The Effects of Plasma Fluctuation in MHD Generators*

(Studies with a Numerical Turbulence Model)

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The effects of the spatiotemporal inhomogeneity of a plasma in thermal equilibrium on the local and output characteristics of a Faraday-type MHD generator were investigated using numerically modeled turbulent velocity and temperature data. The conditions of Saha equilibrium in the range of the mean temperature level and the maximum turbulent frequency were given. The mean conductivity formula was also given as a function of the rms fluctuation. The average correlations between the electric field fluctuations and those of the conductivity in the mean Ohm's law were evaluated in terms of the mean electrical parameters. Rosa's G -factor was modified to include the spatiotemporal conductivity variance. It was shown that the one-dimensional anisotropic model somewhat underestimates the effective resistance of the turbulent MHD plasma with a two-dimensional anisotropy.

Key Words: MHD Power Generation, Combustion Plasma, Turbulence, Correlation, Effective Conductivity

1. Introduction

In many magnetohydrodynamic plasmas, the electrical fields fluctuate in time and space due to fluid-dynamical turbulence and/or combustion. In such circumstances, statistically averaged phenomena must be dealt with. However, the effect of the plasma fluctuation on the mean behavior of the electrical parameters has not yet been considered. We investigate the following subjects in this paper for a plasma in thermal equilibrium using simulated velocity and temperature turbulence data:

(1) The temperature of the electrons and that of the gas atoms are almost always equal in combustion plasmas. However, the ionization and the recombination reactions may not balance instantaneously when temperature is low and the fluctuation is rapid. From this viewpoint, the condition is examined in Sec. 2 under which the Saha equilibrium is established. We also evaluate the error in the average electron density when the density is calculated from Saha's relationship.

(2) The electrical conductivity of the Saha plasma is a strong nonlinear function of the temperature. Therefore, the conductivity calculated from the average temperature may be different from the average of the conductivity itself. To date this effect has been ignored. In Sec. 4, we derive a formula for the average conductivity that will take the temperature fluctuation into account.

(3) The mean behavior of the turbulent plasma should be described by the mean conservation equations for the mass, momentum and energy, the mean generalized Ohm's law and the Maxwell's equations simplified by the MHD approximations.

In most papers published so far⁽¹⁾ the turbulence model appropriate to ordinary fluids has been used approximately in the MHD conservation equations, but no model has been considered for the electrical equations, and instantaneous formulae have been used elsewhere. Clearly, the mean generalized Ohm's law includes the correlation between the conductivity fluctuation and the electrical counterpart. Therefore, it is meaningful to examine the effect of gaseous turbulence on the mean electrical parameter. In Secs. 6 and 7, using a one-dimensional nonuniform plasma, where the exact solutions can be obtained in space and time, we examine the influence of the fluctuation on the local mean electrical phenomena and on the power output characteristics of an MHD generator. The

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mean correlations will also be given explicitly as functions of the mean electric field and the current. In Sec. 8, in order to discuss the applicability of the one-dimensional model to a two-dimensional plasma, we analyze the instantaneous electrical phenomena under 2D turbulent velocity and temperature in the cross section of the MHD channel.

2. Relationship between Electron Number Density and Temperature Turbulence

Here, employing the ionization recombination rate equation for a combustion plasma, we investigate the response of the electron density to the fluctuation of the gaseous temperature and evaluate the influence of the mean temperature level and the fluctuation frequency on the mean electron number density. Denoting the electron density by n_e and the seed atom number density by n_K , the rate equation can be written as follows:

$$\frac{dn_e}{dt} = (\alpha_e + \alpha_g)(n_K K - n_e^2) \quad (1)$$

Here, the ionization equilibrium constant K is a unique function of the temperature and is given by

$$K = \left(\frac{2\pi m_e k T}{h^2} \right)^{3/2} \exp\left(-\frac{e\varepsilon}{kT}\right) \quad (2)$$

(a) $f_{\max}=5$ kHz, $\bar{T}=1800$ K, $p=10^5$ Pa
 $n_0=5.32 \times 10^{18}/\text{m}^3$

Fig. 1(a) Nonequilibrium effect in the electron number density in a plasma with low-frequency temperature turbulence

(b) $f_{\max}=50$ kHz, $\bar{T}=1800$ K, $p=10^5$ Pa
 $n_0=5.67 \times 10^{18}/\text{m}^3$

Fig. 1(b) Nonequilibrium effect in the electron number density in a plasma with high-frequency temperature turbulence

where ε is the ionization potential, and m_e , k and h are the electron mass, Boltzmann's constant and Planck's constant, respectively. The Saha equilibrium value $n_s(t)$ can be calculated from the equation $n_K K - n_s^2 = 0$. The recombination coefficients α_e and α_g are those of the reaction processes, where the third body is the electron and the gaseous particle, respectively. These are given by⁽²⁾ $\alpha_e = 1.09 \times 10^{-20} n_e T^{-4.5}$ and by⁽³⁾ $\alpha_g = 1.7 \times 10^{-28} n_g T^{-3.5}$. Equation (1) can be solved if we know the time-dependent temperature $T(t)$ which is specified by an inverse Fourier transform method as described in Sec. 3. The gas particle density was evaluated by assuming the ideal gas law and the adiabatic condition. The calculated results are shown in Fig. 1(a) for a case with low maximum frequency and in Fig. 1(b) for one with high maximum frequency. The dotted lines are the Saha equilibrium values, which have the same time dependence as the given temperature. The normalization base density n_0 is the maximum n_s value obtained in the time range analysed. Figure 1 shows a part of the whole period analyzed.

From Fig. 1(b) we see that, if the temperature fluctuation is rapid, then the phase of the electron density fluctuation becomes different from that of the given temperature and the turbulence level decreases. Figures 2 and 3 show how the error in n_s changes with T and $f_{\max}$ when the mean density is evaluated as the Saha equilibrium value. The error bars denote the finite number of samples. The error in n_s has maxima with respect to the maximum frequency because the turbulence data have been generated from the spectrum of first-order Markov type, which has a maximum spectrum at zero frequency and has spectrum decreasing to zero at higher frequencies. The results shown above mean that the electron number density can be estimated from Saha's equation if the mean

Fig. 2 Relation between the error in the mean electron density calculated by Saha equilibrium and the mean temperature

Fig. 3 Relation between the error in the mean electron density calculated by Saha equilibrium and the maximum turbulent frequency

Fig. 4 MHD generator channel and the coordinate system

temperature is higher than about 1800 K and the frequency $f_{\max}$ is lower than several kHz. When the temperature is lower than this and the frequency is higher than a few kHz, the Saha equilibrium equation cannot be adopted. In what follows, for simplicity we consider plasmas in the range of the Saha equilibrium conditions.

3. Numerical Data of the Velocity and the Temperature Fluctuation

The MHD generator channel and the coordinate system are shown in Fig. 4. We assume that the magnetic field B is in the z direction and uniform, that the electrodes are positioned at $y=0$ and H and that the insulator walls are at $z=0$ and W . The mean velocity and the temperature are given by the $1/7$ th law within the boundary layers with a thickness δ . In the y direction, only the fluctuation part of the velocity is considered and we assume no velocity component in the z direction, since the latter has primarily no influence on the electrical behavior. The fluctuation of the velocity components in the axial and in the electrode direction is denoted by u' and v' , respectively and that of the temperature by T' . The inverse Fourier transform method may be used for the turbu-

lent power spectrum to calculate approximately the time series data of the fluctuation $\mathbf{g}' = (u', v', T')$. Here, we employ the following spectrum of the first-order Markov type, because the spectrum of the MHD plasma is undetermined.

$$S_g(f) = \overline{g'^2} \frac{\xi}{\xi^2 + 4\pi^2 f^2} \quad (3)$$

In the above, $\overline{g'^2}$ is the variance of the fluctuation and ξ is a constant. Introducing the random phase angle θ_j , the fluctuation data can be calculated from⁽⁴⁾

$$\mathbf{g}'(t) = C_g \sum_{j=1}^n \sqrt{4\pi S_g \Delta f_j} \cdot \cos(2\pi f_j t + \theta_j). \quad (4)$$

In the following analyses, the coefficient C_g was chosen to be $C_u=7$, $C_T=5$ and $C_v=4$ so that the average intensity would equal the empirical turbulence level, and the maximum frequency was assumed to be 1.7 kHz and the minimum to be 35 Hz. In order to make the variance zero on the walls and to obtain a maximum within the boundary layer⁽⁵⁾, we assumed the following weighting function for $\overline{g'^2}$.

$$\sqrt{\overline{g'^2}} = \left(\frac{A_g}{B_g} \right) \left(\frac{2y}{H} \right)^{1/C_g} \exp \left(-B_g \frac{y}{H} \right) \quad (5)$$

Here, H is the electrode height. The constants A_g and B_g can be determined by specifying the maximum and the minimum variance respectively at the point $y_{\max}$ where Eq. (5) reaches the extremum, and at the center $y=H/2$. The signs of the fluctuations were determined statistically so that the signs of the mean correlations $\overline{u'v'}$ or $\overline{v'T'}$ are opposite to that of the gradient of the mean velocity $\overline{u}$ or mean temperature $\overline{T}$.

Examples of the axial velocity and the temperature at a vicinity close to the wall and in the core are shown in Figs. 5 and 6, where Y is the nondimensional distance y/H , and δ/H was assumed to be 0.1818. The mean velocity and the temperature are given by $\overline{u} = \overline{u_c}(y/\delta)^{1/7}$ and $\overline{T} = T_w + (\overline{T_c} - T_w)(y/\delta)^{1/7}$, respectively. The mean correlations $\overline{u'v'}$, $\overline{v'T'}$, $\overline{\sigma'u'}$ and $\overline{\sigma'v'}$ are distributed as shown in Fig. 7. The turbulent conductivity has the same sign as the turbulent temperature, and the correlations $\overline{u'v'}$ and $\overline{v'T'}$ take statistically the opposite sign to those of $d\overline{u}/dy$ and $d\overline{T}/dy$. Therefore, $\overline{\sigma'u'}$ is positive in the whole space, while $\overline{\sigma'v'}$ is negative on the cathode side and positive on the anode side. In the above, the electrical conductivity was approximated by the following formula that fits the chemical equilibrium composition analysis⁽⁶⁾ of a KOH-seeded kerosene-oxygen combustion plasma with 1 wt % of potassium seed.

$$\sigma(T, p) = 89.9 \frac{T^{1.005}}{p^{0.51}} \cdot \exp \left(-\frac{e\epsilon}{2kT} \right) \quad (6)$$

It should be noted that the maxima and minima of the variance calculated as above are smaller than those assumed in Eq. (5) to specify the coefficients A_g and

Fig. 5 Examples of the axial velocity turbulence data of 1D nonuniform model

Fig. 6 Examples of the temperature turbulence data of 1D nonuniform model

B_θ , because of a finite number of samples and finite number of discrete frequencies considered.

4. Effects of Turbulence on Mean Conductivity

The spatial distribution of the conductivity calculated from the data shown in Fig. 6 is given in Fig. 8. Compared with the temperature, the conductivity fluctuation is more noticeable. We note the mean conductivity increase in comparison with $\sigma(\bar{T})$; this increment due to the temperature fluctuation was calculated approximately from Eq.(7). Results are shown in Fig. 9.

$$\bar{\sigma} = \sigma(\bar{T}) + \frac{1}{2} \cdot \frac{d^2 \sigma(\bar{T})}{d\bar{T}^2} \cdot \bar{T}^2 \quad (7)$$

Equation (7) well describes the mean conductivity of an equilibrium turbulent plasma. The conductivity increment depends markedly on the mean temperature, that is, from 10 to 20% in the temperature

Fig. 8 Distribution of the conductivity in space and time

Fig. 9 Relation between the mean conductivity and the mean temperature

occurring in boundary layers and a few percent in the core temperature range. The mean intensity of the turbulence $\sqrt{T^2}$ is approximately proportional to the gradient of the mean temperature, so that if the fluctuation is not taken into account, a significant error may result when performing calculations for regions near the walls.

5. Average Electrical Parameters and Electrical Correlations

In this paper, we deal with the influence of velocity and temperature on the electrical parameters. Therefore, the basic equations are the generalized Ohm's law and Maxwell's equations. Neglecting the ion slip and the induced magnetic field, we can write the instantaneous forms as

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}) - \mu \mathbf{J} \times \mathbf{B} \quad (8)$$

$$\nabla \times \mathbf{E} = 0 \quad \mathbf{E} = -\nabla \phi \quad (9)$$

$$\nabla \cdot \mathbf{J} = 0 \quad \mathbf{J} = \nabla \times \boldsymbol{\psi}, \quad (10)$$

where $\mathbf{E}$ and $\mathbf{J}$ are the electric field and the current density, and ϕ and $\boldsymbol{\psi}$ are the electric potential and the current stream function. The time average of the first-order fluctuation is zero and the fluctuation in mobility can be neglected because of its weak dependence on the temperature. Thus, we can write

$$\bar{\mathbf{J}} = \bar{\sigma}(\bar{\mathbf{E}} + \bar{\mathbf{V}} \times \mathbf{B}) - \mu \bar{\mathbf{J}} \times \mathbf{B} + \bar{\sigma}' \bar{\mathbf{E}}' + \bar{\sigma}' \bar{\mathbf{V}}' \times \mathbf{B}. \quad (11)$$

The averages of Eqs.(9) and (10) take the same form. As shown above, the mean current density $\bar{\mathbf{J}}$

and the mean electric field $\bar{E}$ can be determined if the correlations $\overline{\sigma' E'}$ and $\overline{\sigma' v'}$ can be specified. The latter is related only to the plasma properties, while the former is affected generally not only by the plasma states but also by the external electrical load. Thus, we assume here a Faraday channel with infinitely segmented electrodes and perfectly insulated walls.

Once the mean current is given, the source term in the equation of motion can be written as $\bar{J} \times B$. On the other hand, the energy equation has the mean source term of the form $\bar{J} \cdot \bar{E}$ or $\bar{J}^2/\bar{\sigma}$, which may be written as follows:

$$\begin{aligned} \bar{J} \cdot \bar{E} &= \bar{J} \cdot \bar{E} + \bar{J}' \cdot \bar{E}' \\ \bar{J}^2/\bar{\sigma} &= \bar{J} \cdot \bar{E} + \bar{J}' \cdot \bar{E}' + \bar{J} \cdot (\bar{V} \times B) + \bar{J}' \cdot (\bar{V}' \times B) \end{aligned} \quad (12)$$

Therefore, in order to close the turbulent MHD equations, the evaluation of both $\bar{J}' \cdot \bar{E}'$ and $\bar{J}' \cdot (\bar{V}' \times B)$ is also necessary.

6. Solutions for 1D Nonuniform Plasmas

As seen in Eqs. (8) and (9), the electrical parameters of Saha equilibrium plasmas depend explicitly upon only the spatial coordinates. In this respect, the spatially averaged formulae for the effective current, the electric field and the conductivity have been studied in several papers. Rosa⁽⁷⁾ has proposed an effective conductivity formula of $\sigma_{\text{eff}} = \langle \sigma \rangle / G_*$ based on a one-dimensional plasma nonuniformity in the electrode direction only. Here, the bracket $\langle \rangle$ means the spatial average. With Hall's parameter $\beta = \mu B$, G_* is defined as

$$G_* = \langle \sigma \rangle \left\langle \frac{1 + \beta^2}{\sigma} \right\rangle - \langle \beta \rangle^2. \quad (14)$$

This factor accounts for the increment of the plasma resistance due to the spatial nonuniformity and is always larger than unity except in the case of uniform plasma. In this paper, we employ Rosa's model for the spatial nonuniformity. According to this and to the infinite segmentation of electrodes, E_x and J_y are constant with respect to y , and from the assumption of a Faraday channel, $\langle J_x \rangle = 0$. Then the solutions of the potential and the current stream function can be obtained as follows:

$$\phi = -E_x(t)x - \int_0^y E_y(t, y') dy' + V_c \quad (15)$$

$$\psi = -J_y(t)x + \int_0^y J_x(t, y') dy' \quad (16)$$

The output voltage is given by $V_c - V_A = \phi(t, x, H)$, and $I/W = \phi(t, P, H)$ gives the output current per unit width in the z direction, where P is the electrode pitch. The electric field E_y and the current density J_x can be calculated from

$$E_y = uB + \left(\frac{1 + \beta^2}{\sigma} - \frac{\beta \langle \beta \rangle}{\langle \sigma \rangle} \right)$$

$$- \beta \left(v' - \frac{\langle \sigma v' \rangle}{\langle \sigma \rangle} \right) B \quad (17)$$

$$J_x = \left(\sigma \frac{\langle \beta \rangle}{\langle \sigma \rangle} - \beta \right) J_y + \sigma \left(v' - \frac{\langle \sigma v' \rangle}{\langle \sigma \rangle} \right) B. \quad (18)$$

Hall's electric field E_x is determined by the condition $\langle J_x \rangle = 0$, and the output current density by $J_y = I/PW$. We calculate the time mean values by adding the number of instantaneous solutions for the velocity and the temperature data given in Sec. 3, the fluctuations by subtracting the time mean from the instant value, and the mean correlations by taking the product of any two fluctuations.

Figure 10 shows the solution $\phi(t, 0, y)$, where, $\delta/H = 0.1818$, $\beta = 3.15$, and the load $R_L PW \sigma_c/H = 4.0$. The solution for the case without fluctuation is indicated by ϕ_* . The plot of a constant stream function $\psi = \text{const} (=0)$ under the same plasma and load conditions is shown by x/H as a function of y/H in Fig. 11. Also, the power output per electrode is shown in Fig. 12. The power is supplied to the external load, when the electric field and the current density have opposite sign, so that a lower negative value means a higher power output.

From the results given above, we see that at first the local electric field fluctuates significantly. The open voltage V_0 shows less fluctuation because it is affected by the velocity fluctuation only. However, the behavior of the electrode potential under loaded conditions is greatly affected by the conductivity turbulence. This tendency coincides with the experimental observations. Furthermore, the mean cathode potential decreases in comparison with that of a plasma without fluctuation and the power output decreases considerably. The mean path of the current becomes asymmetrical due to the spatial characteristics of the correlation $\overline{\sigma' v'}$.

7. Correlations and the Effective G-Factor

In this section, we evaluate the mean correlation $\overline{\sigma' E'} = \bar{\sigma} \bar{E} - \bar{\sigma} \bar{E}$ in terms of $\bar{J}$, $\bar{E}$ and $\bar{u}$. We took the conductivity fluctuation up to σ'^3 into account in

Fig. 10 Potential distribution in space and time

Fig. 11 Distribution of the current stream function in space and time

Fig. 12 Electrical power output

substitution of the solutions E_x and E_y given in Sec. 6 into $\overline{\sigma E}$. Considering the results shown in Fig. 7, we can neglect the spatial average $\overline{\sigma'v'} \sim 0$; then we obtain the solutions as Eqs. (19) and (20):

$$\overline{\sigma' E'_x} \approx -\frac{\overline{\sigma' \langle \sigma' \rangle}}{\langle \overline{\sigma} \rangle^2} \beta \bar{J}_y + O(\overline{\sigma'^4}) \quad (19)$$

$$\begin{aligned} \overline{\sigma' E'_y} \approx & -\frac{\overline{\sigma'^2}}{\bar{\sigma}} E_y + \beta \left(\frac{\overline{\sigma' \langle \sigma' \rangle}}{\langle \overline{\sigma} \rangle} - \frac{\overline{\sigma'^2}}{\bar{\sigma}} \right) E_x \\ & + \overline{\sigma' u'} B + \bar{u} B \frac{\overline{\sigma'^2}}{\bar{\sigma}} - \beta \overline{\sigma' v'} B + O(\overline{\sigma'^3}) \end{aligned} \quad (20)$$

As regards the other correlations, it can be shown numerically that $\overline{\mathbf{J}' \cdot \mathbf{E}'} \sim \overline{J'_y E'_y} \sim 0.03 \bar{J}_y \bar{E}_y$, $\overline{v' J'_x} \sim 0$, and $\overline{u' J'_y} \sim 0$. The correlation $\overline{\sigma' E'_y}$ is positive between the electrodes and has a similar distribution in y as $\overline{\sigma' u'}$. The correlation $\overline{\sigma' E'_x}$ is negative near the electrodes and positive in the outer region. Its magnitude is about $\overline{\sigma' E'_y}/20$. It is also shown numerically that $\overline{\sigma' \langle \sigma' \rangle} \sim \overline{\sigma'^2}/10$.

Defining the effective conductivity as

$$\sigma_{\text{eff}} = \frac{\langle \overline{\sigma} \rangle}{\bar{G}} = -\frac{J_y H}{V_0 - V_c} \quad (21)$$

we obtain $\bar{G}$ by the substitution of Eqs. (19) and (20) into Eq. (10) as follows:

$$\begin{aligned} \bar{G} = & \langle \overline{\sigma} \rangle \left\langle \frac{1 + \beta^2}{\bar{\sigma}} \right\rangle - \langle \beta \rangle^2 \\ & + \langle \overline{\sigma} \rangle \left\langle \frac{1 + \beta^2}{\bar{\sigma}} \cdot \frac{\overline{\sigma'^2}}{\bar{\sigma}^2} \right\rangle - \langle \beta \rangle^2 \frac{\overline{\langle \sigma' \rangle^2}}{\langle \overline{\sigma} \rangle^2} \end{aligned}$$

Fig. 13 Effects of the temperature fluctuation on the mean G-factor

Fig. 14 $G(t)$, $\bar{G}$ and G_* (Numerical conditions are the same as for Fig. 13)

$$+ O\left(\beta^2 \frac{\overline{\langle \sigma'^2 \rangle}}{\langle \overline{\sigma} \rangle^4}\right) = G_0 + \Delta G \quad (22)$$

The first and second terms on the right-hand side are replaced by G_0 , which describes the decrease in the conductivity due to the conductivity nonuniformity in the space. The influence of the spatiotemporal variance is accounted for by ΔG . In the case of a plasma without fluctuation, we can set $\bar{G} = G_0 = G_*$.

In Fig. 13, the relation of $\bar{G}/G_0$ to the temperature turbulence is shown for the case of $T_w = 1800$ K, $\delta/H = 0.1818$ and $p = 105$ kPa. It is seen that in a spatially nonuniform plasma $\Delta G/G_0$ is about 10%, and in a plasma that fluctuates in time but is uniform in space, $\Delta G/G_0$ is about 3%. It should be noted that $\bar{\sigma}$ in G_0 includes the effect of the temperature fluctuation. When G_0 is calculated from the mean temperature only, as is generally the case, then for the present plasma model $G_*/\bar{G} \sim 50\%$ as seen in Fig. 14. In this case, the plasma resistance may be underestimated and the power output overestimated to a considerable extent.

8. Effects of Two-Dimensional Turbulence

Although an exact solution can be obtained for a one-dimensional nonuniform plasma, that model cannot describe the real situation accurately. Here, we examine the validity of the 1D model using 2D nonuniformity and fluctuations within the channel

cross-sectional plane. Since axial convection is much larger than the transverse flow in an MHD channel, to a good approximation we can neglect the variation in the x direction. Accordingly, E_x is constant over the cross section. Rearranging Eqs. (8) and (9) in terms of the electric potential, we obtain

$$\phi_{yy} + (1 + \beta^2)\phi_{zz} + \frac{\sigma_y}{\sigma}\phi_y + (1 + \beta^2)\frac{\sigma_z}{\sigma}\phi_z + u_y B - v_y B - \frac{\sigma_y}{\sigma}(\beta E_x - uB + \beta vB) = 0, \quad (23)$$

where the subscripts, except in E_x , denote partial differentiation with respect to y and z . The time series data of the velocity and the temperature were given at the FDM mesh points by adding the fluctuation component to the mean 1/7th profile, where the method explained in Sec. 3, (i.e. the weighting function $\sqrt{y^2}$) was extended to two dimensions. The boundary layer thickness of $\delta/H = \delta/W = 0.15$ and a wall temperature of $T_w = 1800$ K were assumed. Examples of the instantaneous distribution of the velocity and the temperature are given in Figs. 15 and 16, respectively. In these cases, the velocity variance is from 3.4 to 9.7% of u_c and that of the temperature is from 2.7 to 3.8% of the mean core value 2800 K.

Using these data and the boundary conditions $\partial\phi/\partial z = 0$ on the electrodes and $J_z = 0$ on the insulator walls, the distribution of the potential was calculated as shown in Fig. 17, where $V_c/\bar{u}_c B H = 0.54$ and $\beta = 1.93$. The potential fluctuation in the electrode boundary layers is significant, and this means that the 1D model which neglects the variation in the z direction is not appropriate. Although no figures are presented here, it should be mentioned that with decreasing wall temperatures, an eddy current appears in the insulator

boundary layers and results in decreases in J_y , the output current and the open voltage.

A number of the analyses mentioned above were performed and the results were added to obtain the mean plasma resistance. Final results are compared in Fig. 18 with those of the 1D model. It is shown that if we assume the 2D model to be correct, the 1D model overestimates the plasma resistance when the wall temperature is high, while it underestimates the resistance when the wall temperature is low. The actual wall temperature may be less than about 2000 K. Therefore, we conclude that the influence of the plasma turbulence in time and space upon the effective mean values is more critical than is estimated by the 1D turbulent model.

Fig. 16 An example of the two-dimensional turbulence of the temperature [isotherm plot, dark area: $2800 \text{ K} \leq T \leq 3090 \text{ K}$ (= +M point)]

Fig. 15 An example of the two-dimensional turbulence of the axial velocity data [constant $u/\bar{u}_c$ plots, dark area: $1 \leq u/\bar{u}_c \leq 1.17$ (= +M point)]

Fig. 17 Distribution of the equipotential lines in the cross plane

Fig. 18 Errors in the effective resistance in the evaluation with the one-dimensional model

9. Conclusions and Discussions

In order to pursue the influence of the statistical fluctuation of Saha equilibrium plasmas upon the mean performance of a combustion plasma MHD power generator, we first clarified the range of the turbulent maximum frequency and the lowest temperature level for which Saha's equilibrium can be assumed. It was confirmed that it is sufficient to take into account the temperature fluctuation up to the second order in the evaluation of mean scalar conductivity. Next, the electrical behavior was analyzed with the use of statistical numerical data of velocity and temperature, from which it was shown that the deterioration of the output characteristics of an MHD generator due to turbulence is crucial. In order to evaluate this effect in terms of effective conductivity, the mean correlations between the conductivity fluctuation and the electric field counterparts in the mean generalized Ohm's law were explicitly given, and then the conventional G -factor was modified to take the temporal fluctuation into account. Finally, the mean plasma resistance obtained by the potential analysis for two-dimensional turbulent plasmas was compared with the result of the one-dimensional model. It was concluded that the 1D turbulence model somewhat overestimates the power output of an MHD generator.

The influence of the electrical parameter fluctuation upon the plasma behavior was not examined in this paper. In Saha equilibrium plasma with low magnetic Reynolds numbers, the electrical equations have no characteristic time. Therefore, the electrical behavior affects the thermal and fluid-dynamical plasma performance only through the Lorentz force

and Joule heat terms. The former may suppress turbulence u' and v' perpendicular to the magnetic field in the core region and in the electrode boundary layers, where the current density is large. Hence, the turbulence effect is certainly less than those presented in this paper. For quantitative evaluation, we must solve the entire set of time-dependent MHD equations. The effect of Joule heat term may differ depending upon the amount of the local Joule heating and the heat conduction to the walls. When the thermal diffusion is sufficient, the plasma fluctuation induced by the electric field may be ignored. In the opposite case, an electrothermal instability⁽⁸⁾ may be induced. In thermal equilibrium plasmas, this phenomenon occurs dominantly in low-temperature boundary layers near electrodes with high frequencies in general, so that the assumption of Saha equilibrium may not be adopted. Discussion of such turbulent plasmas is beyond the scope of the present paper.

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