Reconsideration of the State-of-the-Art of the Shock Wave Reflection Phenomenon in Steady Flows*

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Based on recent analytical, experimental and numerical studies, a new state-of-the-art of oblique-shock-wave reflections in steady flows has been established. The main findings of these analytical, experimental and numerical studies are summarized. The recently established state-of-the-art is summarized in the conclusion section of this review article.

** Key Words:** Shock Wave Reflection, Principle of Minimum Entropy Production, Steady Flow Numerical Simulations

1. Introduction

As indicated in Ref. (1), two shock-wave-reflection configurations are possible in steady flows, namely, regular reflection (RR) and Mach reflection (MR). Schematic illustrations of the wave configurations of a regular and a Mach reflection, together with the definitions of some flow parameters, are shown in Figs. 1(a) and 1(b), respectively.

The regular reflection (RR) consists of two shock waves, namely, the incident shock wave, \( i \), and the reflected shock wave, \( r \). They meet at the reflection point, \( R \), which is located on the reflecting surface. The flow states are (0) ahead of \( i \), (1) behind it and (2) behind \( r \). The angle of incidence, \( \phi_i \), of a regular reflection is sufficiently small that the streamline deflection, \( \delta_i \), caused by the incident shock wave, \( i \), can be canceled by the opposite streamline deflection, \( \delta_r \), caused by the reflected shock wave, \( r \). Therefore, the boundary condition of a regular reflection is

\[
\delta_i - \delta_r = 0. \tag{1}
\]

The Mach reflection (MR) consists of three shock

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Fig. 1 Schematic illustration of the wave configurations of (a) regular reflection and (b) Mach reflection. \( i \) - incident shock wave, \( r \) - reflected shock wave, \( m \) - Mach stem, \( s \) - slipstream, \( R \) - reflection point, \( T \) - triple point, \( \phi_i \) and \( \phi_2 \) - angles of incidence of \( i \) and \( r \), respectively, \( \delta_i \), \( \delta_2 \) and \( \delta_3 \) - flow deflections while passing through \( i \), \( r \) and \( m \), respectively, \( \omega_i \) and \( \omega_r \) - wave angles of \( i \) and \( r \), respectively. Note that for both configurations \( \omega_i = \phi_i \) and \( \omega_r = \phi_2 - \delta_i \).
waves, namely, the incident shock wave, \( i \), the reflected shock wave, \( r \), and the Mach stem, \( m \), and one slippstream, \( s \). They all meet at a single point known as the triple point, \( T \). The Mach stem, \( m \), is usually a curved shock wave which is perpendicular to the surface of the reflecting wedge at the reflection point \( R \). The flow states are (0) ahead of \( i \) and \( m \), (1) behind \( i \), (2) behind \( r \) and (3) behind \( m \). Unlike the case of a regular reflection where the net deflection of the streamline is zero, in the case of a Mach reflection the net deflection of the streamline is nonzero, in general, and the streamlines behind the triple point are directed towards the reflecting wall. Since the streamlines on both sides of the slippstream must be parallel to each other, the boundary condition of a Mach reflection is

\[
\theta_1 - \theta_2 = \theta_n \quad (2)
\]

It should be noted here that Eqs. (1) and (2) are based on local considerations in the vicinities of the reflection point of an RR and the triple point of an MR, respectively. In order for these conditions to be global, the discontinuities, i.e., shock waves and slippstreams, must be straight so that the flow regions bounded by them are uniform.

Graphical solutions in the pressure–deflection plane [i.e., the \((P, \theta)\)-plane] have been traditionally used to illustrate and better understand the shock wave reflection phenomenon in general, and possible \( RR \rightarrow MR \) transition criteria in particular. Examples of five different \( I-R \) polar combinations for increasing values of \( \phi_i \) are shown in Figs. 2(a) to 2(e). The locus of all the pressures achievable from the free stream of state (0) via an oblique shock wave deflecting the flow through an angle \( \theta \) is given by the \( I \)-polar. Thus, state (1) in Figs. 1(a) or 1(b) maps into point (1) in Figs. 2(a) to 2(e). The locus of all the pressures achievable from the free stream of state (1) via an oblique shock wave deflecting it by an angle \( \theta \) is given by the \( R \)-polar.

The boundary condition of a regular reflection given by Eq. (1) implies that the solution of a regular reflection in the \((P, \theta)\)-plane is at the point where the \( R \)-polar intersects the \( P \)-axis. Two such points are obtained by the \( I-R \) polar combination shown in Fig. 2(a). It is an experimental fact that the one resulting in the higher pressure (marked by an open circle in Fig. 2(a)) is unstable. It was shown recently in Ref. (2) that the higher pressure solution violates the principle of minimum entropy production and hence is aphysical. Consequently, state (2) of Fig. 1(a) maps into point (2) in Fig. 2(a).

The boundary condition of a Mach reflection given by Eq. (2) implies that the solution of a Mach reflection is at the point where the \( I \) and \( R \) polars intersect. Such a point is shown by the \( I-R \) polar combination shown in Fig. 2(e). States (2) and (3) of the Mach reflection, shown in Fig. 1(b), map into that point. Note that state (2) is on the \( R \)-polar and state (3) is on the \( I \)-polar.

Three intermediate \( I-R \) polar combinations are shown in Figs. 2(b) to 2(d). If one starts with initial conditions, i.e., \( M_o \) and \( \phi_i \), appropriate to the regular reflection whose solution is given by the \( I-R \) polar combination shown in Fig. 2(a), and then increases the angle of incidence \( \phi_i \) while keeping the uniform flow Mach number, \( M_o \), constant, the \( I-R \) polar combination shown in Fig. 2(b) is eventually reached. Since for this combination the \( R \)-polar intersects both the \( P \)-axis and the \( I \)-polar, both \( RR \) and \( MR \) are theoretically possible at this intersection point. Furthermore, this \( I-R \) polar combination represents a possible condition for the \( RR \rightarrow MR \) transition. This possible transition, which was first suggested by von Neumann in the early 1940’s (see Ref. (3)) was re-

![Fig. 2 Various I-R polar combinations illustrating (a) a regular reflection, (b) the von Neumann criterion (also known as the mechanical equilibrium criterion), (c) regular and Mach reflection for the same initial conditions, (d) the detachment criterion, and (e) a Mach reflection.](image-url)
introduced by Ref. (4) in which it was called the “mechanical equilibrium” criterion. It is known nowadays as the von Neumann criterion and can be formulated by combining Eqs. (1) and (2) to give

\[ \theta_1 - \theta_2 = \theta_3 = 0. \]  

(3)

As can be seen in Fig. 2(b) the Mach stem in the vicinity of the triple point is normal to the oncoming flow. The incident shock wave angle appropriate to the I-R polar combination shown in Fig. 2(b) will be denoted by \( \omega_i^e \). Note that for \( \omega_i < \omega_i^e \) a Mach reflection wave configuration is impossible, hence \( \omega_i^e \) is the smallest incident shock wave angle for which a Mach reflection wave configuration is possible for a given flow Mach number \( M_b \).

A further increase in the incident shock wave angle eventually results in the I-R polar combination shown in Fig. 2(d), which corresponds to the largest value of \( \omega_i \) for which a regular reflection wave configuration is obtainable for a given flow Mach number \( M_b \). Note that an increase of \( \omega_i \) beyond the value appropriate to that of Fig. 2(d) results in a situation similar to that shown in Fig. 2(e), in which the \( R \)-polar does not intersect the \( P \)-axis and hence a regular reflection wave configuration is impossible. Consequently, the I-R polar combination of Fig. 2(d) represents another possible condition for the RR → MR transition. This possible transition, which was also suggested in Ref. (3) in the early 1940’s, is known as the detachment criterion because it corresponds to the case in which the streamline deflection through the reflected shock wave is maximal. Its mathematical formulation is

\[ \theta_1 - \theta_2 = 0, \]  

(4)

where \( \theta_2 \) is the detachment deflection angle. The incident shock wave angle appropriate to the I-R polar combination shown in Fig. 2(d) will be denoted by \( \omega_i^d \). Note that for \( \omega_i > \omega_i^d \) a regular reflection is impossible, hence \( \omega_i^d \) is the largest incident shock wave angle for which a regular reflection wave configuration is possible for a given flow Mach number \( M_b \).

Based on the foregoing discussion, for all the incident shock wave angles in the range \( \omega_i^1 \leq \omega_i \leq \omega_i^d \), both regular and Mach reflection wave configurations are theoretically possible. A typical I-R polar combination appropriate to this dual-solution domain is shown in Fig. 2(c). The point where the \( R \)-polar intersects the \( P \)-axis indicates a possible RR solution, while the point where it intersects the \( I \)-polar indicates a possible MR solution.

It should be noted here that since both RR and MR wave configurations are theoretically possible in the dual-solution domain given by \( \omega_i^1 \leq \omega_i \leq \omega_i^d \), the RR → MR transition could occur at any value \( \omega_i \) inside that range. Consequently, the von Neumann criterion, \( \omega_i = \omega_i^N \), is, in fact, the lowest possible value of \( \omega_i \) for transition, while the detachment criterion, i.e., \( \omega_i = \omega_i^d \), is, in fact, the largest possible value of \( \omega_i \) for transition.

It should also be noted here that while transition at angles of incidence in the range \( \omega_i^1 < \omega_i \leq \omega_i^d \) involves a sudden pressure jump behind the reflected shock wave (i.e., a positive jump for the MR → RR transition and a negative jump for the RR → MR transition), a transition at \( \omega_i = \omega_i^N \) is continuous as far as the pressure is concerned.

The dual-solution domain for which both regular and Mach reflection wave configurations are possible in the \((M_b, \omega_i)\)-plane is shown in Fig. 3 for \( \gamma = 1.4 \), where \( \gamma \) is the ratio of specific heat capacities. The detachment \( (\omega_i = \omega_i^d) \) and the von Neumann \((\omega_i = \omega_i^N)\) criteria divide the \((M_b, \omega_i)\)-plane into three domains: a domain in which only RR wave configurations are theoretically possible \( (\omega_i < \omega_i^d) \), a domain in which only MR wave configurations are theoretically possible \( (\omega_i > \omega_i^d) \), and a domain in which both RR and MR wave configurations are theoretically possible \( (\omega_i^1 \leq \omega_i \leq \omega_i^d) \).

While the detachment criterion exists for all values of \( M_b > 1 \), the von Neumann criterion does not exist in the range \( M_b < 2.20 \). \( M_b = 2.20 \) is the point, marked by K in Fig. 3, where the two transition lines arising from these two transition criteria meet. Traditionally, it has been used to distinguish between weak and strong incident shock waves. The I-R polar combination appropriate to the conditions of point K is shown in Fig. 4.

Fig. 3 Domains of possible reflection configurations in the \((M_b, \omega_i)\)-plane. \( \omega_i = \omega_i^1 \) and \( \omega_i = \omega_i^d \) are the transition lines corresponding to the detachment and von Neumann criteria, respectively.
1.1 The RR → MR Transition — The state-of-the-art until recently

Until the beginning of the analytical, experimental and numerical studies whose main results will be presented subsequently and which were initiated in the summer of 1993, Ref. (5) was accepted among the scientific community as the state-of-the-art regarding the RR→MR transition in steady flows.

Based on the experimental results of Ref. (5), as well as those presented in Refs. (4), (6) and (7), it was concluded in Ref. (5) that "...in steady flow, the transition from regular to Mach reflection of strong shock waves [i.e., \( M_s > 2.20 \)] occurs at the von Neumann condition [i.e., \( \omega_l = \omega_r^p \)] and not at the detachment condition [i.e., \( \omega_l = \omega_r^p \)]..." as mentioned in well-known textbooks such as Refs. (8) to (10).

The above conclusion reached in Ref. (5) was based on experiments which were conducted in the range \( 2.8 < M_s < 5.5 \) where, as can be seen from Fig. 3, the differences between the transition lines appropriate to \( \omega_l = \omega_r^p \) and \( \omega_l = \omega_r^f \) are sufficiently large to clearly distinguish between them.

Based on the length scale concept developed in Ref. (11), it was hypothesized there that a hysteresis should exist in the RR→MR transition. Consider Fig. 3 and assume that one starts with an MR at \( \omega_l > \omega_l^f \), and then \( \omega_l \) is slowly decreased while \( M_s \) is kept constant. As a result, the Mach stem height decreases until it vanishes at \( \omega_l = \omega_l^f \) where a smooth transition from MR to RR occurs. If the decrease in \( \omega_l \) continues, then RR is maintained. If \( \omega_l \) is now slowly increased, then since the flow field is free of significant disturbances, the RR should be maintained until \( \omega_l = \omega_l^f \) where a sudden transition from RR to MR should occur. The hysteresis loop suggested above is shown in the \( (\omega_l, \theta) \)- and \( (\omega_l, \omega_i) \)-planes in Figs. 5(a) and 5(b), respectively. \( \omega_l \) and \( \omega_i \) are the angles which the incident and reflected shock waves make with the horizontal axis as shown in Figs. 1(a) and 1(b), and \( \theta \) is the Mach stem height. Experimental attempts to confirm this hypothesis regarding the hysteresis in the RR→MR transition failed and it was concluded in Ref. (5) that "the hysteresis effect predicted in Ref. (11) could not be confirmed". For this reason the RR was referred to in the region \( \omega_l^p < \omega_l < \omega_l^f \) as an unstable regular reflection.

Based on the foregoing introduction, the following summarizes the state-of-the-art regarding the RR→MR transition of planar shock waves in steady flows over straight wedges, as it existed until recently.

1. The RR→MR and the MR→RR transitions occur at the von Neumann condition, i.e., at \( \omega_l = \omega_l^f \).
2. RR wave configurations in the dual-solution

Fig. 4 The \( I-R \) polar combination corresponding to point K in Fig. 3

Fig. 5 Schematic illustrations of the possible hysteresis in the RR→MR transition. (a) in the \( (\theta, \omega_i) \)-plane, and (b) in the \( (\omega_l, \omega_i) \)-plane.
domain, i.e., \( \omega^f < \omega_i < \omega^p \), are unstable.

3. A hysteresis phenomenon does not exist in the RR→MR transition.

It is very important to note here that the above conclusions were based on two sets of experiments, one of which was presented in Refs. (4) and (6) and the other in Refs. (5) and (7). Consequently, there were no experimental reasons to question the validity of the above presented state-of-the-art.

2. Recent Analytical, Experimental and Numerical Findings Regarding the RR→MR Transition

2.1 Analytical results

In a recent analytical study presented in Ref. (2), it was shown by applying the principle of minimum entropy production that, contrary to the results of Ref. (5) and its conclusion, RR wave configurations are stable in most of the \( \omega^f \leq \omega_i \leq \omega^p \) domain.

Figure 6 is a reproduction of Fig. 3. In addition to the \( \omega_i = \omega^f \) and \( \omega_i = \omega^p \) transition lines it also includes the curve \( \omega_i = \omega^p \). The domain \( \omega_i \leq \omega^p \) satisfies the principle of minimum entropy production and hence mathematical solutions of regular reflection there are stable. Note that the \( \omega_i = \omega^p \) line is located very close to the \( \omega_i = \omega^f \) line. Consequently, out of the entire \( \omega^f \leq \omega_i \leq \omega^p \) domain for which RR (and MR) is theoretically possible, only in a very narrow domain \( \omega^f < \omega_i < \omega^p \) is RR unphysical (i.e., unstable).

As a result of this finding it was decided to conduct a detailed experimental investigation of the RR→MR transition of planar shock waves over straight reflecting surfaces in steady flows in an attempt to establish a stable regular reflection inside the \( \omega^f < \omega_i < \omega^p \) domain. Details of this study can be found in Ref. (12). In the following only the major findings of this study are presented.

2.2 Experimental results

The first set of experiments was aimed at determining the conditions where the RR→MR transition takes place.

Initially an MR wave configuration at about \( \omega_i \approx 40^\circ \) (in all the experiments \( M_0 = 4.96 \)) was established. Then the angles of attack of the two symmetric reflecting wedges were simultaneously and continuously decreased at a rate of about 0.57 deg/s. As a result, the wave angle, \( \omega_i \), of the incident shock wave decreased. This resulted in a continuous decrease in the height of the Mach stem of the Mach reflection until we reached a situation in which the Mach stem completely vanished. The point where this occurred was defined as the experimental MR→RR transition point.

The angles of attack of the reflecting wedges were then further decreased until the wave angle of the incident shock wave reached a value of \( \omega_i \leq 30^\circ \) which, as can be seen in Fig.3, is well inside the domain in which, theoretically, only RR is possible. Then the direction of the rotation of the reflecting wedges was reversed and their angles of attack were simultaneously and continuously increased.

While increasing the angles of attack it was realized that the RR wave configuration was not terminated at the previously determined MR→RR transition point. Instead the RR wave configuration was maintained for some time until it suddenly changed to an MR wave configuration. The point where this occurred was defined as the experimental RR→MR transition point. The height of the Mach stem of the suddenly formed MR was definitely nonzero and hence, the actual transition violated the mechanical equilibrium concept which was proposed in Ref. (4).

The experimental results described above are shown in Fig.7 in the \( (\omega_i, \omega) \)-plane. MR wave configurations are marked with triangles and RR wave configurations are marked with circles. The von Neumann transition angle for \( M_0 = 4.96 \) is \( \omega^p = 30.88^\circ \) and the detachment transition angle for \( M_0 = 4.96 \) is \( \omega^p = 39.33^\circ \).

The hysteresis phenomena in the RR→MR transition which was shown schematically in Fig.5(b) is clearly evident in Fig.7. While the MR→RR transition occurs at the von Neumann angle, i.e., \( \omega^p (\text{MR} \rightarrow \text{RR}) = \omega^p = 30.9^\circ \), the reverse transition, the RR→MR transition, occurs at \( \omega^p (\text{RR} \rightarrow \text{MR}) = 37.2^\circ \). Thus, it is clear that a hysteresis phenomenon which was hypothesized in Ref. (11), but was not confirmed, does exist in the RR→MR transition in steady flows.

Note also that the experimental results shown in Fig.7 are consistent with the above-mentioned analytical findings of Ref. (2) that RR wave configurations
are stable in the dual-solution domain.

It should also be noted that although the MR→RR transition is seen in Fig. 7 to occur at \( \omega_i > \omega_i^* \) (note that \( \omega_i \approx 32.5^\circ > 30.9^\circ = \omega_i^* \) for the MR wave configuration with the smallest value of \( \omega_i \)), this was not the case. The recorded video films clearly indicated a smooth transition at the point where the Mach stem of the Mach reflection vanished.

In order to investigate the effect of the geometrical setup on the transition process, experiments similar to the one described above were repeated for three different reflecting wedge surface lengths, \( w \) (50, 60 and 70 mm). Each reflecting wedge was used with three different combinations of inlet cross-sectional areas, \( h_m \), i.e., the distance between the leading edges of the reflecting wedges (about 70, 85 and 100 mm). In seven out of the above nine possible combinations of \( w \) and \( h_m \), both the RR→MR and the MR→RR processes were recorded. The results of these experiments, which are shown in Fig. 8, again verify the above-mentioned hysteresis phenomenon in the transition process. All the experiments represented by open symbols for which the MR→RR transition was investigated show transition in the vicinity of the von Neumann transition angle \( \omega_i^* \). On the other hand, all the experiments represented by solid symbols for which the RR→MR transition was investigated indicate that the transition occurred between the von Neumann, \( \omega_i^* \), and the detachment, \( \omega_d \), transition angles, in the vicinity of \( \omega_i = 37^\circ \). In addition, it is clearly indicated in Fig. 8 that the RR→MR transition depends slightly on both the inlet cross section, \( h_m \), and the reflecting wedge surface length, \( w \).

Consider Figs. 9(a) and 9(b) where the dependence of the nondimensional Mach stem height \( l_m/w \) (\( l_m \) is the height of the Mach stem and \( w \) is the length of the reflecting wedge) on the angle of incidence of the oncoming flow \( \omega_i \) is shown for two different inlet cross-sectional areas \( h_m \). The solid lines are curve fits. The value of \( \omega_i \) at the point where the fitted curves approached \( l_m \rightarrow 0 \) is exactly equal to the von Neumann angle \( \omega_i^* \). Hence, based on Figs. 9(a) and 9(b) there is little doubt that the MR→RR transition occurs at the von Neumann angle, i.e., \( \omega_i^*(\text{MR→RR}) = \omega_i^* \). Furthermore, the transition angle seems to be independent of the inlet cross-sectional area \( h_m \). In addition, the possible effect of the viscous growth on both sides of the slipstream on the MR→RR transition seems to be negligible.

It should be noted here that unlike the experimental results of Ref. (5), where \( l_m/w \) was found to depend linearly on the incident shock wave angle \( \omega_i \), the experimental results of Ref. (12) reveal a nonlinear dependence. The reason for this difference lies simply in the fact that the experiments of Ref. (5) were limited to a relatively narrow range of \( \omega_i \). For example, in the experiment with \( M_0 = 5 \), the investigated range of \( \omega_i \) was \( 30.9^\circ = \omega_i^* \leq \omega_i \leq 34^\circ \). The results of Ref. (12) in this narrow domain also appear to show a linear dependence (see the dashed lines in Figs. 9(a) and 9(b)). However, when the entire range \( 30.9^\circ = \omega_i^* \leq \omega_i \leq 39.3^\circ \) is considered, a clear nonlinear dependence of \( l_m \) on \( \omega_i \) is evident. It should also be noted
here that in the attempt of Ref. (13) to analytically predict the Mach stem height of Mach reflection wave configurations in steady flows, a nonlinear dependence of \( I_m \) on \( \omega_i \) was obtained. The analytical curves of Ref. (13) resembled those shown in Figs. 9(a) and 9(b). The actual hysteresis loop in the \((I_m, \omega_i)\)-plane is shown in Fig. 10. Note that unlike the loop shown in Fig. 5(b), here \( I_m \) does not decrease linearly with \( \omega_i \), and the RR → MR transition takes place at \( \omega_i \approx 37.2^\circ \) rather than \( \omega_i = \omega_{i}^* = 39.3^\circ \).

The experimental results described above also contradict the conclusion of Ref. (4) that the RR → MR transition should occur at the von Neumann condition, i.e., \( \omega_i = \omega_{i}^* \), "in such a way that mechanical equilibrium of the system is preserved through the process." The sudden transition from RR to MR at \( \omega_i > \omega_{i}^* \) occurs at a point where the mechanical equilibrium requirement was clearly violated.

Another set of experiments was aimed at investigating which of the two wave configurations, RR or MR, is stable in the dual-solution domain. For this study MR wave configurations were first established in the domain \( \omega_{i}^* < \omega_i < \omega_{i}^*(RR → MR) \), i.e., \( 30.9^\circ < \omega_i < 37.2^\circ \) for \( M_0 = 4.96 \). At this stage the lower reflecting wedge was completely removed from the flow field by tilting its holding arm sideways by 90°. As a result, the MR wave configuration vanished, leaving a straight and attached shock wave emanating from the leading edge of the upper reflecting wedge. Then the lower reflecting wedge was brought back to its original position. When the transient process was over, the stable wave configuration finally obtained was either an RR or an MR, depending on whether \( \omega_i \) was larger or smaller than a critical value, \( \omega_{i}^* \). The exact value of \( \omega_{i}^* \), which was close to 34.5°, has not been determined yet. In summary, MR wave configurations are stable in the dual-solution domain as longs as \( \omega_{i}^* < \omega_i < \omega_{i}^* \), and RR wave configurations are stable when \( \omega_{i}^* < \omega_i < \omega_{i}^*(RR → MR) \). (Recall that for the flow Mach number in the present experiments, i.e., \( M_0 = 4.96 \), \( \omega_{i}^* = 30.9^\circ \), \( \omega_{i}^*(RR → MR) = 37.2^\circ \) and \( \omega_{i}^* \approx 34.5^\circ \).)

It should finally be noted that owing to the very large disturbances associated with the above-mentioned experimental procedure, it is the author’s belief that the results in which RR wave configurations were found to be the stable reflections inside the dual-solution domain contradict the conclusion of Ref. (11) that inside the dual-solution domain, \( \omega_{i}^* < \omega_i < \omega_{i}^* \), large disturbances should be sufficient to cause the reflection to be an MR rather than an RR.

Typical color schlieren photographs of RR and MR wave configurations, which were obtained for almost identical initial conditions, i.e., \( M_0 \) and \( \theta_0 \), and which are inside the dual-solution domain, are shown...
in Figs. 11(a) and 11(b).

2.3 Numerical result

A numerical investigation aimed at verifying the above findings of both the analytical (Ref. (2)) and the experimental (Ref. (12)) studies was reported in Ref. (14). It was clearly demonstrated in this study that the reflection wave configuration, which is established inside the dual-solution domain, depends on the distance from the trailing edge of the reflecting wedge to the bottom surface, $h$. This distance could be considered in the two-dimensional case as the exit cross-sectional area of the 2-D converging nozzle which is formed by the reflecting wedge and the bottom surface.

The numerical method used in Ref. (14) was the LCPFCT algorithm of Ref. (15) which is a flux-corrected transport algorithm for solving generalized continuity equations. This algorithm is one of the latest one-dimensional Flux-Corrected Transport (FCT) algorithms with fourth-order phase accuracy and minimum residual diffusion. Moreover, one-dimensional continuity equation solvers such as LCPFCT can be used repetitively to construct a multi-dimensional program by timestep splitting in the different coordinate directions.

It was shown in Ref. (14) that for a given set of initial conditions, i.e., flow Mach number $M_0$, reflecting wedge angle $\theta_w$ and reflecting wedge length $w$, a reflection, either RR or MR, will occur provided that the exit cross-sectional area at the trailing edge of the reflecting wedge, $h$, is in the interval $h_{\text{min}} < h < h_{\text{max}}$, where $h_{\text{min}}$ and $h_{\text{max}}$ are defined in Fig. 12. It was clearly shown in Ref. (14) that when $h < h_{\text{min}}$, one obtains a situation in which the 2-D converging nozzle formed by the reflecting wedge and the line of symmetry is unstarted, and as a result a stable bow shock wave is finally established ahead of the unstarted 2-D converging nozzle. The flow behind it which eventually passes through the 2-D converging nozzle is subsonic.

When the exit cross-sectional area was in the range $h_{\text{min}} < h < h_{\text{max}}$, it was found that when $h$ was slightly larger than $h_{\text{min}}$, a clear MR wave configuration was established. When $h$ was increased, the triple point moved backwards (downstream) and the height of the Mach stem was decreased. Further increase of $h$ finally resulted in a situation in which the Mach stem vanished and the reflection became regular. Thus, while stable Mach reflection wave configurations were established numerically in the range $h_{\text{min}} < h < h_{\text{tr}}$, stable regular reflection wave configurations were obtained when $h_{\text{tr}} < h < h_{\text{max}}$.

Typical density color plots of RR and MR wave configurations obtained by the above-mentioned numerical simulation for identical conditions, i.e., $M_0$ and $\theta_w$, but different values of $h$ are shown in Figs. 13(a) and 13(b).

The domains of RR and MR wave configurations inside the dual-solution domain in the $(M_0, h)$-plane are shown in Fig. 14. The symbols indicate the numerically obtained value of $h_{\text{tr}}$ for the five cases which were investigated in Ref. (14). In general, the $h=h_{\text{tr}}$ line is seen to decrease linearly with increasing incident flow Mach number.

3. Conclusion—The Newly Established State-of-the-Art

The conclusions of the analytical (Ref. (2)), experimental (Ref. (12)) and numerical (Ref. (14)) investigations described above comprise the new state-of-the-art regarding the reflection of oblique shock waves in steady flows, which can be summarized as follows:

1. Regular reflection (RR) wave configurations are theoretically impossible in the domain $\omega_1 > \omega_0^R$.
2. Mach reflection (MR) wave configurations are theoretically impossible in the domain $\omega_1 < \omega_0^M$.
3. Inside the dual-solution domain, i.e., $\omega_0^R < \omega_1 < \omega_0^M$ both RR and MR wave configurations are stable.
4. The finally established wave configuration inside the dual-solution domain depends on geometri-

Fig. 12 Schematic illustration of the lower (a) and upper (b) limits on the exit cross-sectional area, $h$, i.e., $h_{\text{min}}$ and $h_{\text{max}}$, respectively.
Fig. 11 Color schlieren photographs of (a) an RR with $M_0=4.96$ and $\theta_w=25^\circ$, and (b) an MR with $M_0=4.96$ and $\theta_w=24.3^\circ$ as obtained in the experiments reported in Ref. (12).

5. The RR→MR transition can occur anywhere inside the dual-solution domain.

6. A hysteresis exists in the RR→MR transition.

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Gabi Ben-Dor

Professor Gabi Ben-Dor was born on June 10, 1950 in Beer-Sheva, Israel, to Shoshana and Paul Sandor, who survived the Holocaust and emigrated to Israel from Hungary in 1949.

Professor Ben-Dor received his B.Sc. degree (cum laude) in Mechanical Engineering from the Ben-Gurion University of the Negev in 1975. He obtained his M.Sc. degree (cum laude) in Mechanical Engineering from the Ben-Gurion University of the Negev in 1976. In 1978 he obtained his Ph.D. from the Institute of Aerospace Studies of the University of Toronto where he has also received the G.N. Paterson Award for outstanding performance in graduate studies.

Professor Ben-Dor joined the Department of Mechanical Engineering of the Ben-Gurion University of the Negev in 1979 as a Lecturer. In 1981 he was promoted to the rank of Senior Lecturer, in 1985 to the rank of Associate Professor, and since 1990 he has been a Professor.

From 1987 to 1991 he served as the Head of the Department of Mechanical Engineering and since 1994 he is the Dean of the Faculty of Engineering Sciences.

Since 1978 Professor Ben-Dor has published a book entitled "Shock Wave Reflection Phenomena", more than 110 papers in scientific journals and presented about 150 papers in scientific conferences and symposia. Of his journal papers about 50 are directly related to the reflection phenomenon of shock waves.

In the past 15 years, Professor Ben-Dor held Visiting Professor positions in the Department of Physics of the University of Victoria, Victoria, British Columbia, Canada (12 months), Institute of Fluid Science of Tohoku University, Sendai, Japan (6 months), Laboratoire d'Aerothermique du C.N.R.S., Meudon, France (1 month), Systemes Energetiques et Transferts Thermiques of Université de Provence, Marseille (2 months) and Sosswellenlabor of RWTH, Aachen, Germany (2 months).