Characteristics of Vertical Annular Two-Phase Flow with Local Liquid Fall-Back*

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A new analytical method for two-phase flow is proposed, in which the most easily occurring and, thus, most stable two-phase flow situation is assumed to be realized. Flow structure is considered to be self-controlled so as to realize the most stable two-phase flow situation. In the present study, the flow situation with smaller pressure energy consumption rate for unit mass of penetrating two-phase fluid through each channel cross section is assumed to be more stable. In addition, two-dimensional turbulent flow analysis is applied to the annular flow regime in which partial or total downflow of the liquid film is allowed. Experimental analysis based on the data from an air-water two-phase flow experiment, which was performed at atmospheric pressure and room temperature, indicated the applicability of the proposed analytical method.

Key Words: Nuclear Reactor, Multiphase Flow, Boiling, Annular Flow, Low Mass Velocity, Liquid Film, Liquid Fall-Back, Pressure Energy Consumption Rate, Radial Velocity Distribution, Void Fraction

1. Introduction

A one-dimensional two-fluid two-phase flow model can give relatively accurate predictions for high-mass-velocity two-phase flow. However, in the case of low-mass-velocity two-phase flow, reliability of the analysis becomes very poor. The two major reasons are as follows.

(1) Since the inertial force terms in the equations of motion are much smaller than the corresponding interfacial drag terms, the calculated change in slip ratio and the resultant variation of void fraction are extremely sensitive to the interfacial drag estimation. For example, in analysis of reflooding two-phase flow in a pressurized water reactor (PWR) core during a loss-of-coolant accident (LOCA), in which mass velocity of the in-core two-phase flow is on the order of 10 kg/m²s, an error of order only 0.1% in estimation of interfacial drag can easily lead to an error of order 10% in prediction of void fraction.(1)

(2) The effect of multidimensional fluid behavior caused by the gravity convection due to density distribution in each channel cross section cannot be neglected in low-mass-velocity two-phase flow analysis. For example, even if net liquid flow is upward, downward liquid flow is often observed locally along the channel wall. In such a case, wall shear force is supposed to act upwards on the two-phase fluid even when net flows of both gas and liquid phases are upward.

Many theoretical or semi-empirical estimation methods for interfacial drag have been utilized(2)-(4) in the past. These methods can estimate interfacial drag uniquely as a function of channel and flow conditions. That is, they can be considered quite deterministic estimation methods. However, none of these methods are satisfactory as regards estimation accuracy, especially when mass velocity is low. Therefore, it is necessary to develop a new indeterministic analytical method in which interfacial drag is naturally self-controlled so as to realize the most suitable two-phase flow situation under the given channel and flow conditions.

In order to markedly improve the reliability of
low-mass-velocity two-phase flow analysis using a one-dimensional two-fluid two-phase flow model, we propose a new analytical method for vertical two-phase flow. Special distinctions of this method are as follows.

1. The most stable two-phase flow situation for the given channel and flow conditions is determined to predict the two-phase flow which is actually realized. Here, when pressure energy consumption rate is smaller, the two-phase flow situation is assumed to be more stable.

2. The gravity convection effect due to the density difference between the liquid film adjacent to the channel wall and the two-phase flow core in the central portion of the channel is taken into account in the stability analysis.

In the present paper, the analytical method will be introduced with some results from a verification analysis.

### Nomenclature

- \( A \) : flow area, constant in Eq. (11)
- \( F \) : frictional force/volume (drag force/volume)
- \( g \) : acceleration of gravity
- \( J \) : superficial velocity
- \( L \) : observation elevation
- \( l \) : mixing length
- \( P \) : pressure
- \( r \) : radial location
- \( t \) : film thickness
- \( u \) : velocity
- \( W \) : mass flow rate
- \( x \) : quality
- \( z \) : axial location
- \( a \) : void fraction
- \( x \) : constant in Eq. (11)
- \( \nu \) : dynamic viscosity
- \( \rho \) : density
- \( \tau \) : shear stress

### Subscript

- \( g \) : gas phase
- \( l \) : liquid phase
- \( F \) : liquid film
- \( C \) : two-phase flow core
- \( W \) : channel wall
- \( I \) : liquid film surface
- \( dg \) : gas drift velocity

### 2. Theory

#### 2.1 Two-region model

In low-mass-velocity two-phase flow, the most typical flow regime is annular flow with droplets in the gas flow core, and slug flow or bubble flow can be seen only when gas superficial velocity is very low. In addition, liquid film of the annular flow regime flows along the channel wall sometimes upwards and sometimes downwards, depending on the combination of gas and liquid superficial velocities. There is also a wide region of flow conditions in which only some part of the liquid film immediately adjacent to the channel wall flows downwards and the rest of the liquid film flows upwards. Such behavior of the liquid film of annular flow is suitable for analytical studies of the effect of gravity convection due to density distribution in the channel cross section. Therefore, in the present paper, annular flow is selected as the analytical object. It is, of course, a well known fact that real annular flow exhibits violent waving at the liquid film surface and also shows entrainment and de-entrainment. The effects of these phenomena can be taken into account through the change in the turbulent flow characteristics of liquid film flow.

Figure 1 shows the two-phase flow model which was used in this study. The flow is composed of two regions: one is a single-phase liquid film flow adjacent to the channel wall and the other is a two-phase flow core with uniform void fraction distribution in the central portion of the channel. Gas phase of the two-phase flow core is assumed to be continuous as in droplet flow. The relationship between void fraction \( a \) of the two-phase flow core and channel-averaged void fraction \( \bar{a} \) is given by

\[
a = \left( \frac{r_w}{r_f} \right)^2 \bar{a}. \quad (1)
\]

Liquid flow rate \( W_{ic} \) of the two-phase flow core is assumed to be equal to the supplied liquid flow rate \( W_l \) minus the liquid film flow rate \( W_{fr} \), i.e.,

\[
W_{ic} = W_l - W_{fr}. \quad (2)
\]

Therefore, \( W_{ic} \) becomes larger than \( W_l \) when \( W_{fr} \) is

![Fig. 1 Two-region two-phase flow model](image-url)
negative (downward flow).

2.2 Principle of minimum pressure energy consumption rate

From the point of view of energy balance, the two-phase flow phenomenon is conversion of pressure energy of the two-phase fluid into mechanical work (accelerational work, gravitational work, frictional work, etc.) as illustrated in Fig. 2. Under the given boundary conditions, therefore, two-phase flow with smaller pressure energy consumption rate (i.e., smaller mechanical work rate) is expected to occur more easily than the other cases. In other words, such two-phase flow is considered to be more stable. Therefore, two-phase flow structure (phase mixing pattern, radial distributions of void fraction and phase velocities and characteristics of pulsation and turbulence, etc.) should be adjusted so as to render the pressure energy consumption rate minimum as shown in Fig. 2, as long as the flow structure is not frozen for some reason. Based on these considerations, we propose a new analytical method to predict the characteristics of two-phase flow which are actually realized by determining the minimum pressure energy consumption rate condition.

Let us investigate one-dimensional upward two-phase flow without phase change as an example. Length $L$ from the gas-liquid mixing nozzle to the observation elevation is assumed sufficiently longer than the length of the entrance region and, therefore, the local flow situation at $z=L$ is evaluated approximately as the average flow situation in the section $L$. Then, based on the mechanical energy balance equation for penetrating two-phase fluid\(^{19}\), average pressure energy consumption rate in the section $L$ per unit penetrating mass is described, for the two-phase flow core as:

\[
-\frac{1}{L} \int_0^L \left[ \frac{x_c}{\rho_g} \bar{u}_{gc} + \frac{1-x_c}{\rho_i} \bar{u}_{ic} \right] \frac{dP}{dz} dz
- \frac{1}{2L} \left( \frac{x_c}{\rho_g} \bar{u}_{gc}(\bar{u}_{gc}) + (1-x_c) \bar{u}_{ic}(\bar{u}_{ic}) \right)_{z=L}
- \frac{1}{2L} \left( \frac{x_c}{\rho_g} \bar{u}_{gc}(\bar{u}_{gc}) + (1-x_c) \bar{u}_{ic}(\bar{u}_{ic}) \right)_{z=0}
+ \left[ \left( \frac{x_c}{\rho_g} \bar{u}_{gc} + \frac{1-x_c}{\rho_i} \bar{u}_{ic} \right) \left( \alpha_c \rho_g + (1-\alpha_c) \rho_i \right) g \right]
+ \left[ \left( \frac{x_c}{\rho_g} \bar{u}_{gc} + \frac{1-x_c}{\rho_i} \bar{u}_{ic} \right) \left( \frac{2 \eta}{r_i} \right) \right].
\]  

(3)

The first to the third terms on the right-hand side of Eq. (3) describe accelerational work rate, sum of gravitational work rate and frictional work rate due to interfacial drag at droplet surface, and frictional work rate due to interfacial shear force at liquid film surface, respectively.

The average value symbol on each velocity $\bar{u}$ indicates that it is the average velocity for the region under discussion. For example, $\bar{u}_{ic}$ means average liquid velocity in the two-phase flow core.

Similarly, for liquid film flow,

\[
-\frac{1}{L} \int_0^L \frac{1}{\rho_i} \bar{u}_{if} \frac{dP}{dz} dz = -\frac{1}{2L} \left( \frac{2 \eta}{r_i} \right) \bar{u}_{if}.
\]  

(4)

The first to the third terms of Eq. (4) describe accelerational work rate, gravitational work rate and sum of frictional work rates due to interfacial shear force at the liquid film surface and wall shear force at the channel wall surface, respectively.

By multiplying Eq. (3) by $W_c/W$ and Eq. (4) by $W_f/W$, and summing the results, one can obtain the equation of pressure energy consumption rate for total two-phase flow. The minimum condition for this equation gives the two-phase flow which is actually realized. Here, since the sign of $\bar{u}_{if}$ and that of $W_f$ are always identical, the sign of $\bar{u}_{if} W_f$ does not always represent the direction of liquid film flow. Therefore, the absolute value of $\bar{u}_{if}$ should be taken in Eq. (4) before the procedure described above. In addition, the kinetic energy flow rate at $z=0$ is independent of the minimization of pressure energy consumption rate, because it is given as a boundary condition.

Given the above considerations, the minimum condition of pressure energy consumption rate for total two-phase flow can be expressed as follows:

\[
\frac{1}{2L} \left[ \frac{x_c}{\rho_g} \bar{u}_{gc}(\rho_g \bar{u}_{gc}^2) + (1-x_c) \left( \frac{W_c}{W} \right) \bar{u}_{ic}(\rho_i \bar{u}_{ic}^2) \right]
+ \left( \frac{W_f}{W} \right) \bar{u}_{if} \left( \frac{2 \eta}{r_i} \right)
+ \left[ \left( \frac{x_c}{\rho_g} \bar{u}_{gc} + (1-x_c) \left( \frac{W_c}{W} \right) \bar{u}_{ic} \right) \left( \alpha_c \rho_g \right) \right]
+ \left[ \left( \frac{x_c}{\rho_g} \bar{u}_{gc} + (1-x_c) \left( \frac{W_c}{W} \right) \bar{u}_{ic} \right) \left( \frac{2 \eta}{r_i} \right) \right]
\]  

(5)

Liquid flow rate $W_c$ of the two-phase flow core is finite unless all of the liquid supplied from the bottom
of channel flows up as liquid film. Therefore, \( \bar{u}_{ic} \) becomes infinite and, as a result, the left-hand side of Eq.(5) becomes infinite when \( ac=1 \). Similarly, the left-hand side of Eq.(5) also becomes infinite when \( ac=0 \), because gas flow rate \( W_{gc} \) supplied from the channel bottom is finite. Therefore, the left-hand side of Eq.(5) has a minimum point between \( ac=1 \) and \( ac=0 \).

The theory described above is similar to the minimum entropy production rate theory of Zivi\(^{10} \). However, in the present theory, pressure energy consumption rate (total mechanical work rate including all of the accelerational work, gravitational work and frictional work) is minimized instead of entropy production rate. The theory is applied to a unit mass of penetrating two-phase fluid instead of the existing two-phase fluid. Moreover, two-dimensional turbulent flow analysis is applied to evaluate the gravity convection effect due to density distribution in the channel cross section. These three points are the notable differences between the present theory and Zivi's theory.

2.3 Two-dimensional turbulent flow analysis

In order to evaluate \( \bar{u}_{gc}, \bar{u}_{ic}, \bar{u}_{ir} \) and \( W_{ir} \) in Eq. (5), two-dimensional turbulent flow analysis is performed.

Using the cylindrical coordinate, the force balance of liquid film is described by

\[
-\frac{dP}{dz} = \rho g + \frac{1}{r} \frac{d(r\tau)}{dr} = 0.
\]

(6)

Here, \( r \) can be written as

\[
\tau = \rho \left( \nu_i + \frac{1}{r} \frac{du_{ir}}{dr} \right) \frac{du_{ir}}{dr}.
\]

(7)

By multiplying Eq. (6) by \( rdr \) and integrating from \( r=r \) to \( r=r_w \), one can derive

\[
\frac{dP}{dz} + \rho g = \frac{2(\nu_i + \frac{1}{r} \frac{du_{ir}}{dr})}{r_w^2 - r^2} \frac{du_{ir}}{dr}.
\]

(8)

By this equation, shear stress \( \tau \) in liquid film flow can be calculated as a function of radial position \( r \), for the given \( dP/dz \) and \( \tau_w \).

From Eq.(7), the following equations are derived.

\[
r \geq 0 : \quad \frac{du_{ir}}{dr} = -\nu_i + \frac{\sqrt{\nu_i^2 + 4\frac{\alpha}{2} \tau_i}}{2r_i} \rho_i,
\]

(9)

\[
r < 0 : \quad \frac{du_{ir}}{dr} = \nu_i - \frac{\sqrt{\nu_i^2 - 4\frac{\alpha}{2} \tau_i \rho_i}}{2r_i}.
\]

(10)

Let us assume the van Driest-type\(^{10} \) expression of mixing length \( l_i \), i.e.,

\[
l_i = x(r_w - r) \left[ 1 - \exp \left(-\left(r_w - r\right)/\tau_i/\rho_i/(A \nu_i)\right) \right],
\]

(11)

where \( x = 0.4 \) and \( A = 26 \) for single-phase flow are chosen as the standard case. Using Eq.(9) or (10) and Eqs.(8) and (11), one can calculate \( du_{ir}/dr \) as a function of \( r \), only if \( dP/dz \) and \( \tau_w \) are given. By integrating Eq. (9) or (10) with respect to \( r \), the radial distribution of liquid film velocity \( u_{ir} \) is calculated. And if radial location \( r_i \) of the liquid film surface is known, liquid film flow rate \( W_{ir} \) and liquid film average velocity \( \bar{u}_{ir} \) are calculated, respectively, by

\[
W_{ir} = 2\pi \rho_i \int_{r_i}^{r_w} r u_{ir} dr,
\]

(12)

\[
\bar{u}_{ir} = \frac{W_{ir}}{\pi (r_w - r_i) \rho_i}.
\]

(13)

Similarly, for gas phase of the two-phase flow core, Eqs. (14) and (15) are derived, which correspond to Eqs. (8) and (9) or (10), respectively, i.e.,

\[
r = \frac{r}{r_i},
\]

(14)

\[
r < 0 : \quad \frac{dt_{gc}}{dr} = \frac{\nu_g - \sqrt{\nu_i^2 - 4\frac{\alpha}{2} \tau_i \rho_i}}{2l_{gc}}
\]

(15)

Here, let us assume the Plandtl-type expression of mixing length \( l_{gc} \) for gas phase of the two-phase core, i.e.,

\[
l_{gc} = 0.4(1 - r - r_i) / l_i.
\]

(16)

By integrating Eq.(15) with respect to \( r \), one can determine the radial distribution of \( u_{gc} \). Then, gas flow rate \( W_{gc} \) of the two-phase flow core can be calculated by

\[
W_{gc} = 2\pi \rho_g \int_{0}^{r_w} r u_{gc} dr,
\]

(17)

and this value should be equal to the supplied gas flow rate \( W_g \). That is,

\[
W_{gc} = W_g.
\]

(18)

By this condition, \( r_i \) can be determined for given \( dP/dz \) and \( \tau_w \) utilizing

\[
\frac{dP}{dz} = -(a \rho_g + (1 - a) \rho_i) g + 2\tau_w r_w
\]

(19)

and Eq. (1). Average gas and liquid velocities, \( \bar{u}_{gc} \) and \( \bar{u}_{ic} \), are calculated by

\[
\bar{u}_{gc} = \frac{W_{gc}}{\pi r l_{gc} \rho_g}
\]

(20)

\[
\bar{u}_{ic} = \frac{W_{ic}}{\pi r l_{ic} (1 - ac) \rho_i},
\]

(21)

where \( W_{gc} \) is calculated by Eq. (2).

2.4 Calculation scheme

From Section 2.3, if \( dP/dz \) and \( \tau_w \) are given, the two-phase flow situation can be uniquely determined. As described in Section 2.2, one of these two parameters, for example \( \tau_w \), can be determined from Eq. (5). Therefore, if \( dP/dz \) is known, one can predict the two-phase flow condition uniquely for given gas and liquid flow rates supplied from the bottom of the channel. For \( dP/dz \), an experimental value was used tentatively in the present work. The calculation scheme is shown in Fig. 3.
3. Verification of Theory

3.1 Experiment

In order to verify the proposed analytical method introduced in Section 2, a vertical upward air–water two-phase flow experiment was performed at atmospheric pressure and room temperature. The experimental facility used is illustrated in Fig. 4.

![Fig. 3 Calculation scheme](image)

![Fig. 4 Experimental facility](image)

Inner diameter of the test pipe is 22 mm. A strain gauge type differential pressure meter, two quick–shut valves, an electromagnetic flow meter and a conduction-type liquid film probe are attached to measure pressure drop, average void fraction, average velocity of continuous liquid (liquid film) and liquid film thickness, respectively. Supplied air and water flow rates were measured with a rotameter and another electromagnetic flow meter, respectively.

Only the data of which liquid film average velocity was negative (downward) were selected in this study. Although flow pattern was judged to be an annular flow in these experiments, strong pulsation was observed in most cases.

3.2 Analytical result and discussion

In this section, some results of verification analysis are introduced.

When supplied gas and liquid flow rates are given, most calculated variables such as liquid film flow rate change monotonously with increase in guessed value of liquid film thickness as seen in Fig. 5(a). However, the pressure energy consumption rate term given by the left-hand side of Eq. (5) has a minimum value.
as shown in Fig. 5(b) and this minimum point gives the most stable two-phase flow which is actually realized.

Figure 6 shows some examples of calculated radial distributions of continuous phase velocity. The vertical lines indicate the radial locations of the liquid film surface. Curves to the right of these vertical lines show the velocity distributions of liquid film and curves to the left show those of gas phase in the two-phase flow core. Inner radius of the channel is 0.011 m and the channel wall is indicated by hatching. The major parameter is gas superficial velocity as given in the figure but liquid superficial velocity also varies, being 0.16, 0.17 and 0.07 m/s, respectively, from the bottom line to the top. It can be easily seen from this figure that both liquid film thickness and downward liquid film velocity become larger when gas superficial velocity becomes smaller. Such characteristics can be considered to be quite reasonable.

The relationship between nondimensional liquid film thickness \( \theta \) and liquid film Reynolds number \( Re_f \) is shown in Fig. 7. Here, \( \theta \) and \( Re_f \) are defined, respectively, as:

\[
t\theta = t_f \left( \frac{g}{\nu_f} \right)^{1/3}
\]
\[
Re_f = \frac{\rho g L_f t_f}{\nu_f}
\]

The Nusselt curve\(^6\) for laminar falling film and the Brotz\(^7\) curve for turbulent falling film are shown in the figure for reference. In the case of low mass-velocity two-phase flow with downward liquid film flow, the liquid film surface is pulled up by the two-phase flow core in the direction opposite to that of the liquid film flow itself and, therefore, liquid film thickness becomes larger than the reference curves. This fact seems to cause the lower void fraction of low mass-velocity two-phase flow.

Figures 8 and 9 show relationships between interfacial friction factor \( f_i \) and gas Reynolds number \( Re_g \) and between wall friction factor \( f_w \) and liquid film Reynolds number \( Re_f \), respectively. Here, \( f_i \), \( f_w \) and \( Re_f \) are defined as:

\[
f_i = - \frac{2 \tau_i}{\rho_a H_g}
\]
\[
f_w = \frac{2 \tau_w}{\rho_l H_f}
\]
\[
Re_f = \frac{\rho_l \nu_f}{\nu_g}
\]

Fig. 6  Velocity distribution in continuous phase

Fig. 7  Characteristics of liquid film thickness

Fig. 8  Characteristics of interfacial friction factor

Fig. 9  Characteristics of wall friction factor
Wall friction factor for laminar flow and that for turbulent flow estimated by the Blasius equation are shown for reference. Because of large velocity gradient and resultant high eddy diffusivity of momentum, extremely high interfacial and wall friction factors were calculated.

Shown in Fig. 10 is a comparison between void fraction $a_c$ of the two-phase flow core and channel-averaged void fraction $a$. Since a certain amount of liquid is adjacent to the channel wall in the form of liquid film, void fraction of the two-phase flow core is always higher than the channel-averaged void fraction. In the present analysis, channel-averaged void fraction over 0.9 was not realized. Such limitation of the channel-averaged void fraction is considered a unique characteristic of low-mass-velocity two-phase flow with local liquid fall-back.

In Fig. 11, the calculated liquid film thickness is compared with the measured one using the conduction-type liquid film probe. The parameter in Fig. 11 (a) is $\chi$ in Eq. (11) and that in Fig. 11 (b) is $A$ of the same equation. The standard combination of $\chi = 0.4$ and $A = 26$ for single-phase flow, which is indicated in the figures by solid circles, seems to give a reasonable estimation. However, the optimum combination of $\chi$ and $A$ should be determined after more comprehensive verification studies in the future.

Finally, to examine the consistency between the drift flux model and the present analysis, the calculated average gas velocity $u_g$ was plotted with respect to the corresponding total superficial velocity $J_T$ in Fig. 12. As is clearly observable from this figure, a very good linear relationship between these two parameters was confirmed. Distribution parameter $C_0$ and gas drift velocity $u_{dg}$ based on the present analysis were 0.998 and 1.12 m/s, respectively. Those parameters estimated by the following conventional equation\(^{(18)}\) for annular two-phase flow are 1.0 and 0.3~2.0 m/s, respectively.

$$C_0 = 1.0, \quad u_{dg} = 23\{\mu T/\langle \rho D \rangle \}^{1/2}(\rho_1 - \rho_2)/\rho_c. \quad (27)$$

Such a good agreement with the drift flux model suggests that the principle of minimum pressure energy consumption rate can be applied to the vertical low-mass-velocity two-phase flow.

4. Conclusions

(1) To improve the reliability of two-phase flow
analysis using the two-fluid two-phase flow model, the minimum pressure energy consumption rate principle was introduced.

(2) Two-dimensional turbulent flow analysis was applied to the low-mass-velocity two-phase flow which had a two-region structure of a continuous liquid film adjacent to the channel wall and an upward two-phase flow core in the central portion of the channel.

(3) Through an experimental analysis based on the data from an air-water two-phase flow experiment at atmospheric pressure and room temperature, reasonable predictive ability of the theory was demonstrated. That is, very good agreement with the drift flux model was achieved.

(4) Characteristics of downward liquid film flow of low-mass-velocity two-phase flow can be reasonably analyzed, suggesting the validity of the present analytical method.

References


