Three-Dimensional Calculation of Air-Water Two-Phase Flow in a Centrifugal Pump Based on a Bubbly Flow Model with Fixed Cavity*

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To predict the behavior of air-water two-phase flows in centrifugal pumps, we have proposed a three-dimensional numerical method on the basis of an inviscid bubbly flow model with slippage between two phases. The void fractions calculated distribute unevenly and their maximum exceeds an applicability of the model. To extend its applicability, a newly modified model is proposed in this paper by assuming that the bubbles in such high void fraction regions coalesce with each other and adhere to the neighboring impeller walls so as to form a fixed cavity. Using this model, the flows in a radial-flow pump are solved. The cavity obtained increases progressively from the shroud to the hub in the section just after the impeller inlet when the inlet void fraction exceeds a critical value and finally fills the section, showing close relation with the experiments when the pump loses its function due to an air-filled blockade.

Key Words: Pump, Turbomachinery, Multiphase Flow, Void Fraction, Numerical Analysis, Finite-Element Method, Fixed Cavity, Pump Performance, Blowdown, Oil Field Development

1. Introduction

Related to the safety analysis of nuclear reactors, a number of researchers have investigated experimentally⁽¹⁾ and theoretically⁽²⁾ the characteristics of centrifugal pumps operating under gas-liquid two-phase flow conditions, as reviewed by one of us⁽³⁾. Recently crude oil pumps in submarine fields are being developed to make the pumps maintain their pumping activity even in the presence of gaseous oil containing a fair amount of gas⁽⁴⁾, and effects of design parameters on the characteristics were studied experimentally by Kosmowski^{(5),(6)} and Furukawa and his co-workers⁽⁷⁾.

To comply with industrial demands for designing such a pump, more definite methods for simulating the two-phase mixture flows in the pump should be developed. We have proposed a three-dimensional numerical method to analyze the flows⁽⁸⁾ and a related method for predicting the change in the head devel-

oped by the impeller due to the inlet void fraction⁽⁹⁾ on the basis of an inviscid bubbly flow model with slippage between two phases^{(10),(11)}, in which the mixture is assumed to be a bubbly flow entraining homogeneously fine bubbles. The calculated results agree well with the measured values; however, such a predominating region as the local void fraction is more than four times the inlet void fraction locally appears along the blade in the impeller inlet region when the inlet void fraction increases greatly.

Since such regions generally exceed the applicability of the model, a newly modified model is investigated here by assuming that the bubbles in such predominating regions of high void fractions adhere to the neighboring impeller blade similarly to a fixed cavity under cavitation conditions, namely, such bubbles easily coalesce with each other to grow into a large mass which is not easily swept out and remains in place. Using this model the flows in the radial-flow pump are solved. Though the degradation in the head developed by the impeller and the distributions of pressures and void fractions calculated are quite similar to the results obtained by the previous bubbly flow model, the cavity calculated expands progressively in

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volume and finally fills the section. The experiments conducted earlier by one of us⁽¹²⁾ are in agreement with the limit of pumping activity, when the air blocks the impeller passage and the pump loses its function. This model is therefore applicable to the design of two-phase flow pumps with arbitrary geometry to evaluate the effect of the geometry on the limit of pumping activity.

2. Nomenclature

- A : flow area of impeller
 C_{DTP} : drag coefficient of a bubble in a multi-bubble system
 C_V : virtual mass coefficient
 g : acceleration of gravity
 H_{imp} : head developed by impeller
 $\dot{m}$: mass flow rate
 N : number of blades
 $\mathbf{n}$: unit vector perpendicular to boundary surface
 n_b : number of bubbles in unit volume
 p : absolute pressure without gravity effect
 Re : Reynolds number of bubble $= 2R_b|\mathbf{W} - \mathbf{V}|/\nu$
 R_b : radius of bubble
 r : radial coordinate
 t : time
 u : peripheral speed of impeller
 $\mathbf{V}$: velocity vector of bubble
 V_v : volume of fixed cavity
 $\mathbf{W}$: velocity vector of water
 z : axial coordinate
 α : void fraction
 α_{cr} : critical void fraction for appearance of fixed cavity
 δ : displacement of node
 Γ : circulation $= 2\pi r W_{a\theta}$
 θ : angular coordinate of a bubble $= \theta_0 + \omega t + \Theta$
 Θ : angular coordinate of a bubble in rotating frame
 ν : kinematic viscosity of water
 ρ : density
 ϕ : velocity potential or non-dimensional expression of pump flow rate $= \dot{m}/(\rho_m A_2 u_2)$
 $\Delta\psi_r$: non-dimensional pressure coefficient $= (p - p_0)/(\rho_l u_2^2)$
 ω : angular velocity of impeller
- Subscripts**
- 0: pump inlet or liquid single-phase flow
 1: impeller inlet
 2: impeller outlet
 a : condition in an absolute frame of reference
 g : gas phase
 l : liquid phase
 m : air-water two-phase flow

- r : radial component
 z : axial component
 θ : circumferential component

3. Theoretical Relations

3.1 Assumptions

The following assumptions are employed in this calculation.

(1) The mixture is a bubbly flow entraining homogeneously fine bubbles. The bubble size is small compared to a characteristic length of impeller channel except for fixed cavity regions.

(2) The bubble movements are represented by Basset-Bousinesque-Oseen's formula⁽¹³⁾ except for fixed cavity regions.

(3) The mixture flow is steady in a relative frame of reference, which rotates around an axis with a constant velocity, and inviscid except for the drag force between two phases.

(4) The change of void fractions in an absolute frame of reference is quasi-steady.

(5) Neither fragmentation nor coalescence of bubbles occurs except when the coalescence in such regions as the local void fraction exceeds a critical level.

(6) Bubbles maintain their spherical shape and change their volume adiabatically according to the pressure that surrounds them. The liquid phase is incompressible.

(7) Neither mass nor heat transfer takes place between the two phases.

(8) The mass and momentum of the gas phase are very small and negligible compared with those of the liquid phase. However, the effects of virtual mass and bubble slip are accounted for. Gravity effects on the bubble movement are also ignored.

(9) The drag coefficient of a bubble is the same as that of a solid particle. The influence of interactions between bubbles on their movements is identical to that in a cloud of solid particles.

(10) Bubbles in such regions as the local void fraction exceeds a certain limit coalesce with each other and adhere to the neighboring impeller walls, generating a fixed cavity there. Neither interfacial forces on the cavity surface except for the normal stress (i.e., the pressure) nor bubble entrainment from the cavity into the flow exists.

The above assumptions, except for (10), are the same as those employed for the bubbly flow model^{(10),(11)} and the previous paper⁽⁸⁾.

3.2 Equations of motion of a bubble and void fraction

Using assumptions (1) and (6), the behavior of bubbles can be obtained by tracking a bubble released

from various initial positions on a pump inlet section.

The equation of motion of a bubble (mass M), moving with velocity $\mathbf{V}_a$ in a water flow field $\mathbf{W}_a$, can be given by the following equation⁽¹¹⁾ under assumptions (1), (2), (6) and (8)^{(8),(11)}.

$$d_a(M\mathbf{V}_a)/dt = \mathbf{f}_p + \mathbf{f}_d + \mathbf{f}_v \quad (1)$$

In Eq. (1), the term $\mathbf{f}_p$ is the force due to the pressure gradient, $\mathbf{f}_d$ the drag force, and $\mathbf{f}_v$ the force due to virtual mass. These terms are given by the following equations, respectively,

$$\begin{aligned} \mathbf{f}_p &= -(4/3)\pi R_b^3 \nabla_a p \\ \mathbf{f}_d &= (1/2)\rho_l \pi R_b^2 C_{DTP} |\mathbf{W}_a - \mathbf{V}_a| (\mathbf{W}_a - \mathbf{V}_a) \\ \mathbf{f}_v &= \rho_l C_v (4/3)\pi [D_a(R_b^3 \mathbf{W}_a)/Dt - d_a(R_b^3 \mathbf{V}_a)/dt] \\ D_a/Dt &= \partial_a/\partial t + \mathbf{W}_a \cdot \nabla_a, \quad d_a/dt = \partial_a/\partial t \end{aligned} \quad (2)$$

where the virtual mass coefficient C_v is assumed to be constant ($C_v=0.5$). Since the bubble flows in a cloud of bubbles with void fraction α , the following formula⁽¹⁴⁾ is available for the drag coefficient of a bubble C_{DTP} ,

$$C_{DTP} = (1-\alpha)^{-q} C_D \quad (3)$$

where C_D is the drag coefficient of a single bubble flowing without interaction with other bubbles, and q is a function of bubble Reynolds number Re as,

$$q = \begin{cases} 8.9Re^{-0.1} & (1 < Re < 500) \\ 4.78 & (500 \leq Re < 7000). \end{cases} \quad (4)$$

The use of the coefficient C_D instead of C_{DTP} in Eq. (2), however, hardly affects the numerical results as described in our previous study⁽⁸⁾.

By transforming Eq. (1) into a frame of reference rotating around an axis with a constant angular velocity, we obtain the governing equation of a bubble in a rotating impeller⁽⁸⁾. By integrating numerically the resultant equation from the pump inlet to the outlet for initial conditions of a bubble (i.e., bubble position, velocity and radius at the pump inlet) on the basis of assumptions (5)~(7), trajectories of the bubble as well as the change in the bubble radius along them are known at any location in the flow field.

In this study, a three-dimensional finite-element method using tetrahedral elements is employed to determine the flow properties in the field, and the field is partitioned into hexahedral elements before each of these elements is re-partitioned into six tetrahedrons. These hexahedral elements are used as control volumes to obtain void fraction α , which is defined by the volumetric ratio of the whole bubbles existing in each control volume to this volume itself. Every tetrahedral element belonging to a control volume, therefore, has the same void fraction as the others. Since the solutions of Eq. (1) give bubble trajectories and a mean bubble volume in each control volume $(4/3)\pi \bar{R}_b^3$ along them, the total number of bubbles existing in each control volume can be evaluated. The number,

divided by the volume of the control volume, namely the number density of bubbles, n_b , yields the value of α in any control volume by the following equation^{(8),(11)}.

$$\alpha = (4/3)\pi \bar{R}_b^3 n_b \quad (5)$$

3.3 Governing equations of air-water two-phase mixture

From assumptions (1), (4), (6) and (8), the equation of continuity for the bubbly flow in an absolute frame of reference can be written with liquid-phase velocity $\mathbf{W}_a$ and void fraction α as⁽⁸⁾:

$$\nabla_a \cdot [(1-\alpha)\mathbf{W}_a] = 0. \quad (6)$$

By examining the conservation of circulation in the bubbly flow field⁽⁸⁾, the velocity $\mathbf{W}_a$ in the absolute frame of reference is found to have a potential ϕ if the flow in the pump inlet is assumed to be irrotational. Then, Eq. (6) can be reduced to the quasi-harmonic equation, namely,

$$\nabla_a \cdot [(1-\alpha)\nabla_a \phi] = 0. \quad (7)$$

The equation of motion of the two-phase mixture in a system rotating with a constant angular velocity ω is expressed by the following equation under assumptions (1), (3), (6) and (8)⁽⁸⁾.

$$(1-\alpha)\mathbf{W} \cdot \nabla \mathbf{W} + (1-\alpha)\omega \times (2\mathbf{W} + \omega \times \mathbf{r}) = -\nabla p / \rho_l \quad (8)$$

Divergence of Eq. (8) yields a Poisson equation as

$$\begin{aligned} \nabla^2 p &= -\rho_l \nabla \cdot [(1-\alpha)\mathbf{W} \cdot \nabla \mathbf{W} \\ &\quad + (1-\alpha)\omega \times (2\mathbf{W} + \omega \times \mathbf{r})]. \end{aligned} \quad (9)$$

After both values of the velocity $\mathbf{W}$ and void fraction α are determined for all the nodes, the pressure p can then be evaluated at every node with Eq. (9). The pressure remains constant on the cavity surface, because the pressure gradients there become zero from Eq. (8) for $\alpha=1$.

3.4 Region for calculation and boundary conditions

In this calculation, the velocity potential ϕ in Eq. (7) is solved in the flow field with a given distribution of void fractions using a three-dimensional finite-element method with simplex tetrahedral elements. The flow region for calculation consists of the spaces between blades and the passages before and after them, as shown in Fig. 1.

In the case of radial-flow pumps, the following boundary conditions should be satisfied.

$$(1-\alpha)\mathbf{n} \cdot \nabla_a \phi = \begin{cases} W_{az} & \text{on } B_1 \\ -W_{ar} & \text{on } B_2 \\ \pm \mathbf{n} \cdot \omega \times \mathbf{r} & \text{on } B_3 \text{ and } B_4 \\ 0 & \text{on } B_5 \text{ and } B_6 \end{cases} \quad (10)$$

where B_1 and B_2 are the inlet and outlet sections, B_3 and B_4 the surfaces of the adjacent blades, B_5 and B_6 the hub and shroud surfaces, and $\mathbf{n}$ a unit vector perpendicular to the boundary surface. For the

wall, producing a cavity there.

The cavity is numerically generated as follows. After the void fraction in every control volume is evaluated, the representative value of the void fraction on every nodal point is also deduced by a method

Fig. 2 Distributions of void fractions on section just after impeller inlet
(by bubbly flow model, $r/r_1=1.1$, $\phi=0.08$)
(The hatched region is regarded as a fixed cavity in this study)

Fig. 3 Flow chart of calculation

usually used in finite element methods, for example, the least squares method⁽¹⁹⁾. These values on every node are available to obtain contour lines expressing equi-void fraction curves by using a linear interpolation method, giving the distribution of the void fraction in the flow field. The cavity is thus represented by the high void fraction region enclosed by the line $\alpha = \alpha_{cr}$. Since the cavity surfaces are determined iteratively together with the calculations to obtain a solution of the liquid-flow field, as described in section 4.2, a relaxation factor γ should be used to determine the displacement of the cavity surface as $\gamma\delta$, so as to prevent formation of an exceedingly large cavity and restrain the divergence of the solution, in which δ is the displacement of the cavity surface newly estimated at any nodal point. The cavity generated is regarded as being a part of the impeller wall in the successive calculations. In this study, the relation $\gamma = 0.2$ is used.

4.2 Numerical procedure

Calculations for obtaining the cavity for inlet void fraction α_0 are performed based on the flow chart shown in Fig. 3. After the cavity for α_0 converges, the same procedure is repeated to obtain the solutions for the inlet void fraction increased by $\Delta\alpha_0 (=0.001)$.

(1) As an initial step ($i=0$), we calculate the distributions of α_i with the values p_0 and W_0 and the cavity, which were the converged solutions in the previous calculation.

(2) A cavity is generated as described in section 4.1.

(3) If the cavity has not converged, one may repartition the flow field outside the cavity, and interpolate void fractions in each element α'_i using coordinates of the element centroid and α_i .

(4) Solve Eq. (7) on the field of α'_i by FEM to obtain ϕ_i , and calculate W_i by differentiating ϕ_i . Calculate pressure p_{i+1} with Eq. (9) for the field of α'_i and W_i .

(5) If the pressures on the trailing edge do not satisfy the Kutta condition, we modify circulation Γ_i , and return to step (4) to obtain the flow properties.

(6) We calculate the void fraction α_i on the field of α'_i , W_i and p_i by using Eqs. (1) and (5), and return to step (2).

5. Calculated Results and Discussion

5.1 Change in fixed cavity form due to inlet void fractions

Distributions of the void fractions calculated by the previous bubbly flow model⁽⁸⁾, shown in Fig. 2, agree well with those measured⁽¹⁸⁾; however, the calculated values are 40% smaller than the measured ones. This difference is attributed to neglect of the

effect of fluid viscosity, as pointed out in the previous paper⁽⁸⁾. In this study the cavity is formed based on the void fractions calculated by this model. This means that the void fractions calculated are regarded as being equivalent to the value which increased upon the corresponding reduction in the void fractions (40 %); for example, the calculated void fraction of 0.20 corresponds to the measured value of 0.50.

According to the observations⁽¹⁸⁾ conducted under the same flow condition, an air-filled hollow exists on the shroud when $\alpha_0=0.05$ and the extent of the hollow ranges approximately in the same circumferential length on the shroud with the hatched area in Fig. 2, in which the region of $\alpha \geq 0.2$ is hatched for the case of $\alpha_0=0.05$. In this study, the critical void fraction α_{cr} , beyond which the bubbles are assumed to coalesce and adhere to the neighboring impeller wall to produce a fixed cavity, is thus taken to be 0.2. Effects of the change in α_{cr} are discussed later.

Figure 4 shows the result for the normal flow capacity ($\phi=0.08$) by the cavity model, when $\alpha_0=0.075$, the highest value for obtaining a convergent solution. In the figure we plot the cavity and contour lines of void fractions $\alpha/\alpha_0=\text{constant}$ for various

Fig. 4 Fixed cavity form and distributions of void fractions on meridional sections ($\phi=0.08$)

Fig. 5 Fixed cavity form within impeller

meridional sections, dislocating circumferentially from the suction side of the blade to the pressure one. The cavity represented by a hatched area is seen to exist on the shroud near the suction side just after the impeller inlet, corresponding to the predominant region of the high void fractions calculated by the bubbly flow model (Fig. 2). The cavity is rather thin in the pump axial direction (z -direction), and its maximum thickness is 22% less than the depth of the flow passage. The contour lines of void fractions near the cavity are seen to be almost parallel to the cavity surface.

Figure 5 shows drawings of the cavity calculated for the normal flow capacity, though the scale in the axial direction (z) is taken to be slightly larger than that in the circumferential one (θ) because of the thinness of the cavity in the axial direction. Since the cavity begins to form at $\alpha_0=0.05$, the result for $\alpha_0=0.06$, which is slightly larger than the initial appearance of the cavity, is plotted in Fig. 5(a). The cavity is seen to exist in a region on the shroud from the mid-channel between the blades to the suction side and just after the impeller inlet.

When the inlet void fraction is slightly increased further to $\alpha_0=0.068$, the cavity region increases abruptly near the suction side of the blade in a radial direction and its thickness also increases as shown in Fig. 5(b). When $\alpha_0=0.075$, the highest value of α_0 , the cavity obtained as well as its thickness increase toward the pressure side as shown in Fig. 5(c). Compared with the cavity observed on the shroud under the same pump operating condition⁽¹²⁾, the width appears to be as big as in the circumferential direction, but somewhat smaller in the radial direction.

When α_0 is changed at the flow capacity $\phi=0.06$ or 0.1 different from the normal one, $\phi=0.08$, the extent of the cavity and contour lines of void fractions show almost similar changes to those for $\phi=0.08$ and are thus omitted here. For every ϕ the cavity appears on the suction side of the blade in the same way as observed earlier⁽¹²⁾, and a high void fraction region also exists in the neighborhood of the cavity as expected from the measurements of void fractions^{(18),(20)}.

5.2 Change in volume of fixed cavity

When inlet void fraction α_0 is increased, the cavity volume V_v also increases as shown in Fig. 6, where the volume V_v is normalized by the cube of impeller inlet radius r_1 , and the results for three values of flow capacities, $\phi=0.06$, 0.08 and 0.1, are superimposed with different symbols.

The results for the normal flow capacity ($\phi=0.08$), plotted by the symbol $\bigcirc$, show that the appearance of the cavity takes place at $\alpha_0=0.05$, and that the

cavity volume increases almost linearly with increasing α_0 within the range $0.05 \leq \alpha_0 \leq 0.071$. The volume, however, increases abruptly when $\alpha_0 > 0.071$. In this range, increment of the inlet void fraction $\Delta\alpha_0$ was thus decreased to 0.0005. When $\alpha_0 > 0.075$, however, the cavity region just after the impeller inlet increases monotonously from the shroud to the hub and a converged solution could not be obtained. It is, however, evident from Fig. 6 that the cavity volume increases abruptly nearly at $\alpha_0 = 0.075$. According to previous experiments⁽¹²⁾, the pump rapidly loses its capacity to generate a pressure head at the nearly same value as this ($\alpha_0 = 0.07$) (see Fig. 7) and no longer functions when α_0 is slightly increased further. This rapid degradation is attributable to the fact that an air-filled hollow, generated just after the impeller inlet, expands with an increase of α_0 and finally shuts off the water flow there⁽¹²⁾. The calculated increase of the cavity volume, as shown in Fig. 6, can then be considered to correspond to the loss of pumping

activity.

The results for lower ($\phi = 0.06$) and higher ($\phi = 0.1$) capacities are also shown in Fig. 6 with the symbols $\triangle$ and $\square$, respectively. The volume V_v changes, owing to α_0 , in a manner almost similar to that for $\phi = 0.08$. When ϕ is greater than normal, however, the appearance of the cavity begins at a smaller value of α_0 and V_v increases at a faster rate, so that the abrupt increase is observed at lower values of α_0 . Irrespective of different ϕ , such an abrupt increase occurs at the same α_0 value as when the pump ceases to function, as seen in Fig. 7.

When the critical value of void fractions α_{cr} is altered, similar changes in the cavity volume occur as seen in Fig. 6, where the results for $\alpha_{cr} = 0.18$ and 0.22 for the normal flow capacity ($\phi = 0.08$) are plotted with different kinds of lines. The greater the α_{cr} , the larger the values of α_0 before the appearance of the cavity and the abrupt increase in V_v , though the effects of α_{cr} on the abrupt increase become less with an increase in α_{cr} . Although omitted from the illustration, the same calculations using the value of $\alpha_{cr} = 0.2$ were applied to another pump, which has the same geometry and dimensions except for the number of impeller blades (7 blades). The calculated limit of pumping activity nearly coincided with the measured one. In spite of the sensitivity of α_{cr} on the abrupt increase in V_v , the value $\alpha_{cr} = 0.2$ is thus considered to be applicable to the design problem of the optimum two-phase pump geometry to evaluate the effects of the geometry on the rapid degradation of the pump performances by using the fixed cavity model.

Fig. 6 Change in cavity volume due to α_0 and α_{cr}

Fig. 7 Relationship between the head developed by impeller and α_0

Fig. 8 Pressure distributions on meridional sections ($\phi=0.08$)

Fig. 9 Pressure distributions on blade-to-blade sections ($\phi=0.08$)

5.3 Distributions of pressure

Distributions of pressure calculated for the normal flow capacity ($\phi=0.08$) are shown in Fig. 8, where the non-dimensional isobaric lines for $\alpha_0=0.06$ and 0.075 , connecting points having the same value of $\Delta\psi_r$, on the suction side, those on the mid-channel between the pressure and suction sides, and those on the pressure side, are compared in Figs. 8(a), (b) and (c), respectively. In this figure, the cavity regions are hatched and enclosed with lines corresponding to the isobaric lines. Though the cavity causes severe deformation of the isobaric lines, the deformation is limited to the vicinity of the cavity; thus the pressure drop due to α_0 begins just upstream of the cavity. The deformation, however, scarcely appears in the impeller outlet region, which shows almost the same distributions as those obtained by the bubbly flow model⁽⁸⁾.

Pressure distributions on blade-to-blade sections,

corresponding to those in Fig. 8, are plotted in Fig. 9, where (a) shows the results on the shroud, which is maximally affected by α_0 . Owing to the existence of the cavity (hatched area), the isobaric lines near the cavity shift downstream, and this effect spreads downward, firstly to the suction side of the blade and finally to the center of the flow passage in the course of flow. As the result, the convex shape of the isobaric lines appears in the center of the passage near the impeller outlet, though the bubbly flow model yields only parallel lines⁽⁸⁾. Such changes due to α_0 are in agreement with the measurement⁽¹⁶⁾, though the calculation shows a somewhat smaller shift of the isobaric lines. Figure 9(b) presents the results on a mid-channel between the shroud and the hub, indicating less effects of α_0 .

When the flow capacity is decreased ($\phi=0.06$) or increased ($\phi=0.1$), resultant pressure distributions change in almost the same manner with those obtained when flow capacity is normal ($\phi=0.08$).

5.4 Change in head developed by impeller

As described in section 3.6, calculated velocities and void fractions on the sections just before the impeller inlet and just after the outlet yield the head developed by the impeller H_{imp} even in two-phase flow conditions. H_{imp} decreases slightly with increasing α_0 and remains unaltered in the range greater than a certain value of α_0 , as shown in Fig. 7, where the head divided by that in the single-phase flow H_{imp0} , H_{imp}/H_{imp0} , is plotted against inlet void fraction α_0 by thick solid lines, and compared with that calculated by the bubbly flow model represented by the symbol $\bigcirc$ connected by dotted lines. The results for the normal flow capacity ($\phi=0.08$) are shown in the middle plot of Fig. 7. The results calculated both by this fixed cavity model and by the bubbly flow model agree well even when the cavity exists at $\alpha_0 \geq 0.05$. Such effects of the cavity on the head are quite similar to those of the void caused by cavitation, when the flow around the void is calculated based on the assumptions of steady and inviscid flow by using two-dimensional singularity methods^{(21)–(23)}. According to such calculations, the head obtained remains unaltered owing to the change in extent of the void, as long as the void exists in the neighborhood of the leading edge on the suction side.

Figure 7 also shows experimental results⁽¹⁷⁾ indicated by the symbol $\square$. When $\phi=0.08$, they coincide with the calculations by this fixed cavity model within the range of $\alpha_0 \leq 0.04$. When $\alpha_0 > 0.04$, however, the measured values show a larger decrease. From these observations, the cavity region was found to be present already at $\alpha_0=0.051$, and the rapid drop in pump performance occurred when α_0 was increased beyond 0.071 .

The results for a lower flow capacity ($\phi=0.06$) are shown in the upper plot of Fig. 7. The relationship between the head ratio and α_0 is similar to that for the normal flow capacity. The maximum value of α_0 , at which a converged solution can be obtained, is 0.0805. This value is slightly larger than that obtained with a rapid drop in pump performance ($\alpha_0=0.07$). The results for a higher capacity ($\phi=0.1$) are shown in the lower plot of Fig. 7. The maximum value of α_0 of 0.058 is slightly smaller than that for the rapid drop in performance ($\alpha_0=0.06$). Hence, this calculation can predict the effect of ϕ on the rapid drop.

6. Conclusions

To solve numerically air-water two-phase flows in centrifugal pumps, a bubbly flow model with a fixed cavity has been proposed and applied to a radial-flow low-specific-speed pump. The results obtained are summarized as follows.

(1) On the shroud just after the impeller inlet, a fixed cavity appears at a certain value of α_0 corresponding to the flow capacity ϕ ($0.06 \leq \phi \leq 0.1$), and its volume increases linearly with an increase in α_0 . When α_0 is increased beyond a certain limit, the cavity increases dramatically at the inlet so as to fill the flow passage.

(2) When the cavity volume calculated increases abruptly, the pump performance also deteriorates rapidly at almost the same value of α_0 owing to air blockade of the impeller passage. The limit of this rapid deterioration can be simulated by using the proposed model.

(3) When a fixed cavity exists, pressure rise in a radial direction begins to degrade from the upstream edge of the cavity. With an increase in α_0 the pressure rise along the shroud shows a larger decrease owing to the increase of the cavity region.

(4) The impeller heads calculated both by this fixed cavity model and by the bubbly flow model agree well even when the cavity exists.

References

- (1) Nilsson, K.A., LWR Recirculation Pump Performance Under Two-Phase Flow Conditions, Proc. Cavitation and Polyphase Flow Forum, ASME, (1977), p. 39.
- (2) Furuya, O., An Analytical Model for Prediction of Two-Phase (Noncondensable) Flow Pump Performance, Trans. ASME, J. Fluid Eng., Vol. 107, No. 1 (1985), p. 139.
- (3) Minemura, K. and Murakami, M., Developments in the Research of Air-Water Two-Phase Flows in Turbomachinery, JSME Int. J., Series II, Vol. 31, No. 4 (1988), p. 615.
- (4) De Donno, S., Senni, S., Orlando, G., and Benvenuti, E., The S.B.S. Project-Development of a Subsea Booster System for the Exploitation of Deep Water Oil Fields, Proc. 5th Deep Offshore Technol., (1985), p. 357.
- (5) Kosmowski, I., Problems When Conveying Liquid-Gas Mixtures and Conclusions for the Construction of Centrifugal Pumps, Oper. Probl. Pump Stations Power Plants, Vol. 1 (1982), p. 21.1.
- (6) Kosmowski, I., Einfluss des Gehäuses von Turbopumpen auf die Förderung gasbeladener Flüssigkeiten, Energietechnik, Vol. 34, No. 4 (1984), p. 141.
- (7) Furukawa, A., Sato, S., Ikeda, H. and Takamatsu, Y., Some Experiments on Improving the Liquid/Gas Two-Phase Flow Performance of a Centrifugal Pump, Technology Reports of Kyushu University, (in Japanese), Vol. 63, No. 5 (1990), p. 535.
- (8) Minemura, K. and Uchiyama, T., Three-Dimensional Calculation of Air-Water Two-Phase Flow in Centrifugal Pump Impeller Based on a Bubbly Flow Model, Trans. ASME, J. Fluid Eng., Vol. 115, No. 4 (1993), p. 766.
- (9) Minemura, K. and Uchiyama, T., Prediction of Pump Performance under Air-Water Two-Phase Flow Based on a Bubbly Flow Model, Trans. ASME, J. Fluid Eng., Vol. 115, No. 4 (1993), p. 781.
- (10) Noordzij, L. and van Wijngaarden, L., Relaxation Effects, Caused by Relative Motion, on Shock Waves in Gas-Bubble/Liquid Mixtures, J. Fluid Mech., Vol. 66, Part 1 (1974), p. 115.
- (11) Matsumoto, Y., Nishikawa, H. and Ohashi, H., Performance of Cascade of Blades in Two-Phase Flow (Modeling and Numerical Calculation), JSME Int. J., Series II, Vol. 31, No. 4 (1988), p. 652.
- (12) Murakami, M. and Minemura, K., Effects of Entrained Air on the Performance of a Centrifugal Pump (1st Report, Performance and Flow Conditions), Bulletin of JSME, Vol. 17, No. 110 (1974), p. 1047.
- (13) Soo, S.L., Fluid Dynamics of Multiphase Systems, (1967), 32, Blaisdell Pub. Co.
- (14) Richardson, J.F. and Meikle, R.A., Sedimentation and Fluidization-Part 4; Drag Force on Individual Particles in an Assemblage, Trans. Inst. Chem. Engrs., Vol. 39, p. 357.
- (15) Nagafuji, T., A Flow Analysis in Fluid Machinery (Part-1, A Method of Analysis), JSME, 11th Symposium (No. 760-18), (in Japanese), p. 211.
- (16) Minemura, K., Murakami, M. and Kataoka, T., Pressure Distribution in a Centrifugal Impeller Handling Air-Water Mixtures, Bulletin of JSME, Vol. 26, No. 220 (1983), p. 1727.
- (17) Minemura, K., Murakami, M. and Katagiri, H., Characteristics of Centrifugal Pumps Handling Air-Water Mixtures and Size of Air Bubbles in Pump Impellers, Bulletin of JSME, Vol. 28, No. 244 (1985), p. 2310.

- (18) Minemura, K. and Murakami, M., Distributions of Velocities and Void Fractions in a Radial-Flow Pump Impeller Handling Air-Water Two-Phase Mixtures, Proc. Cavitation and Multiphase Flow Forum, (1986), ASME, p. 35.
 - (19) Daiguji, H., Numerical Analysis of 3-D Potential Flow in Axial Flow Turbomachines, Bulletin of JSME, Vol. 26, No. 215 (1983), p. 763.
 - (20) Sekoguchi, K., Takada, S. and Kanemori, Y., A Study on Air-Water Two-Phase Centrifugal Pump by Measuring Void Fraction with Electric-Resistivity Probe (1st Report, Measurement of Distributions of Void Fraction in Impeller), Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 49, No. 445, B (1983), p. 1859.
 - (21) Nishiyama, T. and Ueki, T., Analyses of Characteristics of Centrifugal Impeller with Leading Edge Cavitation by Singularity Method, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 49, No. 441, B (1983), p. 931.
 - (22) Nishiyama, H., Theoretical Analysis for Passage and Exit Flow of Two Dimensional Centrifugal Impeller with Leading Edge Cavitation (Effects of Blade Number and Blade Angle), Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 54, No. 503, B (1988), p. 1703.
 - (23) Imai, R., Tsujimoto, Y. and Furuya, O., Non-Linear Analyses of Cavitating Flows through a Centrifugal Impeller by a Singularity Method, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 56, No. 522, B (1990), p. 454.
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