution inside the bubble and its influence on the bubble motion. He showed that the motion of a hydrogen gas bubble is close to that of an isothermal bubble because the heat transfer at the bubble wall becomes higher due to the high conductivity. When the initial radius is large, the motion approaches that of an adiabatic bubble, and when the radius is small, the motion approaches that of an isothermal one.

The influence of the initial bubble radius, which is the parameter of nondimensional heat conductivity, on the bubble motion is investigated. Figure 2 shows the time histories of the radius, pressure at the center, mean temperature inside the bubble and dimensionless heat input from the liquid phase into the gas phase in the cases where the initial radii are 1 000 μm , 100 μm and 10 μm . The calculation phase is continued to 6τ . Mean temperature is defined as follows.

$$T_{\text{mean}} = \int_0^1 y^2 \rho_g T_g dy / \int_0^1 y^2 \rho_g dy \quad (19)$$

In this calculation, the mean temperature is proportional to the total internal energy because the mass of gas is constant. The heat input is calculated as

$$q^* = 4\pi\lambda_n^* R^{*2} \frac{\partial T^*}{\partial r^*} \bigg|_{r^*=R^*} \quad (20)$$

Figure 2 shows that the pressure at the center synchronizes with the bubble motion regardless of the initial radius. When the bubble radius takes a maximum or minimum value, the pressure takes a minimum or maximum value, respectively. The mean temperature takes a maximum at almost the same time as when the bubble radius takes the minimum. However it does not take a minimum when the bubble radius takes the maximum. This is because the heat input $\dot{q}^*$ takes a positive or negative values contrary to the work received from the liquid phase, and has a time lag. Therefore the sum of the work which the bubble receives per unit time from the liquid phase and the heat input $\dot{q}^*$ becomes zero before the radius takes the minimum or the maximum value. This is the minimum or the maximum point of the temperature. Upon collapse, the work received from the liquid per

Fig. 1 The precision of numerical solution versus the grid size

Fig. 2 Time histories of radius, pressure at the bubble center, mean temperature and heat input from the liquid phase

unit time decreases so rapidly that the temperature takes the maximum at almost the same time as the radius takes the minimum. On the other hand, upon expansion, the bubble supplies work to the liquid phase and the heat flows outward from the gas phase due to the temperature distribution developed in the collapsing phase. Therefore the temperature decreases rapidly and the gradient of the temperature at the gas-liquid interface become positive. Subsequently, the heat flows inward from the liquid phase and the mean temperature takes the minimum value when the sum of the work received from the liquid per unit time and the heat input becomes equal to zero. The maximum temperature inside the bubble depends on the size of the initial radius. The gradient of temperature at the interface becomes positive early since the initial radius is small. Hence, the sooner the temperature reaches the minimum value, the smaller the initial radius.

Heat input q^* has a time lag to the change of the work which the bubble receives from the liquid phase, as in the above discussion. Figure 3 shows decay of the bubble volume and total internal energy at every point at which the bubble radius takes the maximum. The upper and lower parts show the changes of the bubble volume and total energy, respectively, in the cases where the initial radii are 1 000, 100 and 10 μm . The ordinate of the upper part shows dimensionless volume normalized by the initial volume of the bubble, and that of the lower part, dimensionless total internal energy normalized by the initial internal energy. The abscissa shows the periods. Squares, circles and triangles show the cases where the initial radius is 1 000

Fig. 3 The changes in the maximum volume and total internal energy of bubble

μm , 100 μm or 10 μm , respectively. The figure shows that the bubble size decays in a short time as the initial radius is small. It also shows that when the bubble radius reaches the maximum, the total energy becomes less than the initial energy. This is because the heat input q^* has a time lag to the work received from the liquid phase per unit time.

4.3 Effect of diffusion on bubble motion

4.3.1 Time histories of concentration distribution and mass of vapor inside the bubble Figures 4 and 5 show the time histories of concentration distribution of vapor inside the bubble in the cases where the initial radius is 1000 μm and 10 μm .

When the bubble collapses, partial pressure of vapor inside the bubble increases. Then vapor condenses onto the interface, and the concentration of vapor near the interface decreases because the satur-

Fig. 4 Time history of concentration of vapor distribution (initial radius: 1000 μm)

Fig. 5 Time history of concentration of vapor distribution (initial radius: 10 μm)

ated pressure at the interface is kept almost constant. The vapor flows from the bubble center to the interface by diffusion, and the boundary layer of concentration develops. After rebound of the bubble, evaporation occurs at the interface and the concentration of vapor increases because the partial pressure of vapor near the interface decreases. However, the dimensionless diffusion coefficient, $D_{mg}/R_0 U$, is inversely proportional to the initial radius. The characteristic time for the boundary layer to develop depends greatly on the initial radius, so that the time history of concentration at the center becomes very different depending on the initial radius. In the case where the initial radius is $1\,000\,\mu\text{m}$, the boundary layer does not develop to the center when the radius reaches the minimum value. During the rebounding phase the concentration at the center decreases because of the negative gradient of concentration, even though the concentration at the interface increases. Thus a complicated distribution is formed. On the other hand, in the case where the initial radius is $10\,\mu\text{m}$, the difference between the concentration at the center and that at the interface is much less because the dimensionless diffusion coefficient is larger. After the rebound, the concentration at the center increases quickly due to the development of the boundary layer made formed increment in the concentration of vapor at the interface, though it decreases slightly due to the negative gradient of concentration. Therefore, the distribution of concentration does not become so complicated when the initial radius is $10\,\mu\text{m}$. The dimensionless diffusion coefficient effects the changes in the mass of vapor inside the bubble. Figure 6 shows the time histories of

Fig. 6 Time history of vapor mass inside the bubble

the dimensionless radius and mass of vapor inside the bubble when the initial radius is $10\,\mu\text{m}$, $100\,\mu\text{m}$ and $1\,000\,\mu\text{m}$. The abscissa shows nondimensional time. The ordinate shows the dimensionless radius and mass of vapor normalized by initial mass, where the upper part shows the radius and the lower part shows the mass of vapor. The mass of vapor increases due to evaporation at the interface after the rebound. However, the time at which the mass of vapor starts to increase is later than the time at which the radius reaches the minimum. When the radius reaches minimum, the saturated vapor pressure at the interface is less than the vapor pressure near the bubble wall. Therefore, even if the bubble expands, vapor condenses onto the bubble wall until the vapor pressure near the bubble wall exceeds the saturated vapor pressure. The changes in mass of vapor cause a time lag in bubble motion. The bigger the initial radius, the longer the time lag. This is because the relaxation time becomes longer as the dimensionless diffusion coefficient becomes smaller. When the initial radius is small, the mass of vapor decreases rapidly.

4.3.2 The effect of diffusion in the case of different initial radii The present model is compared with a model in which phase change at the interface is ignored, and with another model in which the diffusion coefficient is much greater than the normal value, and the effect of different initial radii on the bubble motion is investigated. The diffusion coefficient in the latter model is 1 000 times the value of the initial radius. The models are named as follows. CASE A: present model

CASE B: model in which the diffusion coefficient between vapor and noncondensable gas is much greater than the normal value

CASE C: no diffusion between vapor and noncondensable gas and no phase change at the interface

In CASE B, vapor diffuses quickly when the vapor evaporates or condenses at the bubble wall because the diffusion coefficient is large. Thus concentration is uniform and vapor pressure is kept almost constant, except near the minimum point of the radius. CASE B is almost equivalent to the model in which the vapor pressure is constant.

Figure 7 shows the minimum radius and maximum radius after the rebound versus the initial radius. The ordinate shows the dimensionless minimum and maximum radii from the top, and the abscissa shows the initial radius. Solid, dash-dotted and broken lines show CASE A, CASE B and CASE C, respectively. It is shown in Fig. 7 that the present model is located between CASE B and CASE C in terms of the bubble collapse. One of the present authors⁽¹⁰⁾ described that

Fig. 7 Minimum radius and maximum radius after the rebound

noncondensable gas suppresses the phase change at the gas-liquid interface because diffusion between vapor and noncondensable gas has limited velocity. Expressed in another way, in the case where the vapor condenses onto the interface, the concentration of vapor at the interface decreases because it is difficult for the vapor to diffuse from the bubble center to the bubble wall. Hence, the condensation rate decreases. On the other hand, in the case where the vapor evaporates at the interface, the concentration of vapor increases rapidly because it is difficult for vapor to flow toward the bubble center. Therefore, the evaporation rate also decreases. The figure also shows that when the initial radius is small, bubble motion is similar to that in CASE B. As the initial radius becomes large, the motion is similar to that in CASE C. This is because the dimensionless diffusion coefficient is a function of the initial radius. In the case where the dimensionless diffusion coefficient is small, the mass transfer of vapor is so slow that the rate of phase change decreases rapidly. In this case, vapor behaves as noncondensable gas and the motion is close to the motion of CASE C. On the other hand, the vapor diffuses rapidly inside the bubble in the case where the dimensionless diffusion coefficient is large. The vapor pressure does not increase as much as the vapor pressure in the case of no phase change, so that the motion is close to that in CASE B. the difference in the nondimensional diffusion coefficient can be regarded as equivalent to that in initial pressurization ratio.

The relationship between the maximum radius after the rebound and the initial radius is somewhat

complicated. When the initial radius is large, CASE A is located between CASE B and CASE C. However, when the initial radius is smaller than 100 μm , there is little difference between CASE A and CASE B. The effects of phase change at the interface on the decay of bubble motion are to decrease the mass of vapor inside the bubble and increase the pressurization ratio due to the phase change and the diffusion. These effects increase the decay rate of bubble motion. Therefore, the decay in CASE A is larger than that in CASE C.

5. Conclusions

Bubble motion is simulated numerically using the full conservation equations of mass, momentum, and energy for the case of both gas and liquid phases when the surrounding pressure increases stepwise. The following conclusions are obtained.

- (1) Bubble motion is influenced by heat transfer inside and outside the bubble. Dimensionless heat conductivity is inversely proportional to the initial radius. The mean temperature inside the bubble and decaying time of bubble motion depend on the size of initial radius.
- (2) Concentration distribution of vapor is formed inside the bubble due to phase change and diffusion between vapor and noncondensable gas. The dimensionless diffusion coefficient is also inversely proportional to the initial radius. Concentration distribution and mass of vapor depend on the size of the initial radius.
- (3) The difference in the nondimensional diffusion coefficients is equivalent to that in initial pressurization ratios. In the case where the dimensionless diffusion coefficient is small, vapor behaves as a noncondensable gas because it is difficult for vapor to diffuse quickly inside the bubble. On the other hand, in the case where the coefficient is large, vapor pressure inside the bubble does not vary much and the motion of the bubble is close to the motion of a gas bubble without vapor.

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