Diagrammatic Representation of Models for the Burning Velocity and Flame Structure of Premixed Turbulent Flames*

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A method was proposed to represent the model-predicted burning velocities and flame structure parameters diagrammatically in three coordinates (Re–Da, \( \eta_k/\eta_u \square' /S_{10} \) and \( L/\rho_u \square' /S_{10} \)) so far used to illustrate turbulent flame structure phase diagrams; this method was applied to three models by different researchers. By showing the same model in different planes, the model responses to turbulence characteristics and mixture properties were elucidated from different viewpoints, and by showing the turbulent burning velocities and flame structure parameters by the same model in the same plane, their respective relationships implied by the model were clarified.

**Key Words**: Combustion, Premixed Turbulent Flame, Phase Diagram, Diagrammatic Representation, Model, Flame Structure, Burning Velocity

1. Introduction

Turbulent combustion models are usually described by a series of equations. In some cases, predictions of the \( S_1/S_{10} \) ratios for some specified mixtures in the same turbulence field shown as curves in a \( \square'/S_{10} - S_1/S_{10} \) plane are also included to provide a direct view of the model. However, such a method of describing a model cannot provide a thorough view of the model because of the complexity of the equations and deficiency of the \( \square'/S_{10} - S_1/S_{10} \) plane to show the responses of the model predictions to variables other than the \( \square'/S_{10} \) ratios, such as various length scales of turbulence or flames.

On the other hand, in the study of premixed turbulent flame structure, Bray[1], Borghi[2], Abraham et al.[3], Williams[4] and Peters[5] have shown different combustion regimes in the Re–Da, \( \eta_k/\eta_u \square' /S_{10} \) and \( L/\rho_u \square' /S_{10} \) planes or other similar ones. As these planes illustrate the flame structure in phase, they are also referred to as flame structure phase diagrams[6]. As all of the above planes have two coordinate axes representing, simultaneously, both the turbulence characteristics and the mixture properties, which are the two factors dominating the turbulent combustion, responses of the flame structure to these two dominating factors can be qualitatively understood from such phase-diagrams. The purpose of this work is to plot both the turbulent burning velocities and flame structure parameters predicted by models in the above three planes in the form of contours to clarify (1) the responses of the model predictions to the two dominating factors and (2) the relationships between the turbulent burning velocity and flame structure by plotting them in the same plane. For this purpose, the relationships existing among the three planes Re–Da, \( \eta_k/\eta_u \square' /S_{10} \) and \( L/\rho_u \square' /S_{10} \) were clarified first. On the basis of the relationships obtained, the predictions of a model proposed previously by the authors[6] were equivalently shown in the above three planes to validate such a method. A plane showing the contours of turbulent burning velocities predicted by a model was designated as a turbulent burning velocity diagram of the model, or simply, a burning velocity diagram of

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the model, in this work. Similarly, a plane showing those of flame structure parameters was referred to as a flame structure diagram of the model.

The method of plotting the model predictions in the form of diagrams has many benefits, and it applies to most models proposed so far. In order to show this, two other models one by Gouldin[77] and the other by Ballal and Lefebvre[80], were taken for examples. Based on their respective diagrams, the responses of the two models to turbulence characteristics and mixture properties were briefly discussed.

**Nomenclature**

\[ A_0 : \text{Constant in Eq. (1), } A_0 = 0.37 \]
\[ A_1 : \text{Constant in Eq. (2), } A_1 = 11 \]
\[ D_3 : \text{Fractal dimension of flame surfaces} \]
\[ C : \text{Constant in the RIFF model, } C = 4.8 \]
\[ C_1 : \text{Constant in the RIFF model, } C_1 = 0.5 \]
\[ D_a : \text{Damkohler number, } D_a = L \eta_0 / (u' \eta_0) \]
\[ K_a : \text{Karlovitz number, } K_a = (u' \lambda_a) (\eta_0 / L) \]
\[ L : \text{Longitudinal integral length scale of turbulence} \]
\[ Pr : \text{Prandtl number of unburned mixture} \]
\[ Re : \text{Reynolds number, } Re = u' L / \nu \]
\[ S_{lo} : \text{Flame velocity of unstrained laminar flame} \]
\[ S_T : \text{Turbulent burning velocity} \]
\[ u' : \text{Turbulence intensity} \]
\[ v_k : \text{Kolmogorov velocity scale, } v_k = \nu / \eta_k \]
\[ \delta_{lo} : \text{Reaction zone thickness of unstrained laminar flame} \]
\[ \varepsilon : \text{Dissipation rate of turbulence energy} \]
\[ \varepsilon_i : \text{Inner cutoff of fractal flame surfaces} \]
\[ \varepsilon_o : \text{Outer cutoff of fractal flame surfaces} \]
\[ \eta : \text{Preheat zone thickness of flamelets in a turbulent flame} \]
\[ \eta_0 : \text{Preheat zone thickness of unstrained laminar flame; } \eta_0 = \nu / (Pr S_{lo}) \]
\[ \eta_k : \text{Kolmogorov length scale, } \eta_k = (\nu / \epsilon)^{0.25} \]
\[ \lambda_a : \text{Taylor's microscale} \]
\[ \nu : \text{Kinematic viscosity of unburned mixture} \]
\[ \xi : \text{Ratio of reactants burned by the islandlike flamelets to that by the flame front} \]

where \( A_0 \) is an undetermined constant of order one and may take different values for different turbulence fields[89]. Based on the turbulence properties at the centerline of in-pipe flows, Abdel-Gayed et al.[100] recommended \( A_0 = 0.37 \). Kido et al.[11] measured a nearly isotropic turbulence field and recommended that

\[
\varepsilon = A_1 \left( \frac{u'}{L} \right) \left( \frac{\eta_k}{L} \right) 
\]

with \( A_1 \) being a constant of about 11 according to Ref. (12).

Either Eq. (1) or Eq. (2) can be adopted in the transformation of the three coordinate planes, but unless specified, the following equations were based on Eq. (2) to maintain consistency with our previous work[89].

To begin with, the Re-Da plane can be transformed to the other two planes on the basis of Eq. (2) and the definitions of Re, Da and \( \eta_0 \):

\[
u' / S_{lo} = (PrRe / Da)^{0.6} ;
\]

\[
L / \eta_0 = (PrPeDa)^{0.6} ;
\]

\[
\eta_k / \eta_0 = (PrDa)^{0.5} \left( A_1^3 / Re \right)^{0.5}.
\]

By taking \( u' / S_{lo} \) and \( \eta_k / \eta_0 \) as variables, Eqs. (3) to (5) can be solved for Re, Da and \( L / \eta_0 \) so as to transform the \( \eta_k / \eta_0 - u' / S_{lo} \) plane to the other two planes.

\[
Re = A_1^3 \left( \frac{1}{Pr} \right) \left( \frac{u'}{S_{lo}} \right) \left( \frac{\eta_k}{\eta_0} \right)^{2.5} ;
\]

\[
Da = A_1^{1.5} \left( \frac{u'}{S_{lo}} \right)^{0.5} \left( \frac{\eta_k}{\eta_0} \right)^{1.5} ;
\]

\[
L / \eta_0 = A_1^{0.5} \left( \frac{u'}{S_{lo}} \right)^{1.5} \left( \frac{\eta_k}{\eta_0} \right)^{1.5}.
\]

Moreover, treating \( L / \eta_0 \) as a variable in Eq. (8) instead of \( \eta_k / \eta_0 \), one can obtain

\[
Re = \frac{1}{Pr} \left( \frac{u'}{S_{lo}} \right) \left( \frac{L}{\eta_0} \right) ;
\]

\[
Da = \frac{L}{\eta_0} \left( \frac{u'}{S_{lo}} \right) \left( \frac{L}{\eta_0} \right)^{0.4} \left( \frac{u'}{S_{lo}} \right)^{0.6} .
\]

Thus, through Eqs. (3) to (11), the three planes can be transformed from one to another for a given Prandtl number Pr. The relations of the three planes are shown in Fig. 1 with Pr being given as 0.7. With Eq. (1), one can obtain relationships similar to Eqs. (3) to (11). Such relationships are given in Fig. 2 for reference. It is worth mentioning that for models adopting Eq. (2), their burning velocity diagrams must be compared with Fig. 1 to clarify the relationship between the turbulent burning velocity and flame structure, or with Fig. 2 for models adopting Eq. (1).

2. Relationships Between Coordinates of Diagrams

There are three coordinate planes typically used to show the flame structure phase diagrams; these are the Re-Da, \( \eta_k / \eta_0 - u' / S_{lo} \) and \( L / \eta_0 - u' / S_{lo} \) planes. These planes can be transformed from one to another as follows on the basis of Kolmogorov's energy cascade argument.

According to Kolmogorov's energy cascade argument[90], the turbulence energy dissipation rate \( \varepsilon \) is estimated by

\[
\varepsilon = A_0 u^3 / L
\]
Fig. 1 Relationships of the Re–Da, $\eta_u/\eta_f$, $u'/S_{10}$ and $L/\eta_f$–$u'/S_{10}$ coordinate planes with $\epsilon = A_1(u^3/L)(\eta_u/\eta_f)$

Fig. 2 Relationships of the Re–Da, $\eta_u/\eta_f$, $u'/S_{10}$ and $L/\eta_f$–$u'/S_{10}$ coordinate planes with $\epsilon = A_0(u^3/L)$
3. Diagrams of the RIFF Model

3.1 Description of the RIFF model

In order to show the method of plotting the model-predicted turbulent burning velocities and flame structure parameters in the above-mentioned planes as diagrams, we took the authors' proposed RIFF model[6] for example.

On the basis of recent experimental results by the Rayleigh scattering technique[12], schlieren photography[14],[15], laser-sheet tomography[16],[17] and the electrostatic probe method[18], the authors proposed a model for the flame structure and burning velocity of premixed turbulent combustion in a previous work[6], making an attempt to describe the premixed combustion from laminar combustion where the turbulent intensity approaches zero, to high turbulent combustion where flamelets are extinguished. As the model assumes the flamelets to have fractal properties and divides the flamelets into two parts: a continuous flame front and some reactant islands behind the flame front, it is referred here to as the RIFF model for convenience, which stands for reactant islands and fractal flamelets. The validity of the model has been confirmed in Ref. (6) by the good agreement of its predictions of turbulent burning velocities for fifteen mixtures with the ratio \( u'/S_{10} \) varying up to about 10.

According to Ref. (6), for stoichiometric hydrocarbon mixtures, the reduced turbulent burning velocity \( S_{1}/S_{10} \) in this model is calculated by

\[
\frac{S_{1}}{S_{10}} = (1 - Ka) \left( \frac{\xi_{f}}{\xi_{l}} \right)^{1 + \xi_{f}} (1 + \xi),
\]

where \( \xi \) is a parameter characterizing the global flame structure feature and represents the ratio of the combustion fraction by the islandlike flamelets (behind the flame front) to that by the continuous flame front. Therefore, the larger the value of \( \xi \), the more the islandlike flamelets exist in the turbulent flame zone. \( D_{b}, \xi_{b} \) and \( \xi_{l} \) are the fractal dimension, outer and inner cutoffs of the flame surfaces, respectively. \( \xi, \xi_{b}/\xi_{l} \) and \( D_{b} \) are respectively calculated as follows:

\[
\xi = 4 \left( \frac{\delta_{b}}{\delta_{l}} \right)^{2} \left( \frac{\delta_{b}}{\delta_{l}} \right)^{1 + \frac{1}{6} - \frac{1}{2} (1 - Ka) \exp\left(C S_{10}/U'\right)} \right),
\]

\[
\xi_{b} = \left( \frac{L}{\eta} \right) \left( \frac{\eta}{\eta_{k}} \right)^{\xi_{b}/\xi_{l}} + 1,
\]

\[
D_{b} = 2 - 1.53 \exp\left(\ln\left(1 + u'/S_{10}\right)\right),
\]

The \( \delta_{b} \) in Eq. (13) is the reaction zone thickness of laminar flames. As the ratio of \( \delta_{b}/\delta_{l} \) ranges from 4 to 5 approximately for hydrocarbon flames[6], it was given a constant of 4.5 in the present work. The \( \eta \) in Eq. (14) is the preheat zone thickness of flamelets in turbulent flames and is calculated by

\[
\eta = \eta_{k}(1 + C_{v} U/\delta_{l}).
\]

In Eq. (13) and Eq. (16) are two model constants, determined to be 4.8 and 0.5, respectively, in Ref. (6).

As the expressions of the ratios \( u'/S_{10} \) and \( L/\eta_{k} \) in the Re-Da, \( \eta_{k}/u'/u'/S_{10} \) and \( L/\eta_{k} - u'/S_{10} \) coordinate systems are available in section 2, in order to calculate the values of \( S_{1}/S_{10} \) and \( \xi \), only the expressions for \( K, \eta_{k}/u' \) and \( L/\eta \) in each coordinate system are needed.

3.2 Diagrams in the Re-Da plane

According to the definition of \( K \) and Eqs. (3) to (5), \( K, \eta_{k}/u' \), \( L/\eta \) in the Re-Da plane can be calculated by the following equations:

\[
K = \frac{A_{b} R_{e}^{A_{b}}}{(\sqrt{15} D_{a})^{0.5}}.
\]

\[
L = \frac{1 + C_{l} A_{b} R_{e}^{A_{b}} (P_{r} D_{a})^{0.5}}{\eta},
\]

\[
\eta_{k} = \frac{L}{\eta} \left( \frac{1}{\eta} \right) A_{b} R_{e}^{A_{b}}.
\]

For hydrocarbon mixtures, \( P_{r} \) ranges from 0.6 to 0.8 approximately, being given a constant of 0.7 in this work. Therefore, given the values of Re and Da, the values of \( S_{1}/S_{10} \) and \( \xi \) can be calculated. The obtained results are shown in Fig. 3, where two lines designated as \( (S_{1}/S_{10})_{max} \) and \( \xi_{max} \) respectively, indicate where the maximum \( S_{1}/S_{10} \) and maximum \( \xi \) are reached for given Reynolds numbers.

3.3 Diagrams in the \( \eta_{k}/u' - u'/S_{10} \) and \( L/\eta \) - \( u'/S_{10} \) planes

By substituting Eqs. (6) and (7) into Eqs. (17) to (19), the expressions for \( K, L/\eta \) and \( \eta_{k}/u' \) in the \( \eta_{k}/u' - u'/S_{10} \) coordinate system were obtained as follows:

\[
K = \frac{P_{t}}{\sqrt{15} \left( \frac{y_{b}}{\eta} \right)^{2}}.
\]

\[
L = \frac{A_{b}^{2} \left( u'/S_{10} \right)^{1.5} \left( \eta_{k}/u' \right)^{2.5}}{\eta^{2} \left( 1 + C_{l} P_{r} / \left( \eta_{k}/u' \right) \right)}.
\]

\[
\eta_{k} = \frac{\left( \eta_{k}/u' \right)^{2}}{\eta_{k}/u' + C_{l} P_{r}}.
\]

The diagrams plotted in the \( \eta_{k}/u' - u'/S_{10} \) plane are given in Fig. 4, where the \( (S_{1}/S_{10})_{max} \) and \( \xi_{max} \) lines have the same meanings as those in Fig. 3.

Moreover, by substituting Eqs. (9) and (10) into Eqs. (17) to (19), one can obtain the following equations to plot the diagrams in the \( L/\eta - u'/S_{10} \) plane:

\[
K = \frac{A_{b}^{2} \left( u'/S_{10} \right)^{1.5}}{\sqrt{15} P_{r}^{0.5} \left( L/\eta \right)^{0.5}}.
\]

\[
L = \frac{A_{b}^{2} \left( u'/S_{10} \right)^{1.5}}{\left( L/\eta \right)^{0.5} + C_{l} P_{r} \left( u'/S_{10} \right)^{0.5}}.
\]

\[
\eta_{k} = \frac{A_{b}^{2} \left( u'/S_{10} \right)^{1.5}}{\left( L/\eta \right)^{0.5} + C_{l} P_{r} \left( u'/S_{10} \right)^{0.5}}.
\]

Figure 5 shows the diagrams thus obtained. Again, the \( (S_{1}/S_{10})_{max} \) and \( \xi_{max} \) lines in Fig. 5 are the same as in
It is worth mentioning that in Figs. 3 to 5 the ratio of $\delta_{0}/\eta_{b}$ and the Prandtl number $Pr$ have been taken to be constant, being 4.5 and 0.7, respectively. For mixtures whose $\delta_{0}/\eta_{b}$ ratio and Prandtl number $Pr$ are substantially different from such values, the diagrams can be modified according to their values. Nevertheless, as both the $\delta_{0}/\eta_{b}$ ratio and the Prandtl number $Pr$ for hydrocarbon/air mixtures are restricted within very narrow ranges, it is thought that the overall characteristics of the diagrams would not be changed too much even if the diagrams were modified.

Fig. 3 Burning velocity and flame structure diagrams of the RIFF model in the Re-Da coordinate plane.

(b) Flame structure parameter

Fig. 4 Burning velocity and flame structure diagrams of the RIFF model in the $\eta_{b}/\eta_{o}$-$u'/S_{o}$ coordinate plane.

4. Discussion

4.1 Benefits of showing the model predictions as diagrams

As shown in Figs. 3 to 5, representing a model in the form of its burning velocity and flame structure diagrams can reveal many characteristics of the model which can hardly be obtained by other methods. Here we only point out the main benefits of showing the model predictions in the form of diagrams as follows:

First, the diagrams can exhibit the responses of the model predictions (including the $S_{l}/S_{o}$ ratio and any other concerned flame structure parameters predicted) to both the turbulence characteristics and
the mixture properties simultaneously. Usually, the model-predicted turbulent burning velocities are shown in an $S_T/u'$ or $S_{T/Lo}/u'/S_{Lo}$ plane with each curve representing a specific mixture burned in a certain turbulence field. Since the response of $S_T$ to $u'$ (or $S_{T/Lo}$ to $u'/S_{Lo}$) changes from one to another turbulence field or mixture, the curves for certain mixtures are no longer valid for other mixtures with different properties even if they are burned in the same turbulence field, and neither are such curves for other turbulence fields with different characteristics even if the mixtures are the same. Compared with the curves shown in an $S_T/u'$ plane, a burning velocity diagram applies not only to the same mixture in different turbulence fields, but also to different mixtures in the same turbulence field, and even to the case where both the mixture properties and turbulence characteristics are changed. In other words, a burning velocity or flame structure diagram can show the responses of the model predictions to the mixture properties and the turbulence characteristics simultaneously.

Second, a diagram can provide the overall characteristics of the model in addition to the detailed predictions. For instance, Figs. 3 to 5 provide not only the detailed turbulent burning velocities but also the flame structure parameters from nearly laminar combustion (where $S_{T/Lo}$ ≈ 1 and $\xi$ ≈ 0) to combustion with turbulence so high as to quench the flames. Moreover, the predicted maximum burning velocity positions, flame quenching limit, etc., can also be shown in the diagrams. Therefore, one can understand the overall characteristics of the model from its diagrams.

Third, the diagrams of a model can reveal the model-implied relationship between the turbulent burning velocity and the flame structure. In most turbulent burning velocity models proposed so far, flame structure has rarely been explicitly described. Even in such models, the turbulent burning velocity is somehow related to the flame structure, which can be revealed by superimposing the turbulent burning velocity diagrams of the concerned models onto the flame structure phase diagrams shown in Fig. 1 or Fig. 2 according to the adopted assumption on energy dissipation rate (Eq. (1) or Eq. (2)). This will be shown in section 5 by taking Gouldin's model for example. In addition, for the RIFF model, one can also superimpose the diagrams of the $\xi$ parameter (the so-called flame structure diagrams) onto the corresponding burning velocity diagrams to clarify the relationships between $S_{T/Lo}$ and $\xi$.

Fourth, the diagrams of a model can show various conditions to which the model is supposed to apply. Some models are constructed so that for different combustion conditions (or combustion regimes) the expressions for the turbulent burning velocity $S_{T/Lo}$ take different forms; such conditions can be readily included in the diagrams. An example on this point will be given in section 5 by plotting the predictions of the 3-region model by Ballal and Lefebvre as diagrams.

4.2 Analysis of the diagrams

4.2.1 Comparison of the flame structure diagrams with experimental results by authors

In order to test the flame structure predicted by the RIFF model and to identify various combustion regimes shown by Abraham et al., and Peters and other researchers, a passive electrostatic probe of
0.6 mm length and 0.2 mm diameter was used to investigate the structure of premixed turbulent flames in a constant volume combustion chamber by directing the probe to the outwardly propagating turbulent flame front. When the turbulence was intensified to some extent, the obtained flame potential signals\(^{(18)}\) (similar to ion current signals, each spike in the signals exhibiting once a flamelet passing through the probe) showed multiple spikes. Truly multiple spikes in a signal can be explained as a single flamelet passing through the probe repeatedly, but this seems unlikely because the flame propagation speeds (15 m/s to 40 m/s) were much higher than the turbulence intensities (below 3.8 m/s) in all experiments carried out. Therefore, different spikes in a signal were identified with different flamelets passing through the probe. The obtained results are shown in Fig. 6 in comparison with the flame structure parameter \(\xi\) predicted by the RIFF model. The lines indicated by “M 1”, “M 2”, etc., in Fig. 6 represent different mixtures. The state of a turbulent mixture varies from the upper left to the lower right as the turbulence is intensified. In the regime from the upper left to the “○” marks, the detected flame potential signals showed only one spike in each signal. In the regime from the “○” marks to the “●” marks, each signal showed one or two spikes, and below the “●” marks, individual signals having three or more spikes were observed. The “●” marks were considered to be the boundary separating the wrinkled laminar flame regime and reactant island flame regime. As mentioned above, the larger the value of \(\xi\), the more the flamelets exist in a turbulent flame. From Fig. 6 one can find that the number of flamelets obviously depends on the \(\xi\) parameter (also on the \(u’/S_{1,0}\) ratio, though), increasing when the latter is increased. Such a result is considered to support the model-predicted flame structure.

For the flame quenching, the “○” marks in Fig. 6 are the experiment points where 50% of the investigated flames failed to propagate after ignition (the remaining flames were quenched when the turbulence was further intensified to some extent) when the ignition energy was 0.8 J (much higher than that for gasoline engines). Figure 6 shows such points being located in the regime from the \((S_{T}/S_{1,0})_{\text{max}}\) curve to the \(K_{r}=1\) curve. This fact is explained as follows: The model-predicted constant \(S_{T}/S_{1,0}\) or constant \(\xi\) curves are heavily and increasingly concentrated, in the regime from the \((S_{T}/S_{1,0})_{\text{max}}\) curve to the \(K_{r}=1\) curve (coinciding with the \(S_{T}/S_{1,0}=0\) curve), which means that a small variation in the combustion conditions, say, a fluctuation in the turbulence intensity, would bring about substantial changes in the flame structure and the turbulent burning velocity. Therefore, combustion in this regime is unsteady, which can be attributed to local quenching of flamelets. Thus, the fact that the predicted constant \(S_{T}/S_{1,0}\) and constant \(\xi\) curves are increasingly concentrated from the \((S_{T}/S_{1,0})_{\text{max}}\) curve to the \(K_{r}=1\) curve is interpreted as indicating that the flames become more and more easily quenched from the \((S_{T}/S_{1,0})_{\text{max}}\) curve to the \(K_{r}=1\) curve, and up to the \(K_{r}=1\) curve, all flames are quenched. With this interpretation, we concluded that the experimental result supports the model prediction on flame quenching.

4.2.2 Comparison of the diagrams with combustion regimes specified by Abraham et al. Abraham et al.\(^{(10)}\) have specified several combustion regimes in the Re–Da plane, including a weak turbulence regime, a reaction sheet regime, an engine operating condition regime, etc. These regimes were cited in Fig. 7, where the regime surrounded by the large box is the engine operating conditions with the triangle marks being the estimated conditions of experiments performed on engines\(^{(9)}\). In the figure, two lines marked with “A” and “B” are the \(u’/S_{1,0}=1\) and \(\eta_{e}/\rho_{0}\) =1 lines, respectively, in Ref. (3), whose positions are somewhat different from those of the corresponding lines in the present work because of the difference in the Prandtl number Pr as well as the turbulence energy dissipation rate \(\varepsilon\) assumed, Pr being taken to be unity and \(\varepsilon\) being estimated by Eq. (1) in Ref. (3).

From Fig. 7 it is clarified that: (1) the predic-
Fig. 7 Comparison of the diagram in Fig.3 (a) with combustion regimes specified by Abraham et al.\(^\text{(19)}\). Regime in box is the operating conditions of engines, and triangle marks are experiment points\(^\text{(19)}\).

Fig. 8 Experimental results reviewed by Abdel-Gayed et al.\(^\text{(19)}\).

sections of the RIFF model cover almost all the engine operating conditions; (2) in the weak turbulence regime, the RIFF model predicted quite low turbulent burning velocities, the \(S_{f}/S_{lo}\) ratios being lower than three, while in the reaction sheets regime, such ratios can become very high, up to about ten because of greatly increased flame area; and (3) as the \(u'/S_{lo}\) ratio for the transition of wrinkled flames to island-like flames is about two (cf. "●" marks in Fig. 6), the fact that, except for the lowest three points (triangles), all the other experiment points lie near or above the \(u'/S_{lo}=2\) line suggests that island flamelets would not be observed at these points.

4.2.3 Comparison of the diagrams with experimental results reviewed by Abdel-Gayed et al.

In order to obtain the relationships between the turbulent flame structure and burning velocity, Abdel-Gayed et al.\(^\text{(19)}\) reviewed the experimental results on the turbulent burning velocity from 1940 to 1985. With some estimated experimental conditions, the \(S_{f}/S_{lo}\) ratio was taken to be a function of the Karlovitz number \(Ka\) (a flame structure parameter) and the Reynolds number \(Re\) (characterizing the turbulence field). The data thus obtained were further plotted in an \(L/\nu-u'/S_{lo}\) plane\(^\text{(19)}\), as shown in Fig. 8.

In Fig. 8, the turbulent flame structure was roughly classified by three lines of constant Karlovitz numbers\(^1\) \(Ka=1.5, Ka=0.3\) and \(Ka=0.15\). Upwardly from the lower right of Fig. 8, the flame structure becomes more and more distributed with increasing \(u'/S_{lo}\) ratio, and finally all flames are quenched when the \(u'/S_{lo}\) ratio is further increased to some extent. Comparing Fig. 8 with Fig. 5, one finds that the flame structure predicted by the RIFF model is in good agreement with the experimental results, and so is the flame quenching limit.

For the turbulent burning velocity, there are three constant burning velocity curves shown with dots in Fig. 8; these lines agree with the predictions by the RIFF model shown in Fig. 8 quite well qualitatively, and to some extent quantitatively. Nevertheless, Fig. 5 (a) shows the turbulent burning velocities near the flame quenching line (\(Ka=1\)) to decrease with increasing \(u'/S_{lo}\) ratio, although experimental results corresponding to such predictions cannot be found in Fig. 8. Because some experiments\(^\text{(19)}\) have shown the turbulent burning velocity to decrease after reaching maximum when the turbulence is further intensified, this discrepancy is attributed to the lack of sufficient experimental results at the critical conditions under which flames are highly unsteady and easily quenched.

5. Diagrams of Other Models

The method of representing a turbulent burning velocity model in the form of diagrams applies to most models so far proposed. By showing the model

\(^1\) As the Karlovitz number \(Ka\) in Ref. (19) was based on Eq. (1) with \(A_s=0.37\), its values are different from those in the present work, but this does not affect the experimental results plotted in an \(L/\nu-u'/S_{lo}\) plane because the coordinates are independent of the energy dissipation rate.
predictions of turbulent burning velocity as a burning velocity diagram in the coordinate system for the flame structure phase diagrams, the model-implied relationships between the turbulent burning velocity and flame structure can be clarified by superimposing the burning velocity diagram onto the corresponding phase diagram even if the model does not describe the flame structure explicitly. Moreover, the conditions to which a model applies can also be included in the diagram. Such benefits of the burning velocity diagrams are demonstrated in the following by taking Gouldin's model\(^{(7)}\) and the 3-region model by Ballal and Lefebvre\(^{(8)}\) for examples.

5.1 Gouldin's model

Based on the fractal properties of turbulent flame surfaces, Gouldin\(^{(7)}\) proposed a model for the turbulent burning velocity, where the turbulent burning velocity \(S_t/S_{t0}\) is calculated by

\[
\frac{S_t}{S_{t0}} = \left[1 - \left(1 - \frac{A_0^D \text{Re}^{D}}{A_0^D \text{Re}^{D}}\right)\right] \exp\left(-\frac{u'}{S_{t0} \text{Re}}\right)^{D-2},
\]

where \(A_0\) is the constant in Eq. (1), being given as 0.37 and \(D\) is the fractal dimension of flame surfaces, being given as a constant of 2.37 in the model\(^{(7)}\). This model was supposed to apply to turbulent combustion with moderate to low turbulence levels, except extremely low ones\(^{(7)}\). The diagrams of the model are shown in Fig. 9 with \(\varepsilon\) being estimated by Eq. (1). As the turbulence energy dissipation rate \(\varepsilon\) has been based on Eq. (1), the diagrams must be combined with the flame structure phase diagrams\(^{(8)}\) in Fig. 2 to obtain parameters for the flame structure, so as to clarify the model-implied relationship of the turbulent burning velocity to the flame structure. For instance, given a Reynolds number Re and a Damkohler number Da, one can obtain the \(S_t/S_{t0}\) ratio from Fig. 9(a) and flame structure parameters \(L/L_0\), \(\eta/K\), \(K\) as well as \(u'/S_{t0}\) from Fig. 2.

5.2 Diagrams of the 3-region model by Ballal and Lefebvre

By examining the schlieren photographs together with corresponding measurements of the turbulent flame propagation speed, Ballal and Lefebvre\(^{(8)}\) found the flame structure and the influence of turbulence scale on turbulent burning velocity both dependent on the level of turbulence intensity. To account for such observations, the so-called 3-region model of turbulent flames was proposed, in which turbulent combustion was divided into three regions in terms of the scales of \(u'/S_{t0}\) and \(\eta/K\) as follows:

For region 1 (low turbulence), \(u' < 2S_{t0}, \eta > \eta_0\) and

\[
\frac{S_t}{S_{t0}} = \left[1 - \left(1 - \frac{A_0^D \text{Re}^{D}}{A_0^D \text{Re}^{D}}\right)\right] \exp\left(-\frac{u'}{S_{t0} \text{Re}}\right)^{D-2},
\]

Fig. 9 Burning velocity diagram of Gouldin's model\(^{(7)}\); results in high turbulence (beyond model capacity) also included for reference.
\[
\frac{S_T}{S_{Lo}} = \left\{ 1 + 0.125 \left( \frac{u'}{S_{Lo} \eta_0} \right) \right\}^{0.45}
\]  
(27)

For region II (intermediate turbulence), \( u' \approx 2S_{Lo} \eta_k \approx \eta_0 \) and
\[
\frac{S_T}{S_{Lo}} \approx 4.
\]  
(28)

And, for region III (high turbulence), \( u' > 2S_{Lo} \eta_k < \eta_0 \) and
\[
\frac{S_T}{S_{Lo}} = 0.5 \left( \frac{u'}{S_{Lo} \eta_k} \right)
\]  
(29)

In order to evaluate the Kolmogorov scale \( \eta_k \) Eq. (1) was used in the present work with \( A_0 \) being taken to be 0.37 as in Gouldin's model. The obtained results are shown in Fig. 10, where "I", "II" and "III" indicate regions I, II, and III, respectively.

Like diagrams of other models, the diagrams in Fig. 10 can be compared with Fig. 2 to clarify the relationships between the flame structure and turbulent burning velocity which are implied by the 3-region model\(^{40}\) as done for Gouldin's model, or with Fig. 8 to discuss the validity of the model. However, what we are here interested in is that the diagrams in Fig. 10 provide not only the detailed predictions of the model, but also the conditions under which the model is supposed to apply. From Fig. 10, it is obvious that the model covers only parts of combustion conditions.

6. Conclusions

(1) The Ra-Da, \( \eta_k/\eta_0 - u'/S_{Lo} \) and \( L/\eta_0 - u'/S_{Lo} \) coordinate planes were used so far for the flame structure phase diagrams can be transformed from one to another equivalently.

(2) By plotting the model-predicted turbulent burning velocities in the form of turbulent burning velocity diagrams, the detailed responses of the model predictions to both the turbulence characteristics and the mixture properties can be exhibited simultaneously.

(3) The turbulent burning velocity diagrams of a model can provide not only the detailed predictions on the turbulent burning velocity at various combustion conditions, but also the overall characteristics of the model, as well as the combustion conditions under which the model applies.

(4) By combining the turbulent burning velocity diagrams of a model with the corresponding flame structure phase diagrams, the model-implied relationships between the flame structure and turbulent burning velocity can be clarified.

(5) The turbulent burning velocity and flame structure diagrams of the RIFF model\(^{40}\) agree quite well with the experimental results on flame structure transitions and flame quenching obtained by the authors, and agree with the experimental results on turbulent burning velocity and flame structure.
reviewed by Abdel-Gayed et al.\textsuperscript{19}, well qualitatively and to some extent, quantitatively.

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References